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Degree Splitting of Root Square Mean Graphs

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Abstract

Let $f: V(G) \to \{1, 2, \cdots, q+1\}$ be an injective function. For a vertex labeling *f*, the induced edge labeling $f^*(e = uv)$ is defined by, $f^*(e=uv) = \left\lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \right\rfloor$; then, the edge labels are distinct and are from $\{1, 2, \cdots, q\}$. Then *f* is called a root square mean labeling of *G*. In this paper, we prove root square mean labeling of some degree splitting graphs.

Keywords

Graph, Path, Cycle, Degree Splitting Graphs, Root Square Mean Graphs, Union of Graphs

1. Introduction

The graphs considered here are simple, finite and undirected. Let $V(G)$ denote the vertex set and $E(G)$ denote the edge set of *G*. For detailed survey of graph labeling we refer to Gallian [1]. For all other standard terminology and notations we follow Harary [2]. The concept of mean labeling on degree splitting graph was introduced in [3]. Motivated by the authors we study the root square mean labeling on degree splitting graphs. Root square mean labeling was introduced in [4] and the root square mean labeling of some standard graphs was proved in [5]-[11]. The definitions and theorems are useful for our present study.

Definition 1.1: A graph $G = (V, E)$ with *p* vertices and *q* edge is called a root square mean graph if it is possible to label the vertices $x \in V$ with distinct labels $f(x)$ from $1, 2, \cdots, q+1$ in such a way that when

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each edge $e=uv$ is labeled with $f(e=uv)=\left\lceil\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{f(u)^2+f(v)^2}{2}}\right\rfloor$, then the edge labels are distinct and are from $\{1,2,\cdots,q\}$. In this case *f* is called root square mean labeling of *G*.

Definition 1.2: A walk in which $u_1u_2\cdots u_n$ are distinct is called a path. A path on *n* vertices is denoted by P_n.

Definition 1.3: A closed path is called a cycle. A cycle on *n* vertices is denoted by C_n.

Definition 1.4: Let $G=(V,E)$ be a graph with $V=S_1\cup S_2\cup\cdots\cup S_t\cup T$, where each S_i is a set of vertices having at least two vertices and having the same degree and $T=V-\cup S_i$. The degree splitting graph of *G* is denoted by $DS(G)$ and is obtained from *G* by adding the vertices $w_1,w_2,\cdots,w_t$ and joining w_i to each vertex of $S_i, 1\le i\le t$. The graph *G* and its degree splitting graph $DS(G)$ are given in **Figure 1**.

Definition 1.5: The union of two graphs $G_1=(V_1,E_1)$ and $G_2=(V_2,E_2)$ is a graph $G=G_1\cup G_2$ with vertex set $V=V_1\cup V_2$ and the edge set $E=E_1\cup E_2$.

Theorem 1.6: Any path is a root square mean graph.

Theorem 1.7: Any cycle is a root square mean graph.

2. Main Results

Theorem 2.1: $nDS(P_3)$ is a root square mean graph.

Proof: The graph $DS(P_3)$ is shown in **Figure 2**.

Let $G=nDS(P_3)$. Let the vertex set of *G* be $V=V_1\cup V_2\cup\cdots\cup V_n$ where $V_i=\{v_1^i,v_2^i,v_3^i,w_i,1\le i\le n\}$. Define a function $f:V(G)\to\{1,2,\cdots,q+1\}$ by

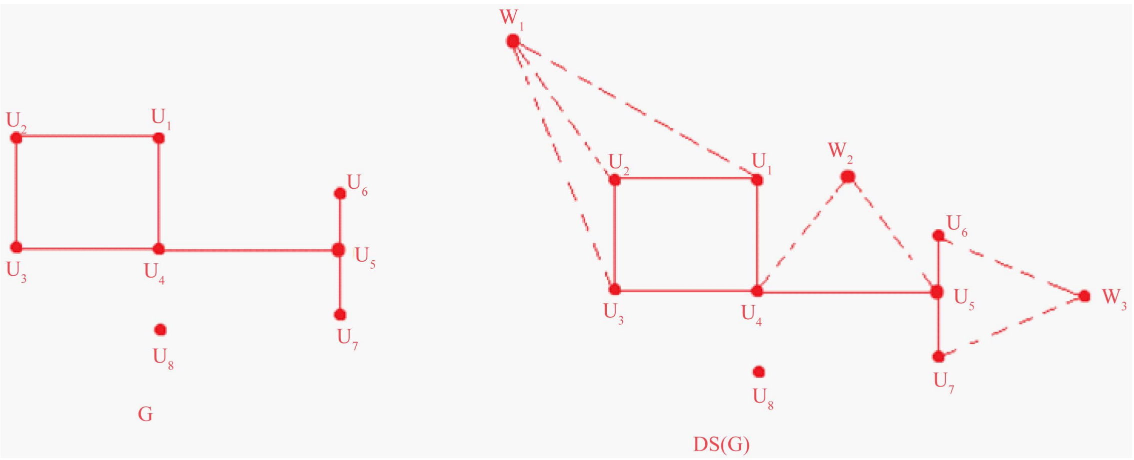

Figure 1. The graph *G* and its degree splitting graph $DS(G)$.

