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Multigrid Method for the Numerical Solution of the Modified Equal Width Wave Equation

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Abstract

Numerical solutions of the modified equal width wave equation are obtained by using the multigrid method and finite difference method. The motion of a single solitary wave, interaction of two solitary waves and development of the Maxwellian initial condition into solitary waves are studied using the proposed method. The numerical solutions are compared with the known analytical solutions. Using L_2, L_∞ error norms and conservative properties of mass, momentum and energy, accuracy and efficiency of the mentioned method will be established through comparison with other methods.

Keywords

Multigrid Method, Finite Difference Method, MEW Equation

1. Introduction

A large system of equations comes out from discretization of the domain of partial differential equations into a collection of points and the optimal method for solving these problems is multigrid method, see [1]-[4].

The modified equal width wave (MEW) equation introduced by Morrison *et al*. [5] is used as a model equation to describe the nonlinear dispersive waves. Gardner and Gardner [6] [7] solved the EW equation with the Galerkin's method using cubic B-splines as a trial and test function. The MEW equation was similar with the modified regularized long wave (MRLW) equation [8] and modified Korteweg-de Vries (MKdV) equation [9]. All the modified equations are nonlinear wave equations with cubic nonlinearities and all of them have solitary wave solutions, which are wave packets or pulses. These waves propagate in non-linear media by keeping wave forms and velocity even after interaction occurs.

Several solutions for MEW had been proposed in [10]-[22]. In Geyikli and Battal Gazi Karakoc [10] [11], the

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solutions are based on septic B-spline finite elements and Petrov-Galerkin finite element method with weight functions quadratic and element shape functions which are cubic B-splines. Esen [12] [13] solved the MEW equation by applying a lumped Galerkin method based on quadratic B-spline finite elements. Saka [14] proposed algorithms for the numerical solution of the MEW equation using quintic B-spline collocation method. Zaki [15] considered the solitary wave interactions for the MEW equation by collocation method using quintic B-spline finite elements and obtained the numerical solution of the EW equation by using least-squares method [16]. Wazwaz [17] investigated the MEW equation and two of its variants by the tanh and the sine-cosine methods. A solution based on a collocation method incorporated cubic B-splines is investigated by Saka and Dağ [18]. Lu [19] presented a variational iteration method to solve the MEW equation. Evans and Raslan [20] studied the generalized EW equation by using collocation method based on quadratic B-splines to obtain the numerical solutions of a single solitary waves and the birth of solitons. Esen and Kutluay [21] studied a linearized implicit finite difference method in solving the MEW equation. Battal Gazi Karakoc and Geyikli [22] solved the MEW equation by a lumped Galerkin method using cubic B-spline finite elements.

An outline of this paper is as follows: We begin in Section 2 by reviewing the analytical solution of the MEW equation. In Section 3, we derive a new numerical method based on the multigrid technique and finite difference method for obtaining the numerical solution of MEW equation. Finally, in Section 4, we introduce the numerical results for solving the MEW equation through some well known standard problems.

2. The Analytical Solution

The modified equal width wave equation which is as a model for non-linear dispersive waves, considered here has the normalized form [5]

$$u_t + 3u^2 u_x - \mu u_{xxt} = 0, \tag{1}$$

with the physical boundary conditions $u \to 0$ as $x \to \pm\infty$, where t is time and x is the space coordinate, μ is a positive parameter. For this study boundary conditions are chosen

$$\begin{aligned} &u(a,t) = 0,\ u(b,t) = 0, \\ &u_x(a,t) = 0,\ u_x(b,t) = 0, \\ &u_{xxt}(a,t) = 0,\ u_{xxt}(b,t) = 0, \end{aligned} \tag{2}$$

and the initial condition as

$$u(x,0) = f(x),\ a \le x \le b,$$

where f is a localized disturbance inside the considered interval.

