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On the Non-Common Neighbourhood Energy of Graphs

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Abstract

In this paper, we introduce a new type of graph energy called the non-common-neighborhood energy $E_{\mathrm{NCN}}(G)$, NCN-energy for some standard graphs is obtained and an upper bound for $E_{\mathrm{NCN}}(G)$ is found when G is a strongly regular graph. Also the relation between common neighbourhood energy and non-common neighbourhood energy of a graph is established.

Keywords

NCN-Eigenvalues (of Graph), NCN-Energy (of Graph), NCN-Adjacency Matrix (of Graph)

1. Introduction

Let G be a simple graph with n vertices, and let $\boldsymbol{A} = \|a_{ij}\|$ be its adjacency matrix. The eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $\boldsymbol{A}$ are the (ordinary) eigenvalues of the graph G [1]. Since $\boldsymbol{A}$ is a symmetric matrix with zero trace, these eigenvalues are real with sum equal to zero.

The energy of the graph G is defined [2] as the sum of the absolute values of its eigenvalues:

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

Details on the theory of graph energy can be found in the reviews [3]-[5], whereas details on its chemical applications in the book [6] and in the review [7]. Let G be simple graph with vertex set $V(G) = \{v_1, v_2, \cdots, v_n\}$. For $i \neq j$, the common neighborhood of the the vertices v_i and v_j, denoted by $\Gamma(v_i, v_j)$, is the set of vertices, different from v_i and v_j, which are adjacent to both v_i and v_j. The common-neighborhood matrix of G is then $\mathbf{CN} = \mathbf{CN}(G) = \|\gamma_{ij}\|$, where

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$$\gamma_{ij} = \begin{cases} \left|\Gamma\{v_i, v_j\}\right| & \text{if } i \neq j \\ 0 & \text{otherwise.} \end{cases}$$

The common-neighborhood energy (or, shorter, CN-energy) of the graph G is

$$E_{\mathrm{CN}} = E_{\mathrm{CN}}(G) = \sum_{i=1}^{n} |\gamma_i|.$$

where $\gamma_1, \gamma_2, \cdots, \gamma_n$ are the eigenvalues of the $\mathbf{CN}(G)$, for more details about CN-energy, see [9]

Theorem 1. [8] For almost all n-vertex graphs

$$E(G) = \left(\frac{4}{3\pi} + o(1)\right) n^{3/2}.$$

Theorem 1 immediately implies that almost all graphs are hyperenergetic, making any further search for them pointless.

In what follows we shall need a few auxiliary results.

Lemma 1. [1] *Let G be a connected k-regular graph with n vertices and $k \geq 3$. Let $k, \lambda_2, \cdots, \lambda_n$ be its eigenvalues. Then the eigenvalues of the line graph of G are $2k-2, \lambda_2 + k - 2, \cdots, \lambda_n + k - 2$, and -2 with multiplicity $n(k-2)/2$.*

Lemma 2. [1] *Let G be a graph with an adjacency matrix A and with eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_p$, then the* $\det A = \prod_{i=1}^{p} \lambda_i$, *for any polynomial $P(x)$, $P(\lambda)$ is an eigenvalue of $P(A)$ and hence* $\det P(A) = \prod_{i=1}^{p} P(\lambda_i)$.

Corollary 1. [9] *Let G be a connected k-regular graph and let $k, \lambda_2, \cdots, \lambda_n$ be its eigenvalues.*

1) The common-neighborhood eigenvalues of the complement of G are

$$(n-k-1)(n-k-2), \lambda_2^2 + 2\lambda_2 - n + k + 2, \cdots, \lambda_n^2 + 2\lambda_n - n + k + 2.$$

2) The common-neighborhood eigenvalues of the line graph $L(G)$ of G are

$$4k^2 - 10k + 6, \lambda_2^2 + (2k-4)\lambda_2 + k^2 - 6k + 6, \cdots, \lambda_n^2 + (2k-4)\lambda_n + k^2 - 6k + 6, 6 - 2k$$

where the CN-eigenvalue $6-2k$ has multiplicity $n(k-2)/2$.

