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Nonexistence of Nontrivial Solutions with Decay Order for a Biharmonic P-Laplacian Equation and System

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Abstract

We use the Morawetz multiplier to show that there are no nontrivial solutions of certain decay order for a biharmonic equation with a p-Laplacian term and a system of coupled biharmonic equations with p-Laplacian terms in the entire Euclidean space.

Keywords

Morawetz Multiplier, Biharmonic, P-Laplacian, Nonexistence

1. Introduction

Recently, there has been an active research on the biharmonic equation with a p-Laplacian term

$$\Delta^2 u + \alpha \nabla \cdot \left(|\nabla u|^{p-2} \nabla u \right) + \beta \cdot u + f(\boldsymbol{x}, u) = 0. \tag{1.1}$$

as well as the evolutionary biharmonic equations with a p-Laplacian term

$$\partial u / \partial t = \Delta^2 u + \alpha \nabla \cdot \left(|\nabla u|^{p-2} \nabla u \right) + \beta \cdot u + f(\boldsymbol{x}, u) \tag{1.2}$$

and

$$\partial^2 u / \partial t^2 = \Delta^2 u + \alpha \nabla \cdot \left(|\nabla u|^{p-2} \nabla u \right) + \beta \cdot u + f(\boldsymbol{x}, u) \tag{1.3}$$

where $\boldsymbol{x} \in \boldsymbol{R}^n, n \geq 2, p > 1$, α and β are real constants.

Equation (1.1) is the stationary state of the Equation (1.2) while the traveling wave solution for (1.3) satisfies an equation of the form (1.1) as was shown in Strauss [1]. For the analysis and applications of (1.1), (1.2), and (1.3), see, for example, [2]-[42]. In this article, we shall use the Morawetz multiplier [43] [44] [45] to show that there are no nontrivial solutions of certain decay order for (1.1) and a system of coupled biharmonic equations with p-Laplacian terms,

$$\Delta^2 u + \alpha\nabla\cdot\left(\left|\nabla u\right|^{p-2}\nabla u\right) + \beta\cdot u + f\left(\boldsymbol{x},u\right) + \left[a\left(\boldsymbol{x}\right)u^2 + b\left(\boldsymbol{x}\right)v^2\right]u = 0, \quad (1.4a)$$

$$\Delta^2 v + \sigma\nabla\cdot\left(\left|\nabla u\right|^{p-2}\nabla v\right) + \mu\cdot v + g\left(\boldsymbol{x},v\right) + \left[c\left(\boldsymbol{x}\right)u^2 + d\left(\boldsymbol{x}\right)v^2\right]v = 0, \quad (1.4b)$$

where $u = u\left(\boldsymbol{x}\right)$, $v = v\left(\boldsymbol{x}\right)$, $a\left(\boldsymbol{x}\right), b\left(\boldsymbol{x}\right), c\left(\boldsymbol{x}\right)$ and $d\left(\boldsymbol{x}\right)$ are real-valued functions, $\boldsymbol{x}\in\boldsymbol{R}^n, n\ge 2$, α,β,σ,μ are all real constants.

As usual, $\boldsymbol{x} = \left(x_1, x_2, \cdots, x_n\right)$, ∇u denotes the gradient of *u*, $\nabla\cdot u$ denotes the divergence of *u*, and $r = \left|\boldsymbol{x}\right|$. Also the subscript denotes the partial derivative, thus $u_s = \partial u/\partial s$. We also use the notation $u_r = \partial u/\partial r = \left(\boldsymbol{x}/r\right)\cdot\nabla u$ and $\partial_j = \partial/\partial x_j$. $F_r\left(\boldsymbol{x},s\right)$ denotes $\partial F\left(\boldsymbol{x},s\right)/\partial r = \left(\boldsymbol{x}/r\right)\cdot\nabla_x F\left(\boldsymbol{x},s\right)$. $C^k\left(\boldsymbol{R}^n\right)$ is the space of functions whose partial derivatives of order up to and including *k* are continuously differentiable.

