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Analysis of US Sector of Services with a New Fama-French 5-Factor Model

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Abstract

In this paper, we empirically test a new model with the data of US services sector, which is an extension of the 5-factor model in Fama and French (2015) [1]. 3 types of 5 factors (Global, North American and US) are compared. Empirical results show the Fama-French 5 factors are still alive! The new model has better in-sample fit than the 5-factor model in Fama and French (2015).

Keywords

Fama-French 5-Factor Model (FF5), Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD), EGARCH

1. Introduction

After the Capital Asset Pricing Model (CAPM) was created by Sharpe (1964) [2] and Lintner (1965) [3], it makes a fundamental contribution to understand the relationship between expected returns and market risks. Fama and French (1993) added size and book-to-market factors into the CAPM, many empirical results show it's capable to explain the stock returns better than the CAPM. After that, many new factor models are developed. Panel A of **Table 1** documents the development of the factor model in stock market. For example, Carhart (1997) [4] introduced a Carhart 4-factor (C) model by augumenting the Fama-French 3-factor (FF3) model with momentum factor which can explain the short-term persistence in expected returns. Chan and Faff (2005) [5] advocated a liquidity-agumented FF3 model by using Australian data and find the liquidity factor is very robust to sensitivity checks. Connor, Hagmann and Linton (2012)

Table 1. Researches about the Fama-French 5-factor Model.

Author (Year)	Research Purpose	Model	Data		
			Country	Factors	Frequency & Period
			Panel A: Development of Factor Model		
Fama *et al.* (1993) [16]	CAPM Extension	FF3	USA	Mkt, SMB, HML, WML	M1963:7-1991:12
Carhart (1997)	FF3 Extension	CAPM, FF3, C	USA	Mkt, SMB, HML, WML	M1962:1-1993:12
Griffin (2002) [17]	FF3 Extension	Domestic or International FF3	Global	Mkt, SMB, HML	M1981:1995:12
Chan *et al.* (2005) [18]	FF3 Extension	FF3 with IML	Australia	Mkt, SMB, HML, IML	M1990:1-1998:12
Fama *et al.* (2012) [19]	Model Comparison	Global or Local CAPM, FF3, C	Global	Mkt, SMB, HML, WML	M1990:11-2011:3
Connor *et al.* (2012)	C Extension	C with VOL	USA	Mkt, SMB, HML, WML, VOL	M1970-2007
Chai *et al.* (2013) [20]	C Extension	C with IML	Australia	Mkt, SMB, HML, WML, IML	M1982:1-2010:12
Fama *et al.* (2013) [21]	FF3 Extension	FF4	USA	Mkt, SMB, HML, RMW	M1963:7-2012:12
Yang (2013)	FF3 Extension	FF3 with SSAEPD, EGARCH	USA	Mkt, SMB, HML	M1926-2011
Hou *et al.* (2014)	Model Comparison	FF5, C, q-factor	USA	Mkt, SMB, RMW, CMA, WML, HML	M1972:1-2011:12
Fama *et al.* (2015a)	FF4 Extension	FF5	USA	Mkt, SMB, HML, RMW, CMA	M1963:7-2013:12
Zhu (2016)	FF5 Extension	FF5 with SSAEPD, EGARCH	USA	Mkt, SMB, HML, RMW, CMA	M1963:7-2013:12
			Panel B: Researches for Fama-French 5-Factor Model		
Fama *et al.* (2014)	Model Comparison	CAPM, FF3, FF4, FF5, FF5 with WML	USA	Mkt, SMB, HML, RMW, CMA, WML	M1963:7-2014:12
Hou *et al.* (2015)	Model Comparison	FF5, C, q-factor	USA	Mkt, SMB, HML, RMW, CMA, WML	M1967:1-2013:12
Harshita *et al.* (2015)	Model Comparison	CAPM, FF3, FF5	India	Mkt, SMB, HML, RMW, CMA	M1999:10-2014:9
Fama *et al.* (2015b)	Empirical Tests	FF5	Global	Mkt, SMB, HML, RMW, CMA	M1990:7-2014:9
Chiah *et al.* (2016)	Empirical Tests	FF3, C, FF5	Australia	Mkt, SMB, HML, PMU, LMH	M:1982:1-2013:12
Bin Guo *et al.* (2017)	Empirical Tests	FF5	China	Mkt, SMB, HML, RMW, CMA,CMAB	M:1995:7-2014:6
Rehab *et al.* (2016)	Empirical Tests	FF5	Egypt	MKT, SMB, HML, HEMLE, HSMLS, HDMLD, IML and WML	M:2005:7-2013:7
Fama *et al.* (2016)	Empirical Tests	FF5	Global	Mkt, SMB, HML, RMW, CMA	M:1990:7-2015:12