The exact solution of equation (1) can be written in the form [15]

$$u(x,t) = A\operatorname{sech}\left(P\left(x - x_0 - vt\right)\right), \tag{3}$$

which represents the motion of a single solitary wave with amplitude A, where the wave velocity $v = A^2/2$ and $P = \sqrt{1/\mu}$. The initial condition is given by

$$u(x,0) = A\operatorname{sech}\left(P\left(x - x_0\right)\right) \tag{4}$$

For the MEW equation, it is important to discuss the following three invariant conditions given in [15], which, respectively, correspond to conversation of mass, momentum, and energy. The analytical values of the invariants are

$$C_1 = \frac{A\pi}{P},\quad C_2 = \frac{2A^2}{P} + \frac{2\mu PA^2}{3},\quad C_3 = \frac{4A^4}{3P}. \tag{5}$$

3. Numerical Method

The basic idea of multigrid techniques is illustrated by Brandt [1]. In this section we apply this method for initial boundary value problem, except that, the upper boundary conditions change with time, in which the initial condition is $u(x,0) = f(x)$ for $0 < t < T$. Dividing the interval of time to K parts, we obtain the solutions of the partial differential equation at time t_1 and use these solutions as initial values for the next level $u(x,0)=u(x,t_1)$, and for the other, we obtain the solutions at time T. The numbers of points in a coarse grid for this domain are two points.

We apply the full multigrid algorithm for the MRLW equation. Assuming the initial condition $u(x,0)=f(x)$ and the solution $u(x,t)$, $a\le x\le b, 0\le t\le T$ has the usual partition with a space step size Δx and a time step size Δt $(t_{K+1}=t_K+\Delta t, K=0,1,2,\cdots)$.

We start handling the non-linear term $3u^2u_x$ by expressing in the form $\frac{\partial u^3}{\partial x}$. The back-time and centre-space difference for Equation (1) is

$$\frac{u_{i,n}^k-u_{i,n-1}^k}{\Delta t}+\frac{\left(u_{i+1,n}^k\right)^3-\left(u_{i-1,n}^k\right)^3}{2\Delta x}-\mu\frac{\left(u_{i+1,n}^k-u_{i+1,n-1}^k\right)-2\left(u_{i,n}^k-u_{i,n-1}^k\right)+\left(u_{i-1,n}^k-u_{i-1,n-1}^k\right)}{(\Delta x)^2(\Delta t)}=0, \tag{6}$$

where $i=1,\cdots,2^k-1, n=1,\cdots,2^k$, $k=1,\cdots,M$ for a set grids $G^1,G^2,\cdots,G^M$.

Step 1: $K=0, u(x,0)=f(x)$.

Step 2: Starting from $k=1$ in the coarse grid, we can calculate the approximate value $u_{i,n}$ at two points using Equation (5) leading to:

$$\begin{aligned} u_{i,n}^1 &= u_{i,n-1}^1+\frac{1}{\left(2(\Delta x)^2+4\mu\right)}\Big[2\mu\left(u_{i+1,n}^1+u_{i-1,n}^1-u_{i+1,n-1}^1-u_{i-1,n-1}^1\right) \\ &\quad -(\Delta x)(\Delta t)\left(\left(u_{i+1,n}^1\right)^3-\left(u_{i-1,n}^1\right)^3\right)\Big]; \qquad i=1,\ n=1,2. \end{aligned} \tag{7}$$

The right hand side for equation (7) can be computed using the initial and boundary conditions.

Step 3: Interpolating the grid functions from the coarse grid to fine grid using linear interpolation I_k^{k+1}, in which

$$u^{k+1}=I_k^{k+1}u^k, \tag{8}$$

that can be written explicitly as:

$$\begin{aligned} &u_{2i,2n}^{k+1}=u_{i,n}^k; && i=1,\cdots,2^k-1,\ n=1,\cdots,2^k, \\ &u_{2i+1,2n}^{k+1}=0.5\left(u_{i,n}^k+u_{i+1,n}^k\right); && i=0,\cdots,2^k-1,\ n=1,\cdots,2^k, \\ &u_{2i,2n+1}^{k+1}=0.5\left(u_{i,n}^k+u_{i,n+1}^k\right); && i=1,\cdots,2^k-1,\ n=0,\cdots,2^k-1, \\ &u_{2i+1,2n+1}^{k+1}=0.25\left(u_{i,n}^k+u_{i+1,n}^k+u_{i,n+1}^k+u_{i+1,n+1}^k\right); && i,n=0,\cdots,2^k-1. \end{aligned} \tag{9}$$

Step 4: Doing relaxation sweep on G^{k+1} using the point relaxation

$$\begin{aligned} u_{i,n}^{k+1} &= u_{i,n-1}^k+\frac{1}{\left(2(\Delta x)^2+4\mu\right)}\Big[2\mu\left(u_{i+1,n}^k+u_{i-1,n}^k-u_{i+1,n-1}^k-u_{i-1,n-1}^k\right) \\ &\quad -(\Delta x)(\Delta t)\left(\left(u_{i+1,n}^k\right)^3-\left(u_{i-1,n}^k\right)^3\right)\Big]; \qquad i=1,\cdots,2^{k+1}-1,\ n=1,\cdots,2^{k+1}. \end{aligned} \tag{10}$$

