Applied Mathematics, 2017, 8, 655-662
http://www.scirp.org/journal/am
ISSN Online: 2152-7393
ISSN Print: 2152-7385

Thermomagnetic Convection of Magnetic Fluid in an Annular Space under a Non-Uniform Magnetic and Thermal Field

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How to cite this paper: Liu, J. (2017) Thermomagnetic Convection of Magnetic Fluid in an Annular Space under a Non-Uniform Magnetic and Thermal Field. *Applied Mathematics*, **8**, 655-662.
https://doi.org/10.4236/am.2017.85052

Received: February 12, 2017
Accepted: May 20, 2017
Published: May 23, 2017

Abstract

Two-dimensional thermomagnetic convection of magnetic fluid possessing internal spin and relaxation of magnetization with high thermal sensitivity is numerically analyzed under a non-uniform magnetic and thermal field, using a spectral difference scheme, magnetic fluid is assumed to be placed between concentric cylinders, an azimuthal magnetic field being produced by an electric current though the inner cylinder, which is an adiabatic wall, whereas a half of the outer cylinder is kept at a high constant temperature and the rest half is at a lower constant temperature. As a result, in case of $\nabla T \times \nabla |H| \neq 0$, thermomagnetic convection is found to be produced, the convection pattern depends on electric current distribution, depending on which multiple or single circulation is produced, thus, it shows that thermomagnetic convection pattern is controlled by changing relative direction of temperature gradient to that of a magnetic field.

Keywords

Magnetic Fluid, Thermomagnetic Convection

1. Introduction

A familiar example of thermal convection is buoyancy-induced convection, where a driving body force resides in a force of gravity, another example is established in a heated magnetic fluid subject to a magnetic body force, which depends on a thermodynamic constitutive equation. The enhancement of convective heat transfer in magnetic fluids could be applied in cooling current-carrying conductors, and transmissionlines, especially in gravity-free space. Characteristics of thermomagnetic convection are investigated by many researchers, e.g. Finlayson (1970) [1], Polevikov & Fertman (1977) [2], Busse & Riahi (1982) [3], Stiles &

Kagan (1990) [4], Zebib (1996) [5], Fruh (2005) [6], Yamaguchi (2009) [7] and Siddiqa (2013) [8].

In this paper, two-dimensional thermomagnetic convection of magnetic fluid possessing internal spin and relaxation of magnetization with high thermal sensitivity is numerically analyzed under a non-uniform magnetic and thermal field, using a spectral finite difference schemes, and the thermomagnetic convection pattern of magnetic fluid within an annulus is numerical investigated with spatial distributions of magnetic field variations.

2. Analysis

2.1. Physical Model

Analyzed is Two-dimensional thermomagnetic convection followed by a steady-state natural convection of magnetic fluid as shown in **Figure 1**, where a conductive wire of radius R_0 with a direct current I_0 is placed in the center of an annular space filled with a magnetic fluid of magnetic permeability μ_0. It is assumed that magnetic fluid is magnetized by a magnetic field $\boldsymbol{H}$ due to an external magnetic field from a wire current and an induction magnetic field of the magnetic fluid, thosemagnetic fluid is assumed to be at rest up to time $t = 0$ with a uniform temperature T_b, and at $t > 0$ a half part of the outside circumference is assumed to be fixed at a high temperature (T_h), and the rest half part of the outside circumference at a low temperature(T_l).

The following assumption applies:

1) The radius of wire is much smaller than that of the inside circle ($R_0 \ll R_1$).

2) The effect of magnetic field is limited within the distance $50R_2$.

3) The magnetic permeability of magnetic fluid is equal to that of vacuum.

4) The magnetic fluid is incompressible fluid.