Define eight sets of functions $A_m\left(\boldsymbol{R}^n\right), B_m\left(\boldsymbol{R}^n\right), D_{h,k}\left(\boldsymbol{R}^n\right), E_m\left(\boldsymbol{R}^n\right), F_n\left(\boldsymbol{R}^n\right)$, $G_n\left(\boldsymbol{R}^n\right)$, $NF\left(\boldsymbol{R}^n\right)$ and $NG\left(\boldsymbol{R}^n\right)$ which we will use in this article:

$$A_m\left(\boldsymbol{R}^n\right) = \left\{a \mid a\in C^1\left(\boldsymbol{R}^n\right), \sup_{|\boldsymbol{x}|\ge\rho}\left(\left|x\right|^{-m}\left|a\left(\boldsymbol{x}\right)\right|\right) < \infty \text{ for some } \rho > 0 \right.$$
$$\left. \text{and } m > 0, \text{ and } ra_r\left(\boldsymbol{x}\right) - \left(n-2\right)a\left(\boldsymbol{x}\right) \le 0\right\}$$

$$B_m\left(\boldsymbol{R}^n\right) = \left\{b \mid b\in C^1\left(\boldsymbol{R}^n\right), \sup_{|\boldsymbol{x}|\ge\rho}\left(\left|\boldsymbol{x}\right|^{-m}\left|b\left(\boldsymbol{x}\right)\right|\right) < \infty \text{ for some } \rho > 0 \right.$$
$$\left. \text{and } m > 0, \text{ and } 2rb_r\left(\boldsymbol{x}\right) - \left(n-3\right)b\left(\boldsymbol{x}\right) \le 0\right\}$$

$$D_{h,k}\left(\boldsymbol{R}^n\right) = \left\{u \mid u\in C^k\left(\boldsymbol{R}^n\right), \lim_{R\to\infty}\left(\sup_{|\boldsymbol{x}|\le R}\left(\left|\boldsymbol{x}^{\alpha}\right|\left|D^{\beta}u\left(\boldsymbol{x}\right)\right|\right)\right) = 0, \right.$$
$$\text{for all multi-indices } \boldsymbol{\alpha} \text{ and } \boldsymbol{\beta}\in\boldsymbol{N}_0^n \text{ such that } \left|\boldsymbol{\alpha}\right|\le h \text{ and}$$
$$\left|\boldsymbol{\beta}\right|\le k, \text{ where } \boldsymbol{\alpha} = \left(\alpha_1,\alpha_2,\cdots,\alpha_n\right), \boldsymbol{\beta} = \left(\beta_1,\beta_2,\cdots,\beta_n\right),$$
$$\left.\left|\boldsymbol{\alpha}\right| = \alpha_1+\alpha_2+\cdots+\alpha_n \text{ and } \left|\boldsymbol{\beta}\right| = \beta_1+\beta_2+\cdots+\beta_n\right\},$$

$$E_m\left(\boldsymbol{R}^n\right) = \left\{a \mid a\in C^1\left(\boldsymbol{R}^n\right), \sup_{|\boldsymbol{x}|\ge\rho}\left(\left|x\right|^{-m}\left|a\left(\boldsymbol{x}\right)\right|\right) < \infty \text{ for some } \rho > 0 \text{ and } m > 0\right\},$$

$$F_n\left(\boldsymbol{R}^n\right) = \left\{u \mid \lim_{R\to\infty}\left(R^n\sup_{|\boldsymbol{x}|=R}\left|F\left(\boldsymbol{x},u\left(\boldsymbol{x}\right)\right)\right|\right) = 0\right\},$$

$$G_n\left(\boldsymbol{R}^n\right) = \left\{v \mid \lim_{R\to\infty}\left(R^n\sup_{|\boldsymbol{x}|=R}\left|F\left(\boldsymbol{x},v\left(\boldsymbol{x}\right)\right)\right|\right) = 0\right\},$$

$$NF\left(\boldsymbol{R}^n\right) = \left\{u \mid \int_{\boldsymbol{R}^n}\left[nF\left(\boldsymbol{x},u\left(\boldsymbol{x}\right)\right) + rF_r\left(\boldsymbol{x},u\left(\boldsymbol{x}\right)\right) - \left(\left(n-1\right)/2\right)f\left(\boldsymbol{x},u\left(\boldsymbol{x}\right)\right)u\left(\boldsymbol{x}\right)\right]\mathrm{d}\boldsymbol{x}\le 0\right\},$$