Step 5: Computing the residuals r^{k+1} on G^{k+1} and inject them into G^k using full weighting restriction I_{k+1}^k to get r^k as:

$$r^k=I_{k+1}^k r^{k+1}, \tag{11}$$

$$\begin{aligned} r_{i,n}^k &= \frac{1}{16}\Big[r_{2i-1,2n-1}^{k+1}+r_{2i-1,2n+1}^{k+1}+r_{2i+1,2n-1}^{k+1}+r_{2i+1,2n+1}^{k+1} \\ &\quad +2\left(r_{2i,2n-1}^{k+1}+r_{2i,2n+1}^{k+1}+r_{2i-1,2n}^{k+1}+r_{2i+1,2n}^{k+1}\right)+4r_{2i,2n}^{k+1}\Big]; \quad i,n=1,\cdots,2^k-1. \end{aligned} \tag{12}$$

Step 6: Computing an approximate solution of error e^k.

Step 7: Interpolating the solution of error e^k onto G^{k+1}, $e^{k+1} = I_k^{k+1} e^k$, and adding it to u^{k+1} which is the approximate value of u on the fine grid with $k = 2$.

By taking this solution on coarse grid and repeating steps 3-7, we obtain the approximate values of u on the grid with $k = 3$ and so $k = 4, 5, \cdots, M$ the final value is the solution at the time level $K + 1$.

Step 8: $K = K + 1$, go to step 2 (lead to the solution at higher time level as needed).

4. Numerical Results

In this section, numerical solutions of MRLW equation are obtained for standard problems as: the motion of single solitary wave, interaction of two solitary waves and development of Maxwellian initial condition into solitary waves. For the MEW equation, it is important to discuss the following three invariant conditions given in [15], which respectively correspond to conversation of mass, momentum and energy:

$$\begin{aligned} C_1 &= \int_a^b u \mathrm{d}x = \Delta x \sum_{i=1}^{N} u_{i,n}, \\ C_2 &= \int_a^b \left(u^2 + \mu (u_x)^2\right) \mathrm{d}x = \Delta x \sum_{i=1}^{N} \left((u_{i,n})^2 + \mu \left((u_x)_{i,n} \right)^2 \right), \\ C_3 &= \int_a^b u^4 \mathrm{d}x = \Delta x \sum_{i=1}^{N} (u_{i,n})^4. \end{aligned} \tag{13}$$

The accuracy of the method is measured by both the L_2 error norm

$$L_2 = \left\| u^{exact} - u_N \right\|_2 = \sqrt{\Delta x \sum_{i=0}^{N} \left| u_i^{exact} - (u_N)_i \right|^2}, \tag{14}$$

and the L_∞ error norm

$$L_\infty = \left\| u^{exact} - u_N \right\|_\infty = \max_i \left| u_i^{exact} - (u_N)_i \right|, \tag{15}$$

to show how good the numerical results in comparison with the exact results.

4.1. The Motion of Single Solitary Wave

Consider Equation (1) with boundary conditions (2) and the initial condition (4). For a comparison with earlier studies [13] [19] [21] [22] we take the parameters $\Delta x = 0.1, \Delta t = 0.05, \mu = 1, x_0 = 30$ and $A = 0.25$ over the interval [0, 80]. To find the error norms L_2, L_∞ and the numerical invariants C_1, C_2 and C_3 at various times we use the numerical solutions by applying the multigrid method up to $t = 20$. As reported in **Table 1**, the error norms L_2, L_∞ are found to be small enough, and the computed values of invariants are in good agreement with their analytical values $C_1 = 0.7853982, C_2 = 0.1666667, C_3 = 0.0052083$. **Table 2** shows a comparison of the values of the invariants and error norms obtained by the present method with those obtained by other methods [13] [19] [21] [22]. It is clearly seen from **Table 2** that the error norms obtained by the present method are smaller than the other methods.