Notes: "-" means that no information is available in this paper; CAPM = Capital Asset Pricing Model; FF3 = Fama and French (1993) 3-factor model; FF4 = Fama and French 4-factor model (2013); FF5 = Fama and French (2015) 5-factor model; C = Carhart (1997) 4-factor; q-factor = Hou, Xue, and Zhang (2012) q-factor model; 14-factor = Harvay and Liu (2015) 14-factor model; Mkt = Market; SMB = Size; HML = Book-to-market; WML = Momentum; IMV = liquidity; Vol = Own-volatility; RMW = Profitability; CMA = Investment; PMU = Profitable Minus Unprofitable; HML = High Minus Low; HAC-adjusted OLS = Newey-West heteroskedasticity; and autocorrelation-adjusted OLS. WLS = Weighted least squares.

[6] considered a five-factor extension of the C model which suggests an own-volatility factor.

In 2015, Fama and French proposed 5 factor model(FF5), it adds profitability and investment factors into their 3-factor model proposed in 1993. Since then, many studies about Fama-French 5-factor (FF5) model have been done. Panel B of **Table 1** presents the researches for the FF5 Model. And these researches mainly apply the FF5 model to empirical stock markets and compare the FF5 model with others.

For example, Hou, Xue and Zhang (2015) [7] found that the 4-factor q-model created by Hou, Xue and Zhang (2014) [8] performs better than the FF5 model in US market. Harshita *et al.* (2015) [9] pointed out that the FF5 model works better in India than CAPM and FF3 model. Fama and French (2015) [9] also showed that the FF5 model can explain quite well for North America and other 3 regions. Mardy *et al.* (2016) [10] empirically investigated the FF5 Model in Australia, finding after adding the profitability and investment factors, FF5 model is really able to explain more asset pricing anomalies than other competing asset pricing models (like Fama-French 3-factor model and Carhart 4-factor model).

Although FF5 model has better performance in many case, it's not adapted to every situation. Fama and French (2017) [11] analyzed the international market and found that the investment factor CMA is redundant for Europe, Japan and Asia Pacific. Meanwhile, Fama and French also found the new factors' performance are different for small and big stock market. And for different regions, factors' performance also exist difference. Besides, Guo *et al.* (2017) [12] found that the profitability factor significantly improves the description of average return, and investment pattern in average returns is weak in China stock market.

In 2017, Li *et al.* [13] added non-normal errors of SSAEPD proposed by Zhu and Zinde-Walsh (2009) [14] and the EGARCH-type volatilities suggested in Nelson (1991) [15] to extend the 5 factor model in Fama and French (2015). They called this new model as FF5-SSAEPD-EGARCH. Both EGARCH equation and SSAEPD can be used to capture the fat-tailedness. SSAEPD can be used to capture the asymmetric kurtosis of data. Thus, in this paper we use the data of US services industry to empirically test the new model and compare it with Fama-French 5 factors (FF5). In this paper, following two hypotheses will be tested:

1) With EGARCH-type volatilities and SSAEPD errors, are Fama-French 5 factors still alive?

2) Can this new model explain services industry better than the 5 factor model in Fama and French (2015)?

To answer these questions, we run simulation to test the validity of MatLab program used in this paper. Then, the industry of services in US are analyzed. Data are downloaded from the French's Data Library, and the sample period is from Jul. 1990 to Feb. 2017. Method of Maximum Likelihood Estimation (MLE) is used to estimate the parameters. Likelihood Ratio test (LR) and Kolmogorov-Smirnov test (KS) are used for model diagnostics. Akaike

Information Criterion(AIC) is used for model comparsion.

We find out the Fama-French 5 factors are still alive. The new model has better in-sample fit than the 5-factor model in Fama and French (2015). The industry of services can earn extra *Alpha* returns since the constant term in the new model is statistically significant. The *Beta* (β_1) coefficient (for US, North American) is very close to 1. We also find out models with GARCH-typed volatility fit data better than those with EGARCH-typed volatility. To capture fat-tailedness, GARCH equation is better than non-normal error terms of SSAEPD.

The organization of this paper is as follows: The model and methodology are discussed in Section 2; Empirical results and the model comparisons will be presented in Section 3; Section 4 is the conclusions and future extensions.