and

$$NG\left(\boldsymbol{R}^n\right) = \left\{v \mid \int_{\boldsymbol{R}^n}\left[nG\left(\boldsymbol{x},v\left(\boldsymbol{x}\right)\right) + rG_r\left(\boldsymbol{x},v\left(\boldsymbol{x}\right)\right) - \left(\left(n-1\right)/2\right)g\left(\boldsymbol{x},v\left(\boldsymbol{x}\right)\right)v\left(\boldsymbol{x}\right)\right]\mathrm{d}\boldsymbol{x}\le 0\right\},$$

where $F(\boldsymbol{x},u)$ and $G(\boldsymbol{x},v)$ are the antiderivative of $f(\boldsymbol{x},u)$ with respect to u and $g(\boldsymbol{x},v)$ with respect to v, respectively, such that $F(\boldsymbol{x},0)=0$ and $G(\boldsymbol{x},0)=0$.

Remark 1. A function u is said to be of decay order (h, k) if and only if $u \in D_{h,k}(\boldsymbol{R}^n)$.

All the functions are assumed to be real-valued.

2. A Biharmonic Equation with a P-Laplacian Term

We consider the equation (1.1) in this section. Multiplying both sides of the equation (1.1) by the Morawetz multiplier $\zeta\left(u_r+\left((n-1)u/(2r)\right)\right)$, where $\zeta \in C^3(\boldsymbol{R}^n)$ and $\zeta(\boldsymbol{x})=\zeta(|\boldsymbol{x}|)=\zeta(r)$, we get

$$\begin{aligned}0&=\left[\Delta^2 u+\alpha\nabla\cdot\left(|\nabla u|^{p-2}\nabla u\right)+\beta\cdot u+f(\boldsymbol{x},u)\right]\zeta\left(u_r+\left((n-1)u/(2r)\right)\right)\\&=\nabla\cdot Y+Z,\end{aligned} \tag{2.1}$$

where Y depends on ζ and u as well as their partial derivatives up to and including the third order and $F(\boldsymbol{x},u)$, and

$$\begin{aligned}Z&=(3\zeta'/2)(\Delta u)^2+A(u_r)^2+B\left(|\nabla u|^2-|u_r|^2\right)+Cu^2+(\zeta-r\zeta')P\\&+\alpha\left\{\left((\zeta/r)-\zeta'\right)|\nabla u|^{p-2}\left[(u_r)^2+\left((n-1)/(2r)\right)u_r u\right]\right.\\&\left.+\left[(\zeta'/p)-(\zeta/(2pr))\left((n+1)p-2(n-1)\right)\right]|\nabla u|^p\right\}\\&-\beta\left\{(1/2)\zeta_r|\nabla u|^2+(\nabla u\cdot\nabla\zeta)u_r-\zeta_r|u_r|^2+\left((1/r)\zeta-\zeta_r\right)\left(|\nabla u|^2-|u_r|^2\right)\right.\\&\left.+\left((n-1)/(2r)\right)\left[(1/r)\zeta_r-(1/2)(\Delta\zeta)+\left((n-3)/(2r^2)\right)\zeta\right]u^2\right\}\\&-\left(\zeta_r+\left((n-1)/r\right)\zeta\right)F(\boldsymbol{x},u)+\left((n-1)/(2r)\right)\zeta f(\boldsymbol{x},u)u-\zeta F_r(\boldsymbol{x},u)\end{aligned}$$

where

$$A=-7\zeta'''/2-(n-1)(n-3)\left(\zeta'-(\zeta/r)\right)/\left(2r^2\right), \tag{2.2}$$

$$B=-3\zeta'''/2+(n-5)\zeta''/r-\left(n^2+2n-19\right)(\zeta'-\zeta/r)/\left(2r^2\right), \tag{2.3}$$