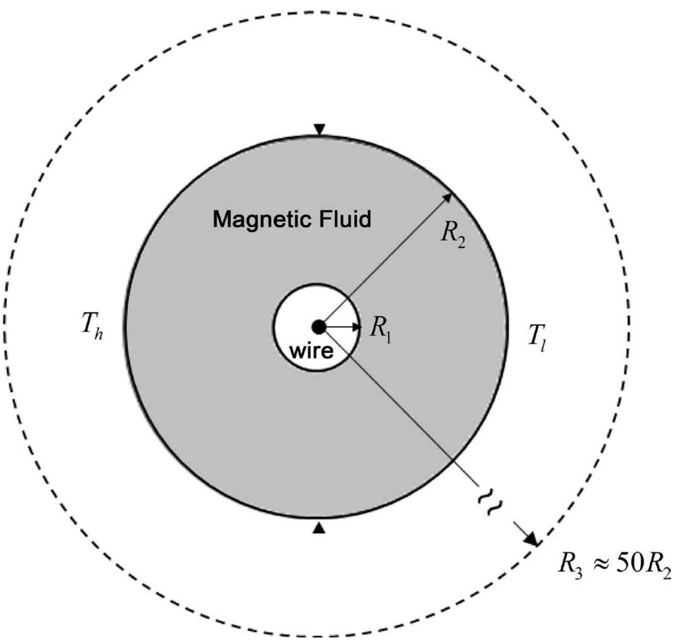

Figure 1. The analytical region.

2.2. Governing Equations of an Incompressible Magnetic Fluid

The equation of continuity is

$$\nabla \cdot \boldsymbol{v} = 0, \tag{1}$$

The momentum equation neglecting gravity force is

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p + \eta \nabla^2 \boldsymbol{v} + \nabla \cdot (\boldsymbol{BH}) + \frac{I}{t_s} \nabla \times (\boldsymbol{\Omega} - \boldsymbol{\omega}), \tag{2}$$

where t is time, $\boldsymbol{v}$: velocity, ρ: mass density of magnetic fluid, p: pressure, η: viscosity of magnetic fluid in absence of a magnetic field, I: average inertia moments of particles per unit volume, t_s: the relaxation time of internal spin rotation, $\boldsymbol{H}$: magnetic field, $\boldsymbol{B}$: magnetic induction, $\boldsymbol{\Omega}$: internal spin rate, $\boldsymbol{\omega}$: effective rate of rotation of a fluid element $\equiv (1/2)\nabla \times \boldsymbol{v}$.

The internal angular momentum equation is

$$I \frac{D\boldsymbol{\Omega}}{Dt} = \gamma I \nabla^2 \boldsymbol{\Omega} - \frac{I}{t_s} (\boldsymbol{\Omega} - \boldsymbol{\omega}) + \mu_0 \boldsymbol{M} \times \boldsymbol{H}, \tag{3}$$

where γ is a dissipation coefficient of inertia spin moment, $\boldsymbol{M}$: magnetization of the magnetic fluid. The magnetization relaxation equation, Shliomis (1974) [9], is

$$\frac{D\boldsymbol{M}}{Dt} = \boldsymbol{\Omega} \times \boldsymbol{M} - \frac{1}{t_b} (\boldsymbol{M} - \boldsymbol{M}_0), \tag{4}$$

where t_b is a relaxation time of the particle rotation by Brownian rotation motion, and $\boldsymbol{M}_0$ is the equilibrium magnetization of the magnetic fluid, and magnitude is given by

$$|\boldsymbol{M}_0|(|\boldsymbol{H}|, T) = \phi M_s(T) \mathrm{L}\left(\frac{\pi \mu_0 M_s(T) d^3 |\boldsymbol{H}|}{6kT} \right), \tag{5a}$$

where $\mathrm{L}(x) = \coth x - 1/x$ is a Langevin function, T: temperature, k: Boltzmann constant, d: diameter of the particle, ϕ: volume fraction of ferromagnetic particles, M_s: magnetic moment of a particle, if $T \le 0.8 T_c$ (T_c: temperature of Curie point),

$$M_s(T) \approx M_s(0)\left(1 - \frac{kT}{nM_B H_M}\right), \tag{5b}$$

where M_B: Bohrmagneton, n: number of spin perunitatom of ferromagntic metal, H_M: molecular field.