Definition. A strongly regular graph with parameters (n, k, λ, μ) is a k-regular graph with n vertices, such that any two adjacent vertices have λ common neighbors, and any two non-adjacent vertices have μ common neighbors.

2. Non-Common Neighbourhood Energy of Graphs

Definition. Let G be simple graph with vertex set $V(G) = \{v_1, v_2, \cdots, v_n\}$. For $i \neq j$, the non-common neighborhood set of the the vertices v_i and v_j, denoted by $\Gamma'(v_i, v_j)$, is the set of vertices, different from v_i and v_j, which are not adjacent to both v_i and v_j. The non-common neighborhood matrix of G is then $\mathbf{NCN} = \mathbf{NCN}(G) = \|a_{ij}\|$, where

$$a_{ij} = \begin{cases} \left|\Gamma'\{v_i, v_j\}\right| & i \neq j \\ 0 & \text{otherwise.} \end{cases}$$

According to the above definition, the non-common neighborhood matrix is a real symmetric $n \times n$ matrix. Therefore its eigenvalues $\gamma_1, \gamma_2, \cdots, \gamma_n$ are real numbers. Since the trace of $\mathbf{NCN}(G)$ is zero, the sum of its eigenvalues is also equal to zero. the eigenvalues $\gamma_1, \gamma_2, \cdots, \gamma_n$ of the matrix $\mathbf{NCN}(G)$ are called the **CNC**-eigenvalues of G

Definition. The non-common neighborhood energy (or, shorter, **CNC**-energy) of the graph G is

$$E_{\mathrm{NCN}} = E_{\mathrm{NCN}}(G) = \sum_{i=1}^{n} |\gamma_i|.$$

We will Denote by $\boldsymbol{I}_n$ the unit matrix of order n, and by $\boldsymbol{J}_n$ the square matrix of order n whose all elements are equal to unity. Let further $\mathbf{0}$ stand for a matrix (or pertinent dimension) whose all elements are equal to zero.

Proposition 2. $E_{\mathrm{NCN}}(K_n) = E(\overline{K_n}) = 0$, *where* K_n *is the complete graph of order n.*

Proof. Observing that $\mathbf{NCN}(K_n) = \boldsymbol{A}(\overline{K_n})$, we get $E_{\mathrm{NCN}}(K_n) = E(\overline{K_n}) = 0$.

Proposition 3. $E_{NCN}(K_{a,b}) = 2[(a-1)(a-2)+(b-1)(b-2)]$, *where* $K_{a,b}$ *is the complete bipartite graph of order* $a+b$.

Proof. First observe that if the vertices of $K_{a,b}$ are labeled so that all vertices $v_1,\cdots,v_a$ are adjacent to all vertices $v_{a+1},\cdots,v_{a+b}$, then

$$\mathbf{NCN}(K_{a,b}) = \begin{pmatrix} \mathbf{0} & (a-2)(\boldsymbol{J}_a - \boldsymbol{I}_a) \\ (b-2)(\boldsymbol{J}_b - \boldsymbol{I}_b) & \mathbf{0} \end{pmatrix}.$$

Observing that $\boldsymbol{J}_a - \boldsymbol{I}_a = \boldsymbol{A}(K_a)$ and $\boldsymbol{J}_b - \boldsymbol{I}_b = \boldsymbol{A}(K_b)$, we have

$$\mathbf{NCN}(K_{a,b}) = \begin{pmatrix} \mathbf{0} & (a-2)\boldsymbol{A}(K_a) \\ (b-2)\boldsymbol{A}(K_b) & \mathbf{0} \end{pmatrix}$$

Which implies $E_{\mathrm{NCN}}(K_{a,b}) = (a-2)E(K_a) + (b-2)E(K_b) = 2[(a-1)(a-2)+(b-1)(b-2)]$.