2. Model and Methodology

2.1. Models

2.1.1. Fama-French 5-Factor Model (FF5-Normal)

Fama and French(2015) propose a 5-factor model (denoted as FF5) to explain market, size, value, profitability, and investment patterns in expected stock returns, and show this model empirically outperforms their 3 factor model. The 5-factor model is:

$$R_t - R_{ft} = \beta_0 + \beta_1 * \left(R_{mt} - R_{ft}\right) + \beta_2 * SMB_t + \beta_3 * HMLO_t \tag{1}$$

$$+\beta_4 * RMW_t + \beta_5 * CMA_t + u_t, u_t \sim \text{Normal}\left(\mu, \sigma^2\right). \tag{2}$$

where $\theta = \left(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \mu, \sigma\right)$ are parameters to be estimated in this model. R_t is the rate of return for stock portfolio. R_{ft} is the rate of return for the risk-free asset. R_{mt} is the rate of return for the market. SMB_t stands for small market capitalization minus big market capitalization. $HMLO_t$ is the high book-to-market ratio minus low book-to-market ratio orthogonalized[1]. RMW_t stands for robust operating profitability portfolios minus weak operating profitability portfolios. CMA_t stands for conservative investment portfolios minus aggressive investment portfolios. The error term u_t is distributed as the Normal. $t = 1, 2, \cdots, T$.

2.1.2. FF5-SSAEPD-EGARCH Model

Li *et al.* (2016) extend Fama-French(2015) five-factor model by introducing a Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD)

[1] $HMLO_t$ is the sum of the intercept and the residual from the regression of HML_t on $R_{mt} - R_{ft}, SMB_t, RMW_t, CMA_t$. The reason we use $HMLO_t$ is that Fama and French (2015) show HML_t (the high book-to-market ratio minus low book-to-market ratio) is redundant in following 5-factor model.

$$R_t - R_{ft} = \beta_0 + \beta_1 * \left(R_{mt} - R_{ft}\right) + \beta_2 * SMB_t + \beta_3 * HML_t + \beta_4 * RMW_t + \beta_5 * CMA_t + u_t, \tag{3}$$

$$u_t \sim \text{Normal}\left(\mu, \sigma^2\right), t = 1, 2, \cdots, T. \tag{4}$$

errors and the EGARCH -type volatilites. The new model we proposed is (denoted as the FF5-SSAEPD-EGARCH model):

$$R_t - R_{ft} = \beta_0 + \beta_1\left(R_{mt} - R_{ft}\right) + \beta_2 SMB_t + \beta_3 HMLO_t \tag{5}$$

$$+\beta_4 RMW_t + \beta_5 CMA_t + u_t,$$

$$u_t = \sigma_t z_t,\ z_t \sim SSAEPD\left(\alpha, p_1, p_2\right), \tag{6}$$

$$\ln\left(\sigma_t^2\right) = a + \sum_{i=1}^{s} g\left(z_{t-i}\right) + \sum_{j=1}^{m} b_j \ln\left(\sigma_{t-j}^2\right), \tag{7}$$

$$g\left(z_{t-i}\right) = c_i z_{t-i} + d_i\left[\left|z_{t-i}\right| - E\left(\left|z_{t-i}\right|\right)\right]$$

$$= \begin{cases} \left(c_i + d_i\right) z_{t-i} - d_i E\left(\left|z_{t-i}\right|\right), & \text{if } z_{t-i} \ge 0, \\ \left(c_i - d_i\right) z_{t-i} - d_i E\left(\left|z_{t-i}\right|\right), & \text{else.} \end{cases} \tag{8}$$

where $\theta = \left(\beta_0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \alpha, p_1, p_2, a, \left\{b_j\right\}_{j=1}^{m}, \left\{c_i\right\}_{i=1}^{s}, \left\{d_i\right\}_{i=1}^{s}\right)$ are the parameters to be estimated. Definitions of variables are the same as before. σ_t is the conditional standard deviation, *i.e.*, volatility. The error term z_t is distributed as the Standardized Standard Asymmetric Exponential Power Distribution (SSAEPD) proposed in Zhu and Zinde-Walsh (2009).

•Standardized Standard AEPD (SSAEPD)