$$\begin{aligned}C&=\left((n-1)/2\right)\left[\zeta''''/(2r)+(n-3)\zeta'''/\left(r^2\right)+(n-3)(n-7)\zeta''/\left(2r^3\right)\right.\\&\left.-3(n-3)(n-5)(\zeta'-\zeta/r)/\left(2r^4\right)\right],\end{aligned} \tag{2.4}$$

and

$$P=(2/r)\left[\sum\left(S_{ij}u\right)^2-\sum\left(\sum\left(x_j/r\right)S_{ij}u\right)^2\right]\geq 0, \tag{2.5}$$

where

$$\boldsymbol{S}_{ij}u=\left(x_i/r^3\right)\sum\left[x_k\left(x_k\partial_j-x_j\partial_k\right)u_r\right]+\partial_j\sum\left[\left(x_k/r^2\right)\left(x_k\partial_i-x_i\partial_k\right)u\right].$$

Note that we use the Einstein summation notation in the expressions for P and $\boldsymbol{S}_{ij}u$.

Theorem 1

Let u be a c^4 solution of (1.1) such that $u \in D_{n,3}(\boldsymbol{R}^n)\cap F_n(\boldsymbol{R}^n)\cap NF(\boldsymbol{R}^n)$.

Assume $\beta \le 0$.

(a) If $p \ge (2n)/(n+1)$ and $\alpha \le 0$, then $u \equiv 0$.

(b) If $1 < p \le (2n)/(n+1)$ and $\alpha \ge 0$, then $u \equiv 0$.

Proof:

Let $R > 0$. Integrating both sides of (2.1) in $|\boldsymbol{x}| \le R$ and using the Divergence theorem, we get

$$\int_{|\boldsymbol{x}|=R} Y \cdot (\boldsymbol{x}/R) \mathrm{d}\boldsymbol{s} + \int_{|\boldsymbol{x}|\le R} Z \mathrm{d}\boldsymbol{x} = 0.$$

Let $R \to \infty$. We get

$$\lim_{R\to\infty} \int_{|\boldsymbol{x}|=R} Y \cdot (x/R) \mathrm{d}\boldsymbol{s} + \int_{R^n} Z \mathrm{d}\boldsymbol{x} = 0.$$

Thus

$$\begin{aligned}
0 = &\lim_{R\to\infty} \int_{|\boldsymbol{x}|=R} Y \cdot (\boldsymbol{x}/R) \mathrm{d}\boldsymbol{s} \\
&+ \int_{R^n} \Big\{ (3\zeta'/2)(\Delta u)^2 + A(u_r)^2 + B\left(|\nabla u|^2 - |u_r|^2\right) + Cu^2 + (\zeta - r\zeta')P \\
&+ \alpha\Big\{ ((\zeta/r) - \zeta')|\nabla u|^{p-2}\left[(u_r)^2 + ((n-1)/(2r))u_r u\right] \\
&+ \left[(\zeta'/p) - (\zeta/(2pr))((n+1)p - 2(n-1))\right]|\nabla u|^p \Big\} \qquad (2.6) \\
&- \beta\Big\{ (1/2)\zeta_r |\nabla u|^2 + (\nabla u \cdot \nabla \zeta)u_r - \zeta_r |u_r|^2 + ((1/r)\zeta - \zeta_r)\left(|\nabla u|^2 - |u_r|^2\right) \\
&+ ((n-1)/(2r))\left[(1/r)\zeta_r - (1/2)(\Delta\zeta) + ((n-3)/(2r^2))\zeta\right]u^2 \Big\} \\
&- (\zeta_r + ((n-1)/r)\zeta)F(\boldsymbol{x},u) + ((n-1)/(2r))\zeta f(\boldsymbol{x},u)u - \zeta F_r(\boldsymbol{x},u) \Big\} \mathrm{d}\boldsymbol{x},
\end{aligned}$$

where A, B, C, P are defined as in (2.2)–(2.5).