Corollary 2. $E_{\mathrm{NCN}}(P_3) = E_{\mathrm{NCN}}(K_{2,2}) = E_{\mathrm{NCN}}(P_2) = 0$

Proposition 4. $E_{\mathrm{NCN}}(K_{a,a}) = (n-2)(n-4)$, *where* $K_{a,a}$ *is the complete bipartite graph of order* $a+a=n$.

Proposition 5. *For any totally disconnected graph* $\overline{K_n}$, $E_{\mathrm{NCN}}\overline{K_n} = 2(n-1)(n-2)$.

Proof. Observing that for any two vertices *u* and *v* in $\mathbf{NCN}(\overline{K_n})$ there are $n-2$ vertices not adjacent to both vertices *u* and *v*. Therefore $\mathbf{NCN}(\overline{K_n}) = (n-2)\boldsymbol{A}(K_n)$, where $\boldsymbol{A}(K_n)$ is the adjacency matrix of the complete graph with *n* vertices. Hence, $E_{\mathrm{NCN}}(\overline{(K_n)}) = (n-2)E(K_n) = 2(n-1)(n-2)$.

The complete multipartite graph $K_{p_1,p_2,\cdots,p_r}$ is a graph on $n = \sum_{i=1}^{r} p_i$ vertices. The set of vertices is partitioned into parts of cardinalities $p_1, p_2, \cdots, p_r$; an edge joins two vertices if and only if they belong to different parts. Thus $K_{1,1,\cdots,1}$ is the complete graph K_n. In the following proposition we get the CNC-energy of the multipartite graph $K_{p_1,p_2,\cdots,p_r}$.

Proposition 6. *Let* $K_{p_1,p_2,\cdots,p_r}$ *be The complete multipartite graph on* $n = \sum_{i=1}^{r} p_i$ *vertices, where* $p_i \ge 3$. *Then,*

$$E_{\mathrm{NCN}}(K_{p_1,p_2,\cdots,p_r}) = \sum_{i=1}^{r}((p_i-2)E(K_{p_i})) = \sum_{i=1}^{r}(E_{\mathrm{CN}}(K_{p_i})).$$

Proof. Let $G = K_{p_1,p_2,\cdots,p_r}$ be a complete multipartite graph with $n = \sum_{i=1}^{r} p_i$ vertices. From the definition of complete multipartite graph we observe for any two distinct vertices v_s, v_t if they belong to the same partite set S_{p_i} with $|S_{p_i}| = p_i$, then $\Gamma'(v_s, v_t) = p_i - 2$. But if the two vertices belongs to different partite sets we have $\Gamma'(v_s, v_t) = 0$. Hence the NCN-matrix of $K_{p_1,p_2,\cdots,p_r}$ is of the following form.

$$\begin{pmatrix} (p_1-2)\boldsymbol{A}(K_{p_1}) & 0 & 0 & \cdots & 0 \\ 0 & (p_2-2)\boldsymbol{A}(K_{p_2}) & 0 & \cdots & 0 \\ 0 & 0 & (p_3-2)\boldsymbol{A}(K_{p_3}) & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & (p_r-2)\boldsymbol{A}(K_{p_r}) \end{pmatrix},$$

where $\boldsymbol{A}(K_{p_i}); i = 1,2,\cdots,r$ is the adjacency matrix of the complete graphs $K_{p_i}; i = 1,2,\cdots,r$. Hence,

$$\begin{aligned} E_{\mathrm{NCN}}(K_{p_1,p_2,\cdots,p_r}) &= (p_1-2)E(K_{p_1}) + (p_2-2)E(K_{p_2}) + \cdots + (p_r-2)E(K_{p_r}) \\ &= \sum_{i=1}^{r}((p_i-2)E(K_{p_i})) = \sum_{i=1}^{r}(E_{\mathrm{CN}}(K_{p_i})). \end{aligned}$$

Corollary 3. *For graph* $G = K_{r\times m}$ *we have,*

$$E_{\mathrm{NCN}}(K_{r\times m}) = 2r(m-1)(m-2).$$

Corollary 4. *For any cocktail party graph G which is the complement of* $(n/2)K_2$,

$$E_{\mathrm{NCN}}(G) = 0.$$

The proof of the following result is straightforward.