4.2. Interaction of Two Solitary Waves

Consider the interaction of two positive solitary waves as a second problem. For this problem, the initial condition is given by:

$$u(x, 0) = \sum_{j=1}^{2} A_j \operatorname{sech}\left(P\left(x - x_j\right)\right). \tag{16}$$

For the computational discussion, firstly we use parameters $\Delta x = 0.1, \Delta t = 0.025, \mu = 1, A_1 = 1, A_2 = 0.5, x_1 = 15$ and $x_2 = 30$ over the range $[0, 80]$ to coincide with those used in [22].

In [20] the analytic invariants are $C_1 = \pi(A_1 + A_2) = 4.7123889$, $C_2 = (8/3)(A_1^2 + A_2^2) = 3.3333333$, $C_3 = (4/3)(A_1^4 + A_2^4) = 1.4166667$. The experiment is run from $t = 0$ to $t = 55$ and values of the invariant quantities C_1, C_2 and C_3 are listed in **Table 3**.

Table 3 shows a comparison of the values of the invariants obtained by present method with those obtained in

Table 1. Invariants and error norms for single solitary wave when $A = 0.25$, $\Delta x = 0.1, \Delta t = 0.05, 0 \le x \le 80$.

t	C_1	C_2	C_3	$L_2 \times 10^5$	$L_\infty \times 10^5$
0	0.7853966199	0.1666662968	0.005208333331	0.000000000	0.000000
2	0.7853966246	0.1666660511	0.005208317956	0.0518705479	0.05440
4	0.7853966176	0.1666658044	0.005208302547	0.1038794545	0.10890
6	0.7853966097	0.1666655554	0.005208286962	0.1560469898	0.16359
8	0.7853966066	0.1666653078	0.005208271505	0.2080329043	0.21810
10	0.7853966012	0.1666650571	0.005208255823	0.2601313073	0.27283
12	0.7853965918	0.1666648091	0.005208240334	0.3122731279	0.32747
14	0.7853965793	0.1666645594	0.005208224692	0.3643751855	0.38216
16	0.7853965769	0.1666643124	0.005208209260	0.4164201991	0.43656
18	0.7853965785	0.1666640667	0.005208193877	0.4684782742	0.49095
20	0.7853965668	0.1666638167	0.005208178255	0.5208044265	0.54566

Table 2. Comparison of errors and invariants for single solitary wave at $t = 20$.

Method	C_1	C_2	C_3	$L_2 \times 10^5$	$L_\infty \times 10^5$
Analytical	0.7853982	0.1666667	0.0052083	0	0
Present	0.7853966	0.1666638	0.0052082	0.520804	0.54566
[13]	0.7853898	0.1667614	0.0052082	7.969400	4.65523
[19]	0.7849545	0.1664765	0.0051995	29.05166	24.98925
[21]	0.7853977	0.1664735	0.0052083	26.92812	25.69972
[22]	0.7853967	0.1666663	0.0052083	8.009800	4.606180

Table 3. Comparison of invariants for the interaction of two solitary waves with results from [22] ($\Delta x = 0.1, \Delta t = 0.025$, $A_1 = 1, A_2 = 0.5, 0 \le x \le 80$).

	Present method			[22]		
t	C_1	C_2	C_3	C_1	C_2	C_3
0	4.712379141	3.333328364	1.416669724	4.7123732	3.3333253	1.4166643
5	4.712378542	3.333075164	1.416419304	4.7123861	3.3333482	1.4166852
10	4.712378533	3.332822094	1.416169046	4.7123959	3.3333621	1.4166982
15	4.712378539	3.332569179	1.415918945	4.7124065	3.3333785	1.4167141
20	4.712378504	3.332316280	1.415668885	4.7124249	3.3334164	1.4167521
25	4.712378509	3.332063538	1.415418955	4.7124899	3.3335832	1.4169238
30	4.712378541	3.331810944	1.415169189	4.7127643	3.3333557	1.4177617
35	4.712378593	3.331558498	1.414919571	4.7130474	3.3352500	1.4188849
40	4.712378583	3.331306069	1.414669976	4.7124881	3.3336316	1.4171690
45	4.712378540	3.331053726	1.414420484	4.7123002	3.3331878	1.4167580
50	4.712378546	3.330801521	1.414171139	4.7122479	3.3330923	1.4167142
55	4.712378563	3.330632678	1.413975397	4.7122576	3.3331149	1.4167237

[22]. It is seen that the numerical values of the invariants remain almost constant during the computer run.