According to Zhu and Zinde-Walsh (2009), the AEPD density has following form[2]:

$$f_{\text{AEPD}}\left(x\right) = \begin{cases} \left(\dfrac{\alpha}{\alpha^*}\right)\dfrac{1}{\sigma} K\left(p_1\right) \exp\left(-\dfrac{1}{p_1}\left|\dfrac{x-\mu}{2\alpha^*\sigma}\right|^{p_1}\right), & \text{if } x \le \mu, \\ \left(\dfrac{1-\alpha}{1-\alpha^*}\right)\dfrac{1}{\sigma} K\left(p_2\right) \exp\left(-\dfrac{1}{p_2}\left|\dfrac{x-\mu}{2\left(1-\alpha^*\right)\sigma}\right|^{p_2}\right), & \text{if } x > \mu. \end{cases} \tag{10}$$

where $\theta = \left(\alpha, p_1, p_2, \mu, \sigma\right)$ is the parameter vector. $\mu \in R$ and $\sigma > 0$ represent location and scale, respectively[3]. $\alpha \in \left(0,1\right)$ is the skewness parameter. $p_1 > 0$ and $p_2 > 0$ are the left and the right tail parameters, respectively. $K\left(p\right)$ and α^* are defined as

$$K\left(p\right) = \frac{1}{2p^{1/p}\Gamma\left(1+1/p\right)}, \tag{11}$$

[2]A convenient reparametrization of Equation (10) is obtained by rescaling, where

$$f_{\text{AEPD}}\left(x\right) = \begin{cases} \dfrac{1}{\sigma}\exp\left(-\dfrac{1}{p_1}\left|\dfrac{x-\mu}{2\alpha\sigma K\left(p_1\right)}\right|^{p_1}\right), & \text{if } x \le \mu, \\ \dfrac{1}{\sigma}\exp\left(-\dfrac{1}{p_2}\left|\dfrac{x-\mu}{2\left(1-\alpha\right)\sigma K\left(p_2\right)}\right|^{p_2}\right), & \text{if } x > \mu. \end{cases} \tag{9}$$

$$\theta = \left(\alpha, p_1, p_2, \mu, \sigma\right)$$

[3]In this case, μ and σ are not the notations for the population mean and the population standard deviation.

$$\alpha^* = \frac{\alpha K(p_1)}{\alpha K(p_1) + (1-\alpha) K(p_2)}. \tag{12}$$

If we set the location parameter $\mu = 0$ and the scale parameter $\sigma = 1$, then we say X is a random variable distributed as Standard AEPD, denote it as $X \sim \text{SAEPD}(\alpha, p_1, p_2, 0, 1)$. Its PDF[4], mean and variance are

$$f_{\text{SAEPD}}(x) = \begin{cases} \left(\frac{\alpha}{\alpha^*}\right) K(p_1) \exp\left(-\frac{1}{p_1}\left|\frac{x}{2\alpha^*}\right|^{p_1}\right), & \text{if } x \le 0, \\ \left(\frac{1-\alpha}{1-\alpha^*}\right) K(p_2) \exp\left(-\frac{1}{p_2}\left|\frac{x}{2(1-\alpha^*)}\right|^{p_2}\right), & \text{if } x > 0, \end{cases} \tag{14}$$

$$E(X) = \frac{1}{B}\left[(1-\alpha)^2 \frac{p_2 \Gamma(2/p_2)}{\Gamma^2(1/p_2)} - \alpha^2 \frac{p_1 \Gamma(2/p_1)}{\Gamma^2(1/p_1)}\right], \tag{15}$$

$$\begin{aligned} Var(X) = \frac{1}{B^2}\Bigg\{ & (1-\alpha)^3 \frac{p_2^2 \Gamma(3/p_2)}{\Gamma^3(1/p_2)} + \alpha^2 \frac{p_1^2 \Gamma(3/p_1)}{\Gamma^3(1/p_1)} \\ & - \left[(1-\alpha) \frac{p_2 \Gamma(2/p_2)}{\Gamma^2(1/p_2)} - \alpha^2 \frac{p_1 \Gamma(2/p_1)}{\Gamma^2(1/p_1)}\right]^2 \Bigg\}. \end{aligned} \tag{16}$$

Then, if we standardize X with its mean and standard deviation, we can get $Z = \frac{X - E(X)}{\sqrt{Var(X)}}$, which we call Standardized Standard AEPD (SSAEPD). The PDF of Z can be got by transformation.

$$f_{\text{SSAEPD}}(Z) = |J|\, f_{\text{SAEPD}}\left(E(X) + Z\sqrt{Var(X)}\right) \tag{17}$$

$$= \delta f_{\text{SAEPD}}(\omega + Z\delta) \tag{18}$$

where $\omega = E(X)$, $|J| = \delta$ and $\delta = \sqrt{Var(X)}$, we can get the probability density function (PDF) of the SSAEPD

$$f_{\text{SSAEPD}}(z) = \begin{cases} \delta\left(\frac{\alpha}{\alpha^*}\right) K(p_1) \exp\left(-\frac{1}{p_1}\left|\frac{\omega + z\delta}{2\alpha^*}\right|^{p_1}\right), & \text{if } z \le -\frac{\omega}{\delta}, \\ \delta\left(\frac{1-\alpha}{1-\alpha^*}\right) K(p_2) \exp\left(-\frac{1}{p_2}\left|\frac{\omega + z\delta}{2(1-\alpha^*)}\right|^{p_2}\right), & \text{if } z > -\frac{\omega}{\delta}. \end{cases} \tag{19}$$