Proposition 7. *If the graph G consists of (disconnected) components* $G_1, G_2, \cdots, G_p$, *then*

$$E_{\mathrm{NCN}}(G) = E_{\mathrm{NCN}}(G_1) + E_{\mathrm{NCN}}(G_2) + \cdots + E_{\mathrm{NCN}}(G_p).$$

Theorem 8. *Let G be a graph on n vertices, and let* $\boldsymbol{A}(G)$ *is the adjacency matrix of G, and* $B(G) = \|b_{i,j}\|$, *where*

$$b_{i,j} = \begin{cases} \deg(v_i) + \deg(v_j), & \text{if } i \neq j \text{ and } v_i v_j \in E; \\ \deg(v_i) + \deg(v_j) + 2, & \text{if } i \neq j \text{ and } v_i v_j \notin E; \\ 0, & \text{otherwise.} \end{cases}$$

and Let $\boldsymbol{D}(G) = \mathrm{diag}\left[\deg(v_1), \deg(v_2), \cdots, \deg(v_n)\right]$. Then,

$$\mathbf{NCN}(G) = n(\boldsymbol{J}_n - \boldsymbol{I}_n) - \boldsymbol{B}(G) + \boldsymbol{A}(G)^2 - \boldsymbol{D}(G).$$

Proof. Since $(\mathbf{NCN}(G))_{ij}$ is equal to size of the set $\Gamma'(v_i, v_j)$. Therefore $\mathbf{NCN}(G) = n(\boldsymbol{J}_n - \boldsymbol{I}_n) - \boldsymbol{B}(G) + \mathbf{CN}(G)$ and as we know that $\mathbf{CN}(G) = \boldsymbol{A}(G)^2 - \boldsymbol{D}(G)$. Hence

$$\mathbf{NCN}(G) = n(\boldsymbol{J}_n - \boldsymbol{I}_n) - \boldsymbol{B}(G) + \boldsymbol{A}(G)^2 - \boldsymbol{D}(G).$$

Lemma 3. *Let* $G = (V, E)$ *be k-regular graph and* $\boldsymbol{B}(G) = \|b_{i,j}\|$, *where*

$$b_{i,j} = \begin{cases} \deg(v_i) + \deg(v_j), & \text{if } i \neq j \text{ and } v_i v_j \in E; \\ \deg(v_i) + \deg(v_j) + 2, & \text{if } i \neq j \text{ and } v_i v_j \notin E; \\ 0, & \text{otherwise.} \end{cases}$$

Then

$$\boldsymbol{B}(G) = (2k+2)(\boldsymbol{J}_n - \boldsymbol{I}_n) - 2\boldsymbol{A}(G).$$

Proof. Observing that if $G = (V, E)$ is k-regular, then $\boldsymbol{B}(G) = \|b_{i,j}\|$, where

$$b_{i,j} = \begin{cases} 2k, & \text{if } i \neq j \text{ and } v_i v_j \in E; \\ 2k+2, & \text{if } i \neq j \text{ and } v_i v_j \notin E; \\ 0, & \text{otherwise.} \end{cases}$$

Therefore,

$$\boldsymbol{B} = 2k\boldsymbol{A}(G) + (2k+2)(\boldsymbol{J}_n - \boldsymbol{I}_n - \boldsymbol{A}(G)).$$

Hence

$$\boldsymbol{B} = (2k+2)(\boldsymbol{J}_n - \boldsymbol{I}_n) - 2\boldsymbol{A}(G).$$