The energy equation can be derived by Cowley and Rosensweig [10] as

$$\rho C_m \frac{DT}{Dt} - \mu_0 \rho TK \left(\frac{D|\boldsymbol{H}|}{Dt} \right) = k_1 \nabla^2 T + \Phi, \tag{6}$$

where C_m is the heat capacity at constant volume with a magnetic field, K: pyromagnetic coefficient of magnetic fluid, k_1: the thermal conductivity, Φ: a viscous dissipation term usually negligible. The Maxwell's equation for a non-conducting fluid with no displacement current becomes

$$\nabla \times \boldsymbol{H} = 0, \ \nabla \cdot \boldsymbol{B} = 0, \tag{7}$$

$$\boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M} \right). \tag{8}$$

2.3. Coordinate System

Cartesian coordinate (x, y) is fixed in the center of an annular pipe, the y-axis being vertically upward, Relationship between cylindrical and Cartesian coordinates is

$$x + iy = Re^{i\theta}, \tag{9}$$

where, a spectral finite difference method is applied for solving Equation (1) - Equation (8), and primary variables are expressed in a Fourier series to the circumferential θ-direction.

2.4. Equation of the Distribution of Magnetic Field

Introduced are a magnetic induction stream functions, ψ_i^m:

$$B_{ix} = \frac{\partial \psi_i^m}{\partial y}, \quad B_{iy} = -\frac{\partial \psi_i^m}{\partial x}, \tag{10}$$

$$\psi_i^m \equiv \overline{\psi_i^m} - \frac{\mu_0 I_0}{2\pi} \ln R, \tag{11}$$

where $i = 1, 2, 3$ correspond to the following three regions:

- $R_0 \le R \le R_1$:

$$\nabla^2 \overline{\psi_1^m} = 0, \tag{12a}$$

- $R_1 \le R \le R_2$:

$$\frac{1}{\mu_0} \nabla^2 \overline{\psi_2^m} + \left(\frac{M_\theta}{R} + \frac{\partial M_\theta}{\partial R} - \frac{1}{R} \frac{\partial M_R}{\partial \theta} \right) = 0, \tag{12b}$$

- $R_2 \le R \le 50R_2$:

$$\nabla^2 \overline{\psi_3^m} = 0. \tag{12c}$$

where M_R and M_θ are R- and θ-component of magnetization of the magnetic fluid, **M**.

2.5. Boundary Conditions

The boundary conditions used are as follows:

- At $R = R_1$, no-slip, no-spin, and thermal insulation:

$$[\boldsymbol{v}]_{R=R_1} = 0, \quad [\boldsymbol{\Omega}]_{R=R_1} = 0, \quad \left(\frac{\partial T}{\partial R} \right)_{R=R_1} = 0, \tag{13}$$

- At R=R_2, no-slip, no-spin, and Dirichlet temperature condition:

$$[\boldsymbol{v}]_{R=R_2} = 0, \quad [\boldsymbol{\Omega}]_{R=R_2} = 0, \tag{14}$$

$$T\left(\theta, R_2\right) = T_h \quad \text{for } \theta \in \left(\frac{\pi}{2}, \frac{3}{2}\pi \right), \tag{15a}$$

$$T\left(\theta, R_2\right) = T_l \quad \text{for } \theta \in \left[0, \frac{\pi}{2} \right] \cup \left[\frac{3}{2}\pi, 2\pi \right]. \tag{15b}$$

The normal component of the magnetic induction and the tangential component of the magnetic field are continuous across the interface between dissimilar regions,

$$\text{at } R = R_1 : [B_1]_n = [B_2]_n, \ [H_1]_t = [H_2]_t, \tag{16a}$$

$$\text{at } R = R_2 : [B_2]_n = [B_3]_n, \ [H_2]_t = [H_3]_t, \tag{16b}$$

where the subscript n and t denote normal and tangential component. Then the boundary condition of Equations 12(a)-(c) are given by

$$\overline{\varphi_1^m} = 0, \quad \frac{\partial \overline{\varphi_1^m}}{\partial R} = 0 \ (R = 0), \tag{17a}$$

$$\frac{\partial \overline{\varphi_1^m}}{\partial R} = \frac{\partial \overline{\varphi_2^m}}{\partial R} + \mu_0 M_\theta \ (R = R_1), \tag{17b}$$

$$\frac{\partial \overline{\varphi_3^m}}{\partial R} = \frac{\partial \overline{\varphi_2^m}}{\partial R} + \mu_0 M_\theta \ (R = R_2), \tag{17c}$$

$$\overline{\varphi_3^m} = 0 \ (R = 50R_2). \tag{17d}$$

3. Results

Assuming that the magnetic particle of magnetic fluid is iron oxide, and that the carrier liquid is hydrocarbon, the following constants and parameters of magnetic fluid are used