[4]A convenient reparametrization of Equation (14) is obtained by rescaling, where

$$f_{\text{SAEPD}}(x) = \begin{cases} \exp\left(-\frac{1}{p_1}\left|\frac{x}{2\alpha K(p_1)}\right|^{p_1}\right), & \text{if } x \le 0, \\ \exp\left(-\frac{1}{p_2}\left|\frac{x}{2(1-\alpha) K(p_2)}\right|^{p_2}\right), & \text{if } x > 0. \end{cases} \tag{13}$$

$$\theta = (\alpha, p_1, p_2, 0, 1)$$

$E(z)=0, Var(z)=1$. With $\alpha=0.5$, $p_1=p_2=2$, SSAEPD reduces to Normal (0,1).

2.2. Method of Maximum Likelihood Estimation (MLE)

We estimate the FF5-SSAEPD-EGARCH model with the method of Maximum Likelihood Estimation (MLE). The likelihood function is

$$L\left(\left\{R_t-R_{ft}, R_{mt}-R_{ft}, SMB_t, HMLO_t, RMW_t, CMA_t\right\}_{t=1}^{T};\theta\right) \tag{20}$$

$$=\prod_{t=1}^{T} f\left(R_t-R_{ft}\right)$$

$$=\prod_{i=1}^{n}\begin{cases}\frac{\delta}{\eta}\left(\frac{\alpha}{\alpha^*}\right)K(p_1)\exp\left(-\frac{1}{p_1}\left|\frac{\omega+\delta z_t}{2\alpha^*}\right|^{p_1}\right), & z_t\le-\frac{\omega}{\delta},\\ \frac{\delta}{\eta}\left(\frac{1-\alpha}{1-\alpha^*}\right)K(p_2)\exp\left(-\frac{1}{p_2}\left|\frac{\omega+\delta z_t}{2\left(1-\alpha^*\right)}\right|^{p_2}\right), & z_t>-\frac{\omega}{\delta}.\end{cases} \tag{21}$$

where

$$z_t=\frac{R_t-R_{ft}-\beta_0-\beta_1\left(R_{mt}-R_{ft}\right)-\beta_2 SMB_t-\beta_3 HMLO_t-\beta_4 RMW_t-\beta_5 CMA_t}{\sigma_t}, \tag{22}$$

$$\ln\left(\sigma_t^2\right)=a+\sum_{i=1}^{s} g\left(z_{t-i}\right)+\sum_{j=1}^{m} b_j \ln\left(\sigma_{t-j}^2\right), \tag{23}$$

$$g\left(z_{t-i}\right)=cz_{t-i}+d_i\left[\left|z_{t-i}\right|-E\left(\left|z_{t-i}\right|\right)\right], \\ =\begin{cases}\left(c_i+d_i\right)z_{t-i}-d_iE\left(\left|z_{t-i}\right|\right), & \text{if } z_{t-i}\ge 0,\\ \left(c_i-d_i\right)z_{t-i}-d_iE\left(\left|z_{t-i}\right|\right), & \text{else.}\end{cases} \tag{24}$$

3. Empirical Analysis

3.1. Data

In this paper, the sector of services in US is analyzed. Monthly return and 3 types of 5 factors (US 5 factors, North American 5 factors and Global 5 factors)[5] are downloaded from French's Data Library[6]. Sample period is from July 1990 to Feb. 2017. 3 types of 5 factors (US, north American, global) are compared.

Table 2 lists the descriptive statistics calculated by Matlab[7]. The values of skewness are not equal to 0 and those of Kurtosis are not 3. Especially, kurtosis values are all greater than 3. P-values of JB tests are 0, which are smaller than

[5]Global 5 factors are constructed by 23 countries, there are Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, Great Britain, Greece, Hong Kong , Ireland, Italy, Japan, Netherlands, Norway, New Zealand, Portugal, Sweden, Singapore, United States. North American 5 factors are construted by Canada and United States. US 5 factors are constructed by United States.

[6]Data source is mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

[7]Excess returns are got by portfolio returns minus the risk free rate.

Table 2. Descriptive statistics (1990:7 to 2017:2, monthly).