The above equation (2.6) can be written as

$$\begin{aligned}
0 = &\lim_{R\to\infty} \int_{|\boldsymbol{x}|=R} Y \cdot (\boldsymbol{x}/R) \mathrm{d}\boldsymbol{s} \\
&+ \int_{R^n} \Big\{ (3\zeta'/2)(\Delta u)^2 + A(u_r)^2 + B\left(|\nabla u|^2 - |u_r|^2\right) + Cu^2 + (\zeta - r\zeta')P \\
&+ \alpha\Big\{ ((\zeta/r) - \zeta')|\nabla u|^{p-2}\left[(u_r)^2 + ((n-1)/(2r))u_r u\right] \\
&+ \left[(\zeta'/p) - (\zeta/(2pr))((n+1)p - 2(n-1))\right]|\nabla u|^p \Big\} \\
&- \beta\Big\{ (1/2)\zeta_r |\nabla u|^2 + (\nabla u \cdot \nabla \zeta)u_r - \zeta_r |u_r|^2 + ((1/r)\zeta - \zeta_r)\left(|\nabla u|^2 - |u_r|^2\right) \\
&+ ((n-1)/(2r))\left[(1/r)\zeta_r - (1/2)(\Delta\zeta) + ((n-3)/(2r^2))\zeta\right]u^2 \Big\} \mathrm{d}\boldsymbol{x} \\
= &\int_{\boldsymbol{R}^n} (\zeta_r + ((n-1)/r)\zeta)F(\boldsymbol{x},u) - ((n-1)/(2r))\zeta f(\boldsymbol{x},u)u + \zeta F_r(\boldsymbol{x},u) \Big\} \mathrm{d}\boldsymbol{x},
\end{aligned}$$

Let $\zeta(\boldsymbol{x}) = \zeta(|\boldsymbol{x}|) = \zeta(r) = r$.

Since u is assumed to be of decay order $(n,3)$ and $u \in F_n(\boldsymbol{R}^n)$, $\lim_{R\to\infty} \int_{|\boldsymbol{x}|=R} Y \cdot (\boldsymbol{x}/R) \mathrm{d}\boldsymbol{s} = 0$, after substituting ζ by r.

Thus

$$\begin{aligned}&\int_{\mathbf{R}^n}\left\{(3/2)(\Delta u)^2+\alpha\left[(1/p)-(1/(2p))((n+1)p-2(n-1))\right]|\nabla u|^p-(\beta/2)|\nabla u|^2\right\}\mathrm{d}\mathbf{x}\\&=\int_{\mathbf{R}^n}\left[nF(\mathbf{x},u)-((n-1)/2)f(\mathbf{x},u)u+rF_r(\mathbf{x},u)\right]\mathrm{d}\mathbf{x}.\end{aligned}\tag{2.7}$$

To prove the assertion (a), since $\beta\le 0$, we have

$$\begin{aligned}&\int_{\mathbf{R}^n}\left\{(3/2)(\Delta u)^2+\alpha\left[(1/p)-(1/(2p))((n+1)p-2(n-1))\right]|\nabla u|^p\right\}\mathrm{d}\mathbf{x}\\&\le\int_{\mathbf{R}^n}\left[nF(\mathbf{x},u)-((n-1)/2)f(\mathbf{x},u)u+rF_r(\mathbf{x},u)\right]\mathrm{d}\mathbf{x}\end{aligned}$$

Thus

$$\begin{aligned}&\int_{\mathbf{R}^n}\left\{(3/2)(\Delta u)^2+\alpha\left[(2n-(n+1)p)/(2p)\right]|\nabla u|^p\right\}\mathrm{d}\mathbf{x}\\&\le\int_{\mathbf{R}^n}\left[nF(\mathbf{x},u)-((n-1)/2)f(\mathbf{x},u)u+rF_r(\mathbf{x},u)\right]\mathrm{d}\mathbf{x}\end{aligned}$$

Since $\alpha\le 0, p\ge(2n)/(n+1)$, and $u\in NF(\mathbf{R}^n)$, we have

$$0\le\int_{\mathbf{R}^n}(3/2)(\Delta u)^2\,\mathrm{d}\mathbf{x}\le\int_{\mathbf{R}^n}\left[nF(\mathbf{x},u)-((n-1)/2)f(\mathbf{x},u)u+rF_r(\mathbf{x},u)\right]\mathrm{d}\mathbf{x}\le 0,$$

Thus $\int_{\mathbf{R}^n}(3/2)(\Delta u)^2\,\mathrm{d}\mathbf{x}=0$. Since $u\in D_{n,3}(R_n), u\equiv 0$

Assertion (b) follows with a similar argument from (2.7).