Proposition 9. *For any k-regular graph G,*

$$\mathbf{NCN}(G) = \boldsymbol{A}(G)^2 + 2\boldsymbol{A}(G) + (n-2k-2)\boldsymbol{J}_n - (n-k-2)\boldsymbol{I}_n.$$

Proof. By Theorem 8 and Lemma 3, we have

$$\mathbf{NCN}(G) = \boldsymbol{A}^2 + 2\boldsymbol{A}(G) + (\boldsymbol{J}_n - \boldsymbol{I}_n) - (2k+2)(\boldsymbol{J}_n - \boldsymbol{I}_n) - k\boldsymbol{I}_n.$$

Hence

$$\mathbf{NCN}(G)=\boldsymbol{A}(G)^2+2\boldsymbol{A}(G)+(n-2k-2)\boldsymbol{J}_n-(n-k-2)\boldsymbol{I}_n.$$

Theorem 10. *For any graph G,* $E_{\mathrm{NCN}}(G)=E_{\mathrm{CN}}(\bar{G})$.

Proof. Since $\mathbf{NCN}(G)_{ij}$ for $i\neq j$ is the number of vertices which not adjacent to both v_i and v_j and it is equal to the number of vertices which adjacent to both v_i and v_j in $\bar{G}$, that means $(\mathbf{NCN}(G))_{ij}=(\mathbf{CN}(\bar{G}))_{ij}$.

Theorem 11. *Let G be a connected k-regular graph with eigenvalues* $k,\lambda_2,\cdots,\lambda_n$. *Then the* **NCN**-*eigenvalues of G are* $(k-n+1)(k-n+2),(\lambda_2)^2+2\lambda_2-n+k+2,\cdots,(\lambda_n)^2+2\lambda_n-n+k+2$.

Proof. Theorem 11 follows from Proposition 8 and Lemma 2 or by applying Theorem 10 and Corollary 1

Theorem 12. *Let G be a connected k-regular graph and let* $k,\lambda_2,\cdots,\lambda_n$ *be its eigenvalues. Then The NCN-eigenvalues of the line graph* $L(G)$ *of G are*

$$2(k-n+1)(k-n+2)-2,(\lambda_2)^2+2\lambda_2-n+2k,\cdots,(\lambda_n)^2+2\lambda_2-n+2k.$$

Proof. Theorem 12 follows from Proposition 8 and Lemma 2 or by applying Theorem 10 and Corollary 1

Theorem 13. *For any connected graph G,* $E_{\mathrm{NCN}}(G)=0$ *if and only if G is complete multi bipartite graph* $K_{n_1,n_2,\cdots,n_m}$ *for some positive integer* $m\geq 2$, *where* $n_i\leq 2$ *for* $i=1,2,\cdots,m$.

Proof. Let $G\cong K_{n_1,n_2,\cdots,n_m}$ for some positive integer $m\geq 2$, where $n_i\leq 2$ for $i=1,2,\cdots,m$. Suppose that *u* and *v* any two vertices in *G*, if *u* and *v* adjacent, then does not exists any vertex in *G* which is not adjacent to both of *u* and *v*, similarly if *u* and *v* are not adjacent that means $\mathbf{NCN}(G)$ is zero matrix. Therefore $E_{\mathrm{NCN}}(G)=0$ and Corollary 2 are spacial cases of multi bipartite graph $K_{n_1,n_2,\cdots,n_m}$ for some positive integer $m\geq 2$, where $n_i\leq 2$ for $i=1,2,\cdots,m$.

Conversely, if *G* is connected graph and $E_{\mathrm{NCN}}(G)=0$, then by Theorem 10 $E_{\mathrm{CN}}(\bar{G})=0$. Therefore $G\cong\overline{(\alpha K_1\cup\beta K_2)}$ for some positive integers $\alpha,\beta\geq 0$. Hence *G* is complete multi bipartite graph $K_{n_1,n_2,\cdots,n_m}$ for some positive integer $m\geq 2$, where $n_i\leq 2$ for $i=1,2,\cdots,m$.