	Mean	Med.	Max.	Min.	St.De.	Ske.	Kur.	P
Panel A: Excess Returns of US Sector of Services								
US	1.00	1.74	22.22	−19.16	5.98	−0.30	3.97	0.00
Panel B: US 5 factors								
ME	0.65	1.19	11.35	−17.23	4.27	−0.68	4.26	0.00
SMB	0.19	0.07	18.73	−15.28	3.11	0.47	8.05	0.00
HML	0.27	−0.05	12.91	−11.25	3.03	0.15	5.57	0.00
RMW	0.33	0.37	13.52	−19.11	2.75	−0.45	13.86	0.00
CMA	0.26	0.09	9.55	−6.88	2.10	0.56	5.22	0.00
Panel C: North American 5 factors								
ME	0.65	1.14	11.54	−18.42	4.25	−0.72	4.55	0.00
SMB	0.18	0.12	16.48	−13.54	2.79	0.33	7.59	0.00
HML	0.24	0.19	16.75	−13.36	3.24	0.56	7.69	0.00
RMW	0.32	0.28	13.13	−15.32	2.43	0.14	12.25	0.00
CMA	0.30	0.02	14.23	−10.03	2.67	0.93	7.63	0.00
Panel D: Global 5 factors								
ME	0.45	0.86	11.45	−19.54	4.33	−0.70	4.62	0.00
SMB	0.12	0.08	8.00	−8.43	1.98	−0.34	5.20	0.00
HML	0.33	0.22	11.65	−9.54	2.29	0.54	8.17	0.00
RMW	0.34	0.34	6.10	−5.44	1.46	−0.04	5.06	0.00
CMA	0.26	0.06	9.60	−6.55	1.89	0.66	6.92	0.00

Notes: Med. = Median; Max. = Maximum; Min. = Minimum; St.De. = Standard Devistion; Ske. = Skewness; Kur. = Kurtosis; P = englishP-value of Jarque-Bera Test; ME = Market Excess Return; SMB = Small minus Big; HML = High minus Low; RMW = Robust minus Weak; CMA = Conservation minus Aggressive; The null hypothesis of JB test is H_0: Data are distributed as Normal(0,1).

0.05. That means, under 5% significance level, we can reject the null hypothesis and conclude that data do not follow Normal distribution. Hence, non-Normal error of SSAEPD may be proper. And from Table 2 we can see US 5 factors are very similar to North American 5 factors.

3.2. Estimation Results

The estimates are listed in Table 3. For FF5-SSAEPD-EGARCH, the Alpha returns for Global five factors is 0.78, bigger than the ones calculated from both US five factors and North American five factors (0.35 and 0.27, respectively). And the values of *Beta* (β_1 coefficient) for US five factors is close to that from the North American five factors, which is very close to 1. Meanwhile, the value of *Beta* (β_1) for Global five factors is 0.81, which is the smallest. It is interesting to find the coefficient of β_2 is negative for global five factors, which means the small-size effect documented in US market can not be found in the global market. Similar conclusions can be found from model of FF5-Normal.

Table 3. Estimates.

	β_0	β_1	β_2	β_3	β_4	β_5	α	p_1	p_2	μ	σ	a	b	c	d
Panel A: FF5-SSAEPD-EGARCH															
US 5 factors	0.35	1.00	0.11	−0.26	−0.31	−0.70	0.43	1.68	1.74	−0.03	1.93	0.12	0.91	0.10	0.09
North American 5 factors	0.27	0.97	0.04	−0.29	−0.25	−0.65	0.43	1.70	1.79	0.03	2.01	0.10	0.94	−0.01	0.16
Global 5 factors	0.78	0.81	−0.26	−0.53	−0.81	−1.02	0.43	1.70	1.80	0.54	3.02	0.06	0.97	−0.02	0.21
Panel B: FF5-Normal															
US 5 factors	0.33	1.02	0.12	−0.26	−0.27	−0.77	-	-	-	0	1.92	-	-	-	-
North American 5 factors	0.31	1.00	0.10	−0.29	−0.16	−0.72	-	-	-	0	1.99	-	-	-	-
Global 5 factors	0.94	0.79	−0.13	−0.27	−0.59	−1.21	-	-	-	0	2.95	-	-	-	-

Notes: FF5-Normal is the model used in Fama-French (2015); FF5-SSAEPD-EGARCH is the new 5-factor model suggested by Zhu and Li (2016) supposing the error term meet the EGARCH-type volatilities and SSAEPD errors.

3.2.1. Fama-French 5 Factors Still Alive

- **Parameter Restriction Tests**

Likelihood Ratio test (LR)[8] is used to test the significance of regressors in these models. The P-values for Likelihood Ratio tests are listed in Table 4. We find out with non-Normal errors such as SSAEPD and EGARCH-type volatilities, the Fama-French 5 factors are still alive for the sector of services.