$C_m = 2.09\times10^3\ \mathrm{J\cdot K^{-1}\cdot kg^{-1}}$, $d = 1.0\times10^{-8}\ \mathrm{m}$, $H_M = 1.7\times10^9\ \mathrm{A\cdot m^{-1}}$, $I = 1.0\times10^{-4}\ \mathrm{kg\cdot m^{-1}}$, $k = 1.38\times10^{-23}\ \mathrm{J\cdot K^{-1}}$, $k_1 = 0.22\ \mathrm{J\cdot K^{-1}\cdot m^{-1}\cdot s^{-1}}$, $K = 220\ \mathrm{A\cdot m^{-1}\cdot K^{-1}}$, $M_B = 1.17\times10^{-29}\ \mathrm{A\cdot m^{-1}}$, $M_s(0) = 3.18\times10^4\ \mathrm{A\cdot m^{-1}}$, $n = 2.2$, $\phi = 0.041$, $\eta = 0.2\ \mathrm{Pa\cdot s}$, $t_b = 7.6\times10^{-7}\ \mathrm{s}$, $t_s = 4.17\times10^{-11}\ \mathrm{s}$, $\mu_0 = 4\pi\times10^{-7}\ \mathrm{H\cdot m^{-1}}$, $\rho = 1.22\times10^3\ \mathrm{kg\cdot m^{-3}}$.

3.1. Magnetic Body Force of Magnetic Fluid

Since the relaxation time t_s ($\sim10^{-11}$ s) of internal spin rotation and the dissipation coefficient γ ($\approx t_b/2 \sim 10^{-7}\ \mathrm{s^{-1}}$) of inertia spin moment is very small, Equation (3) can be reduced as

$$\frac{I}{t_s}(\boldsymbol{\Omega} - \boldsymbol{\omega}) = \mu_0 \boldsymbol{M} \times \boldsymbol{H}, \tag{18}$$

and substituting Equation (18) into Equation (2) leads to an approximate momentum equation:

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p + \eta \nabla^2 \boldsymbol{v} + \mu_0 (\boldsymbol{M}\cdot\nabla)\boldsymbol{H}, \tag{19}$$

and then the curl of the magnetic body force is given by

$$\nabla \times \mu_0 (\boldsymbol{M}\cdot\nabla)\boldsymbol{H} = \mu_0 \frac{\partial \chi}{\partial T}\left[\nabla T \times (\boldsymbol{H}\cdot\nabla)\boldsymbol{H}\right], \tag{20}$$

where χ $(= \chi(T, |\boldsymbol{H}|))$ is the magnetic susceptibility of magnetic fluid, and the magnetization of the magnetic fluid can be written as $\boldsymbol{M} = \chi\boldsymbol{H}$. As for the Equation (20), the producing condition of thermomagnetic convection is derived in the gravity-free space as $\nabla T \times \nabla|\boldsymbol{H}| \neq 0$.

3.2. Numerical Results

In the case of $I_0 = 10$ A, $T_h - T_l = 20$ K and $R_1/R_2 = 0.2$, the isotherms, streamlines and dimensionless magnetization field strengths with the position of electric current wire varies are displayed as shown in Figures 2-4, where all quantities are made dimensionless base on a length L_0 $(=R_2)$, a temperature T_0 $(=T_h - T_l)$, and reference velocity V_0 $(=H_0\sqrt{\mu_0/\rho})$, H_0 $\left(=\dfrac{I_0}{2\pi L_0}\right)$ is representative magnetic field. It shows that the convection pattern depends on electric current distribution, depending on which multiple or single circulation is produced. Thus, it is found that thermomagnetic convection is controlled by changing relative direction of temperature gradient to that of a magnetic field.

4. Conclusions

1) For producing thermomagnetic convection of a magnetic fluid, a spatially non-uniform of temperature and a spatially non-uniform of external magnetic field are required so that $\nabla T \times \nabla |\boldsymbol{H}| \neq 0$.