Finally, we have studied the interaction of two solitary waves with the following parameters $\Delta x = 0.1$, $\Delta t = 0.025, \mu = 1, A_1 = -2, A_2 = 1, x_1 = 15$ and $x_2 = 30$ in the range [0,150].

The analytical invariants can be found as in [22] $C_1 = -3.1415927$, $C_2 = 13.3333333$, $C_3 = 22.6666667$. The experiment is run from t = 0 to t = 55 and values of the invariant quantities C_1, C_2 and C_3 are listed in **Table 4**.

4.3. The Maxwellian Initial Condition

Last study, we consider the numerical solution of the equation (1) with the Maxwellian initial condition

$$u(x,0) = e^{-x^2}, \tag{17}$$

and the boundary conditions $u(-20,t) = u_x(-20,t) = u(20,t) = u_x(20,t) = 0$.

Table 4. Invariants for the interaction of two solitary waves ($\Delta x = 0.1, \Delta t = 0.025, A_1 = -2, A_2 = 1, 0 \le x \le 150$).

t	C_1	C_2	C_3
0	−3.141588324	13.33240988	22.66661773
5	−3.141587221	13.31632255	22.60298870
10	−3.141587293	13.30034113	22.53980538
15	−3.141587369	13.28446423	22.47706277
20	−3.141587465	13.26869077	22.41475674
25	−3.141587571	13.25301907	22.35288150
30	−3.141587642	13.23744806	22.29143237
35	−3.141587711	13.22197615	22.23040435
40	−3.141587744	13.20660137	22.16979117
45	−3.141587842	13.19132246	22.10958842
50	−3.141587954	13.17613770	22.04979002
55	−3.141587989	13.15897649	22.01765488

Table 5. Invariants of MEW equation using the Maxwelliancondition $\mu = 1, 0.5, 0.1, 0.05, 0.02, 0.005$.

t	μ	C_1	C_2	C_3	μ	C_1	C_2	C_3
0		1.772450389	2.507031350	0.8862269258		1.772450389	2.507031350	0.8862269258
3		1.772450324	2.506562241	0.8859617965		1.772450391	2.503301389	0.8812594008
6	1	1.772450355	2.506093335	0.8856969273	0.05	1.772450389	2.508930167	0.8763574457
9		1.772450370	2.505624532	0.8854322207		1.772450398	2.523558486	0.874805062
12		1.772450368	2.505155693	0.8851675910		1.772450397	2.546856704	0.8665905426
0		1.772450389	2.507031350	0.8862269258		1.772450389	2.507031350	0.8862269258
3		1.772450359	2.505856717	0.8855663594		1.772450391	2.505380411	0.8790965969
6	0.5	1.772450306	2.504698981	0.8849067496	0.02	1.772450395	2.523617080	0.8721185562
9		1.772450294	2.503557663	0.882481160		1.772450301	2.561417967	0.8651660441
12		1.772450275	2.502431939	0.8835903023		1.772450306	2.587705888	0.8581212805
0		1.772450389	2.507031350	0.8862269258		1.772450389	2.507031350	0.8862269258
3		1.772450382	2.503247004	0.8830002438		1.772450388	2.509358370	0.8771798047
6	0.1	1.772450373	2.503178816	0.8797994434	0.005	1.772450389	2.545688708	0.8684172340
9		1.772450375	2.506705762	0.8766134367		1.772450390	2. 619139180	0.8596519203
12		1.772450380	2.513694005	0.8734315977		1.772450390	2. 671737369	0.8551833457

It is known that the behavior of the solution with the Maxwellian condition (17) depends on the values of μ. So we have considered various values for μ. The computations are carried out for the cases $\mu = 1, 0.5, 0.1, 0.05, 0.02$ and 0.005 which are used in the earlier papers [15] [19]. The numerical conserved quantities with $\mu = 1, 0.5, 0.1, 0.05, 0.02$ and 0.005 are given in **Table 5**. It is observed that the obtained values of the invariants remain almost constant during the computer run.

5. Conclusion

In this paper we study the MEW problem by extending the use of multigrid technique. We checked our scheme through single solitary wave in which the analytic solution is known. Our scheme was extended to study the interaction of two solitary waves and Maxwellian initial condition where the analytic solutions are unknown during the interaction. The performance and accuracy of the method were explained by calculating the error norms L_2, L_∞ and conservative properties of mass, momentum and energy. The computed results showed that our scheme is a successful numerical technique for solving the MEW problem and can be also efficiently applied for solving a large number of physically important non-linear problems.

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