Panel A of Table 4 lists the test results for the FF5-SSAEPD-EGARCH model. For example, the P-values of the joint significance test (see column T1)[9] for all 3 types of 5 factors (US, North Ameirican and Global) are approximately equal to 0, which means the coefficient of $\beta_1, \beta_2, \beta_3, \beta_4$ and β_5 are statistically joint significant under 5% significance level.

For 3 kinds of five factors, the individual significance tests show β_1 is statistically significant (see column T3). That is, market returns have significant effect on this sector returns. Same is true for β_3, β_4 and β_5. For 2/3 types of 5 factors, β_0 and β_2 are statistically significant (see column T2 and T4, respectively).

Panel B of Table 4 lists the test results for the FF5-Normal model. For this model, this sector doesn't have a statistically significant coefficient β_0 under 5% significance level (see column T2) which means they can not earn statistically signicant Alpha returns. But with FF5-SSAEPD-EGARCH model, this sector in both US and Global market have a statistically significant coefficient β_0 under 5% significance level, expecially we can earn more Alpha return from Global market because β_0 in this market is 0.78 (see Table 3, column 2), the highest among 3 types of 5 factors. Furthermore, the size factor seems statistically significant for 2/3 kinds of 5 factors and is not significant in Global Market[10].

[8]LR formula is from Neyman and Pearson (1993) [22]. The equation is: $\text{LR} = -2\ln(\text{likelihood for null}) + 2\ln(\text{likelihood for alternative})$.

[9]The null hypothesis of the joint significance test is $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$.

[10]See column T4.

Table 4. P-values of likelihood ratio test (LR).

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11	T12	T13	T14	T15	T16	T17
Panel A: FF5-SSAEPD-EGARCH																	
US 5 factors	0*	0*	0*	0*	0*	0*	0*	0.04*	0.06	0.03*	0.01*	0.11	0*	0*	0*	0.01*	0.99
North Ame. 5 factors	0*	0.13	0*	0.99	0*	0*	0*	0.07	0.99	0.03*	0*	0*	0*	0*	0*	0.99	0.99
Global 5 factors	0*	0*	0*	0*	0*	0*	0*	0.01*	0.99	0*	0*	0.07	0*	0*	0*	0.70	0.99
Panel B:FF5-Normal																	
US 5 factors	0*	1	0*	0*	0*	0*	0*	-	-	-	-	-	-	-	-	-	-
North Ame. 5 factors	0*	1	0*	0.03*	0*	0*	0*	-	-	-	-	-	-	-	-	-	-
Global 5 factors	0*	1	0*	0.13	0.01*	0*	0*	-	-	-	-	-	-	-	-	-	-

Notes:

T1 means $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ T2 means $H_0: \beta_0 = 0$.

T3 means $H_0: \beta_1 = 0$. T4 means $H_0: \beta_2 = 0$.

T5 means $H_0: \beta_3 = 0$. T6 means $H_0: \beta_4 = 0$.

T7 means $H_0: \beta_5 = 0$. T8 means $H_0: \alpha = 0.5, p_1 = p_2 = 2$.

T9 means $H_0: \alpha = 0.5$. T10 means $H_0: p_1 = p_2 = 2$.

T11 means $H_0: p_1 = 2$. T12 means $H_0: p_2 = 2$.

T13 means $H_0: b = c = d = 0$. T14 means $H_0: a = 0$.

T15 means $H_0: b = 0$. T16 means $H_0: c = 0$.

T17 means $H_0: d = 0$.

*means the null hypothesis is rejected under 5% significance level; North Ame. = North American

For the FF5-SSAEPD-EGARCH model, among both US 5 factors and Global 5 factors, all individual coefficients in the mean equation are statistically significant. Hence, we conclude that Fama-French 5 factors are alive even if EGARCH and SSAEPD considered.

In this part, some restrictions on the parameters in the EGARCH equation are also tested with Likelihood Ratio test (LR). And the results are also listed in **Table 4**. Results show the EGARCH-type volatility should be included in Fama-French 5 factor model. For instance, we do the joint significance test for hypothesis $H_0: b = c = d = 0$. The P-value of the LR are all smaller than the significance level 5%, which means our EGARCH-type volatilities is necessary, english ARCH and GARCH terms should be added into Fama-French 5-factor model since they are all statistically signicant (see column T13).