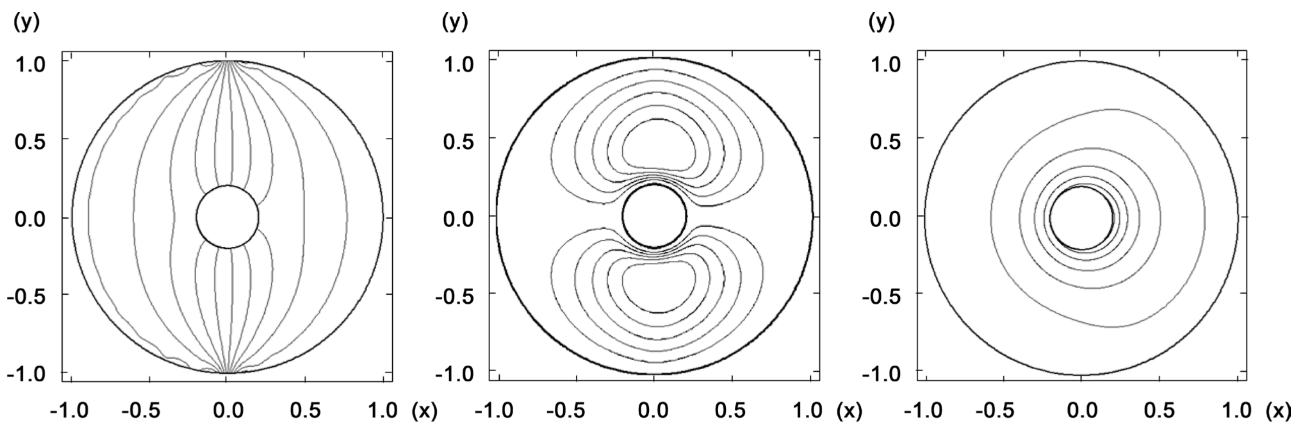

Figure 2. Isotherms (at an interval of 0.1), streamlines(at an interval of 1.0×10^{-7} stream function) and dimensionless magnetization field strengths (at an interval of 1.0×10^{-3} with from 0.001 to 0.007) in the case of the electric current wire fixed at $(x, y) = (0, 0)$, where the direction of velocity along the center is leftward.

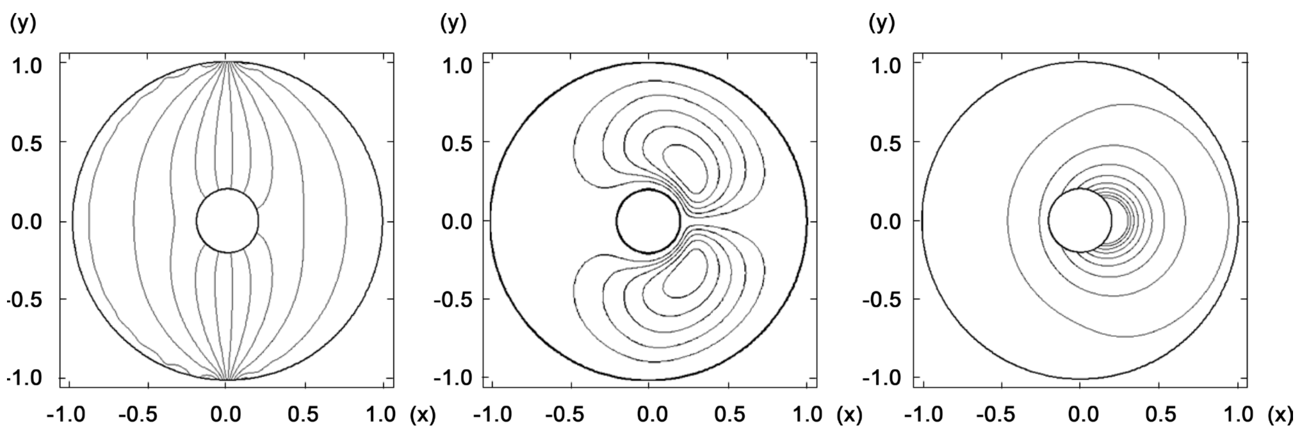

Figure 3. Isotherms (at an interval of 0.1), streamlines (at an interval of 2.0×10^{-8} stream function) and dimensionless magnetization field strengths (at an interval of 2.0×10^{-3} with from 0.002 to 0.02) in the case of the electric current wire fixed at $(x, y) = (0.15, 0)$, where the direction of velocity along the center is leftward.