Remark 2. As an example for $f(\mathbf{x},u)$, let $f(\mathbf{x},u)=q(\mathbf{x})|u|^{s-1}u, s\ge 1$. Then $F(\mathbf{x},u)=(1/(s+1))q(x)|u|^{s+1}$, where $q(\mathbf{x})=q(|\mathbf{x}|)=q(r)$.

Assume $q\in E_m(\mathbf{R}^n)$.

For u to be in $F_n(\mathbf{R}^n)$, we need

$$\lim_{R\to\infty}\left(R^n\sup_{|\mathbf{x}|=R}\left|F(\mathbf{x},u(\mathbf{x}))\right|\right)=0,$$

that is,

$$\lim_{R\to\infty}\left(R^n\sup_{|\mathbf{x}|=R}\left(q(|\mathbf{x}|)|u(\mathbf{x})|^{s+1}\right)\right)=0,$$

This would be satisfied if

$$\lim_{R\to\infty}\left(R^n\sup_{|\mathbf{x}|=R}\left(|\mathbf{x}|^m|u(\mathbf{x})|^{s+1}\right)\right)=0.\tag{2.8}$$

The above condition (2.8) would be satisfied if u is of decay order $(n+m,0)$.

As for u to be in $NF(\mathbf{R}^n)$, since

$$\begin{aligned}&nF(\mathbf{x},u)+rF_r(\mathbf{x},u)-((n-1)/2)f(\mathbf{x},u)u\\&=\left[(n/(s+1))q+(r/(s+1))q_r-((n-1)/2)q\right]|u|^{s+1},\end{aligned}$$

u would be in $NF(\mathbf{R}^n)$ if $(n/(s+1))q+(r/(s+1))q_r-((n-1)/2)q\le 0.$

Thus, if $rq_r\le((ns-n-s-1)/2)q$, then u is in $NF(\mathbf{R}^n)$.

Therefore, if u is of decay order $(n+m,1)$ and $rq_r\le((ns-n-s-1)/2)q$, u satisfies the assumptions of Theorem 1 on u.

Remark 3. A similar conclusion can be obtained for $f(\mathbf{x},u)=q_1(\mathbf{x})|u|^{a-1}u+q_2(\mathbf{x})|u|^{b-1}u$, where $a>b\ge 1$.

3. A System of Biharmonic Equations with p-Laplacian Terms

We consider the system (1.4.a) and (1.4.b) in this section. Let $F(\boldsymbol{x},u)$ and $G(\boldsymbol{x},v)$ be the antiderivatives of $f(\boldsymbol{x},u)$ with respect to *u* and $g(\boldsymbol{x},v)$ with respect to *v*, respectively, such that $F(\boldsymbol{x},0)=0$ and $G(\boldsymbol{x},0)=0$.

Assume also $b(\boldsymbol{x})=c(\boldsymbol{x})$. Multiplying both sides of (1.4.a) by $\zeta\left(u_r+\left((n-1)u/(2r)\right)\right)$ and both sides of (1.4.b) by $\zeta\left(v_r+\left((n-1)v/(2r)\right)\right)$, then adding them up, we get

$$\begin{aligned}0&=\left(\Delta^2u+\alpha\nabla\cdot\left(|\nabla u|^{p-2}\nabla u\right)+\beta\cdot u+f(\boldsymbol{x},u)+\left[a(\boldsymbol{x})u^2+b(\boldsymbol{x})v^2\right]u\right)\zeta\left(u_r+\left((n-1)u/(2r)\right)\right)\\&\quad+\left(\Delta^2v+\sigma\nabla\cdot\left(|\nabla v|^{p-2}\nabla v\right)+\mu\cdot v+g(\boldsymbol{x},v)+\left[c(\boldsymbol{x})u^2+d(\boldsymbol{x})v^2\right]v\right)\zeta\left(v_r+\left((n-1)v/(2r)\right)\right)\\&=\nabla\cdot Y+Z,\end{aligned}$$