Next, we test the parameters in the SSAEPD with same method and the results show englishparameter α is statistically significant equal to 0.5, so skewnessenglish is not documentedenglish. Non-Normality is confirmed (see column T8, T10, T11, T12). The left and the right tail parametersenglish (p_1 and p_2) are jointly statistically different from 2 (see column T10). And leftenglish tail is statistically different from 2 in all markets (see column T11) but right tail is only statistically different from 2 in North American market (see column T12). *i.e.*, strong left fat-tailedness is documented. Therefore. this new 5-factor model can capture the fat-tailedness better than FF5-Normal model.

- **Kolmogorov-Smirnov Test for Residuals**

We check the residuals for models with Kolmogorov-Smirnov test (KS). The P-values of KS test are listed in Table 5, the P-value of the Global five factors is 0.07, greater than 5%, which means under 5% significance level, the null hypothesis is not rejected and the residuals from FF5-SSAEPD-EGARCH do follow the SSAEPD. 2/3 markets support this result. For the FF5-Normal model[11], the P-values of the KS test are also listed in Table 5. All of them have smaller P-values than 0.05, which means reject the nulls. Hence, the residuals of the FF5-Normal model don't follow Normal distribution. And the FF5-Normal model is not adequate for the data.

3.2.2. Model Comparison

We compare models with AIC. Results in Table 6 show FF5-SSAEPD-EGARCH model has smaller AIC value than FF5-Normal model. That is, this new model is better than the one in Fama and French(2015). However, FF5-GARCH model or FF5-SSAEPD-GARCH model seem to be the best because it has the smallest AIC values. That means, models with GARCH is better than the ones with EGARCH. Also, since AIC values are the same for both FF5-GARCH and FF5-SSAEPD-GARCH, we conclude that, to capture fat-tailedness, GARCH equation is better than SSAEPD, which is consistent with what we found out in our previous researches.

4. Conclusions and Future Extensions

In this paper, US sector of services is studied. A new Fama-French 5-factor model (denoted as FF5-SSAEPD-EGARCH) is empirically tested. This new model uses the non-normal error term of SSAEPD of Zhu and Zinde-Walsh

Table 5. P-values of KS test.

Model	FF5-SSAEPD-EGARCH	FF5-Normal
US 5factors	0.04	0.00
North American 5 factors	0.05	0.00
Global 5 factors	0.07	0.00

Note: The null hypothesis of KS test is H_0: Data follow a specified distribution. We set the significance level of all tests at 5%. If the P-value of KS test is bigger than 5%, then we do not reject the null hypothesis. Otherwise, we reject the null hypothesis. For example, We apply KS test for the FF5-SSAEPD-EGARCH model residuals with the null hypothesis of H_0: FF5-SSEAPD-EGARCH model residuals are distributed as $\text{SSAEPD}(\hat{\alpha}, \hat{p}_1, \hat{p}_2)$. For Global five factors, its P-value is 0.07, which is bigger than 0.05. That means, under 5% significance level, we cannot reject the null hypothesis and conclude that the residuals from FF5-SSEAPD-EGARCH model follow SSAEPD.

[11]The null hypothesis H_0: FF5-Normal residuals are distributed as $\text{Normal}(\hat{\mu}, \hat{\sigma}^2)$.

Table 6. AIC values.

Model	FF5-SSAEPD-EGARCH	FF5-Normal	FF5-SSAEPD	FF5-GARCH	FF5-SSAEPD-GARCH	FF5-EGARCH
US 5 factors	4.16	4.19	4.90	4.14	4.14	4.18
North Ame. 5 factors	4.30	4.27	5.06	4.22	4.22	4.31
Global 5 factors	5.09	5.05	6.59	4.91	4.91	5.12

Note: North Ame.=North American.

(2009) and EGARCH-type volatility of Nelson (1991) to extend the 5 factor model of Fama and French (2015). The return of services industry and 3 types of 5 factors (US five factors, North American five factors, Global five factors) from French's Data Library are analyzed and compared. Sample period is from Jul. 1990 to Feb. 2017. Likelihood Ratio test (LR) is used for parameter restriction test, Kolmogorov-Smirnov test (KS) for residual check and AIC for model comparison. Maximum Likelihood Estimation method (MLE) is used to estimate models via MatLab.

Empirical results show: 1) With EGARCH-typed volatilities and non-normal errors, the Fama-French 5 factors are still alive; 2) The new model fits the data better than Fama-French (2015)'s 5-factor model; 3) Models with GARCH-typed volatility are a little bit better than the ones with EGARCH-typed volatility; 4) To capture fat-tailedness, GARCH equation is better than SSAEPD; 5) Using SSAEPD, model can capture stronger left fat-tailedness.

Future extensions will include but not limited to the following: First, we can construct a new index for services industry; Second, other sectors can be analyzed; Last, different factors can be considered.

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