Lemma 4. *If G is a strongly regular graph with parameters* (n,k,λ,μ), *then*

$$\sum_{i=1}^{n}|\gamma_i|^2=n\left[k(n-2k+\lambda)^2-(k-n+1)(n-2k+\mu-2)^2\right]. \tag{1}$$

Proof. If v_i and v_j are adjacent vertices of *G*, then $\gamma_{ij}=n-2k+\lambda$. If v_i and v_j are non-adjacent vertices of *G*, then $\gamma_{ij}=p-2k+\mu-2$. Since *G* has $nk/2$ pairs of adjacent vertices, and $\binom{n}{2}-nk/2$ pairs of non-adjacent vertices,

$$\sum_{i=1}^{n}|\gamma_i|^2=Tr(\mathbf{NCN}(G))^2=\sum_{i=1}^{n}\sum_{j=1}^{n}\gamma_{ij}\gamma_{ji}=\sum_{i=1}^{n}\sum_{j=1}^{n}(\gamma_{ij})^2$$
$$=2\left[\frac{nk}{2}\right](n-2k+\lambda)^2+2\left[\binom{n}{2}-\frac{nk}{2}\right](n-2k+\mu-2)^2,$$

from which Equation (1) follows straightforwardly.

Theorem 14. *If G is a strongly regular graph with parameters* (n,k,λ,μ), *then*

$$E_{\mathrm{NCN}}(G)\leq(k-n+1)(k-n+2)$$
$$+\sqrt{(n-1)\left[n\left[k(n-2k+\lambda)^2-(k-n+1)(n-2k+\mu-2)^2\right]-(n-1)\left[(k-n+1)(k-n+2)\right]^2\right]}. \tag{2}$$

Proof. follows an idea first used by Koolen and Moulton [10] [11]. Let $\gamma_1,\gamma_2,\cdots,\gamma_n$ be the common neighborhood eigenvalues of *G*, and let γ_1 be the greatest eigenvalue. Because the greatest ordinary eigenvalue of *G* is equal to *k*, by Theorem 14, $\gamma_1=(k-n+1)(k-n+2)$.

The Cauchy-Schwarz inequality states that if $(a_1,a_2,\cdots,a_n)$ and $(b_1,b_2,\cdots,b_n)$ are *n*-vectors, then

$$\left(\sum_{i=1}^{n} a_i b_i\right)^2 \le \left(\sum_{i=1}^{n} a_i^2\right)\left(\sum_{i=1}^{n} b_i^2\right).$$

Now, by setting $a_i = 1$ and $b_i = |\gamma_i|$, $i = 2,3,\cdots,n$, in the above inequality, we obtain

$$\left(\sum_{i=2}^{n} |\gamma_i|\right)^2 \le \left(\sum_{i=2}^{n} 1^2\right)\left(\sum_{i=2}^{n} |\gamma_i|^2\right).$$

Therefore

$$\sum_{i=2}^{n} |\gamma_i| \le \sqrt{(n-1)\sum_{i=2}^{n} |\gamma_i|^2}$$

i.e.,

$$\sum_{i=1}^{n} |\gamma_i| - (k-n+1)(k-n+2) \le \sqrt{(n-1)\left[\sum_{i=1}^{n} |\gamma_i|^2 - \left[(k-n+1)(k-n+2)\right]^2\right]}$$

i.e.,

$$E_{\text{NCN}}(G) \le (k-n+1)(k-n+2) + \sqrt{(n-1)\left[\sum_{i=1}^{n} |\gamma_i|^2 - \left[(k-n+1)(k-n+2)\right]^2\right]}.$$

By using Lemma 4,

$$E_{\text{NCN}}(G) \le (k-n+1)(k-n+2)$$
$$+\sqrt{(n-1)\left[n\left[k(n-2k+\lambda)^2 - (k-n+1)(n-2k+\mu-2)^2\right] - (n-1)\left[(k-n+1)(k-n+2)\right]^2\right]}.$$

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