where Y depends on ζ, u, and v as well as their partial derivatives up to and including the third order, $F(\boldsymbol{x},u)$, $G(\boldsymbol{x},v)$, α,β,σ,μ, $a(\boldsymbol{x})$, $b(\boldsymbol{x})$, $c(\boldsymbol{x})$, and $d(\boldsymbol{x})$. Here we assume $b(\boldsymbol{x})=c(\boldsymbol{x})$. Z is similar to Section 2 with appropriate modification to allow terms containing v,σ,μ, $a(\boldsymbol{x})$, $b(\boldsymbol{x})$, $c(\boldsymbol{x})$, and $d(\boldsymbol{x})$.

Theorem 2

Let u and v be C^4 solutions of the system (1.4.a) and (1.4.b) with $b(\boldsymbol{x})=c(\boldsymbol{x})$. Assume $a\in A_{k,2}(\boldsymbol{R}^n)$, $b\in B_{m,2}(\boldsymbol{R}^n)$ and $d\in A_{h,2}(\boldsymbol{R}^n)$. Let $s=\max\{k,m,h,n\}$. Assume $\beta\le 0$ and $\mu\le 0$.

Assume further that $u\in D_{s,3}(\boldsymbol{R}^n)\cap F_n(\boldsymbol{R}^n)\cap NF(\boldsymbol{R}^n)$ and $v\in D_{s,3}(\boldsymbol{R}^n)\cap G_n(\boldsymbol{R}^n)\cap NG(\boldsymbol{R}^n)$.

(a) If $p\ge(2n)/(n+1),\alpha\le 0$ and $\sigma\le 0$, then $u\equiv 0$ and $v\equiv 0$.

(b) If $1<p\le(2n)/(n+1),\alpha\ge 0$, and $\sigma\ge 0$, then $u\equiv 0$ and $v\equiv 0$.

Proof:

Let $\zeta(\boldsymbol{x})=\zeta(|\boldsymbol{x}|)=\zeta(r)=r$. Following the same steps as in Theorem 1, we get

$$\begin{aligned}0&\le\int_{\boldsymbol{R}^n}(3/2)\left((\Delta u)^2+(\Delta v)^2\right)\mathrm{d}\boldsymbol{x}\\&\le\int_{\boldsymbol{R}^n}\Big\{\left[nF(\boldsymbol{x},u)-((n-1)/2)f(\boldsymbol{x},u)u+rF_r(\boldsymbol{x},u)\right]\\&\quad+\left[nG(\boldsymbol{x},v)-((n-1)/2)g(\boldsymbol{x},v)v+rG_r(\boldsymbol{x},v)\right]\\&\quad+(1/4)\left[ra_r(\boldsymbol{x})-(n-2)a(\boldsymbol{x})\right]u^4\\&\quad+(1/4)\left[rd_r(\boldsymbol{x})-(n-2)d(\boldsymbol{x})\right]v^4\\&\quad+(1/2)\left[rb_r(\boldsymbol{x})-((n-3)/2)b(\boldsymbol{x})\right]u^2v^2\Big\}\mathrm{d}\boldsymbol{x}\\&\le 0,\end{aligned}$$

since $a\in A_k(\boldsymbol{R}^n),b\in B_m(\boldsymbol{R}^n),d\in A_h(\boldsymbol{R}^n),u\in D_{s,3}(\boldsymbol{R}^n)\cap F_n(\boldsymbol{R}^n)\cap NF(\boldsymbol{R}^n)$ and $v\in D_{s,3}(\boldsymbol{R}^n)\cap G_n(\boldsymbol{R}^n)\cap NG(\boldsymbol{R}^n)$.

Thus

$$\int_{\boldsymbol{R}^n}(3/2)\left((\Delta u)^2+(\Delta v)^2\right)\mathrm{d}\boldsymbol{x}=0.$$

Since u and $v \in D_{s,3}\left(\boldsymbol{R}^n\right)$, $u \equiv 0$ and $v \equiv 0$.

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