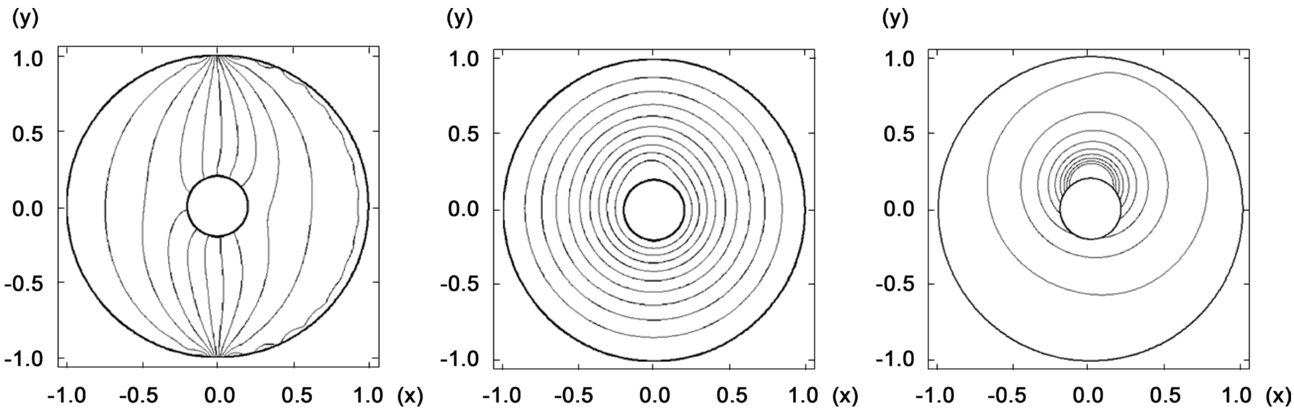

Figure 4. Isotherms (at an interval of 0.1), streamlines (at an interval of 1.0×10^{-7} stream function) and dimensionless magnetization field strengths (at an interval of 2.0×10^{-3} with from 0.002 to 0.02) in the case of the electric current wire fixed at $(x, y) = (0, 0.15)$, where the direction of flow is clockwise.

2) In the case that the electric current wire is fixed in center of annular pipe, two symmetrical circulating flows are produced.

3) If the position of electric current wire is different from the center, the center of the circulating flows moves to the side of lower or higher temperature depending on whether the location of electric current wire is to the lower or higher temperature side. Especially in the case of the position of electric current wire deviates greatly from the median line of temperature, only one circulating flow zone is produced.

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Nomencalature

$\boldsymbol{B}$: magnetic induction

C_m: heat capacity at constant volume with a magnetic field

d: diameter of the particle

$\boldsymbol{H}$: magnetic field

$\boldsymbol{H}_0$: external magnetic field

H_M: molecular field

I: average inertia moments of particles per unit volume

I_0: a direct current

k: Boltzmann constant

k_1: thermal conductivity

K: pyromagnetic coefficient of magnetic fluid

$\boldsymbol{M}$: magnetization of the magnetic fluid

$\boldsymbol{M}_0$: equilibrium magnetization of the magnetic fluid

M_B: Bohr magneton

M_s: magnetic moment of a particle

n: number of spin per unit atom of ferromagntic metal

p: pressure

R_0: radius of conductive wire

R, θ: cylindrical coordinate system

t: time

T: temperature

T_c: temperature of Curie point

t_b: relaxation time of the particle rotation by Brownian rotation motion

t_s: relaxation time of internal spin rotation

$\boldsymbol{v}$: velocity

x, y: Cartesian coordinate system

γ: dissipation coefficient of inertia spin moment

η: viscosity of magnetic fluid in absence of a magnetic field

μ_0: permeability of the magnetic fluid

ρ: mass density of magnetic fluid

ψ : stream function

ψ_i^m : a magnetic induction stream functions

ϕ : volume fraction of ferromagnetic particles

Φ: viscous dissipation term

χ: magnetic susceptibility of magnetic fluid

$\boldsymbol{\omega}$: effective rate of rotation of a fluid element

$\boldsymbol{\Omega}$: internal spin rate