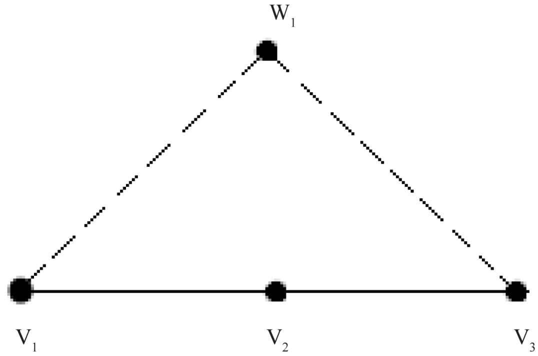

Figure 2. The graph $DS(P_3)$.

$$f\left(v_1^i\right) = 4i - 3, 1 \le i \le n$$

$$f\left(v_2^i\right) = 4i - 2, 1 \le i \le n$$

$$f\left(v_3^i\right) = 4i - 1, 1 \le i \le n$$

$$f\left(w_i\right) = 4i, 1 \le i \le n$$

Then the edges are labeled as

$$f\left(v_1^i v_2^i\right) = 4i - 3, 1 \le i \le n - 1$$

$$f\left(v_2^i v_3^i\right) = 4i - 1, 1 \le i \le n - 1$$

$$f\left(v_1^i w_i\right) = 4i - 2, 1 \le i \le n - 2$$

$$f\left(v_3^i w_i\right) = 4i, 1 \le i \le n - 2$$

Then the edge labels are distinct and are from $\{1, 2, \cdots, q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.2: Root square mean labeling of $4DS\left(P_3\right)$ is shown in **Figure 3**.

Theorem 2.3: $4DS\left(P_4\right)$ is a root square mean graph.

Proof: The graph $DS\left(P_4\right)$ is shown in **Figure 4**.

Let $G = nDS\left(P_3\right)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \cdots \cup V_n$ where $V_i = \left\{v_1^i, v_2^i, v_3^i, v_4^i, w_1^i, w_2^i, 1 \le i \le n\right\}$. Define a function $f : V\left(G\right) \to \{1, 2, \cdots, q + 1\}$ by

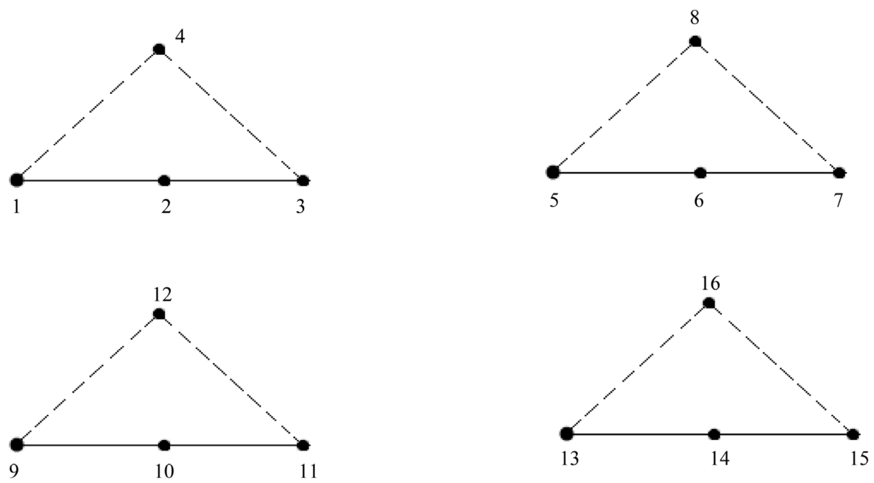

Figure 3. Root square mean labeling of $4DS\left(P_3\right)$.

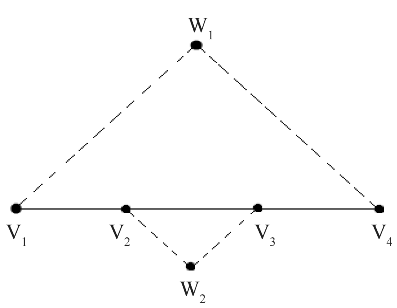

Figure 4. The graph $DS\left(P_4\right)$.

$$f\left(v_1^i\right)=7i-5, 1\le i\le n$$

$$f\left(v_2^i\right)=7i-3, 1\le i\le n$$

$$f\left(v_3^i\right)=7i-1, 1\le i\le n$$

$$f\left(v_4^i\right)=7i-4, 1\le i\le n$$

$$f\left(w_1^i\right)=7i-6, 1\le i\le n$$

$$f\left(w_2^i\right)=7i, 1\le i\le n$$

Then the edges are labeled as

$$f\left(v_1^iv_2^i\right)=7i-4, 1\le i\le n$$

$$f\left(v_2^iv_3^i\right)=7i-2, 1\le i\le n$$

$$f\left(v_3^iv_4^i\right)=7i-3, 1\le i\le n$$

$$f\left(v_1^iw_1^i\right)=7i-6, 1\le i\le n$$

$$f\left(w_1^iv_4^i\right)=7i-5, 1\le i\le n$$

$$f\left(v_2^iw_2^i\right)=7i-1, 1\le i\le n$$

$$f\left(v_3^iw_2^i\right)=7i, 1\le i\le n$$

Then the edge labels are distinct and are from $\{1,2,\cdots,q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.4: Root square mean labeling of $4DS(P_3)$ is shown in **Figure 5**.

Theorem 2.5: $nDS(P_2\odot K_1)$ is a root square mean graph.

Proof: The graph $DS(P_2\odot K_1)$ is shown in **Figure 6**.

Let $G=nDS(P_2\odot K_1)$. Let the vertex set of G be $V=V_1\cup V_2\cup\cdots\cup V_n$ where $V_i=\{v_1^i,v_2^i,v_3^i,v_4^i,w_1^i,w_2^i,1\le i\le n\}$. Define a function $f:V(G)\to\{1,2,\cdots,q+1\}$ by

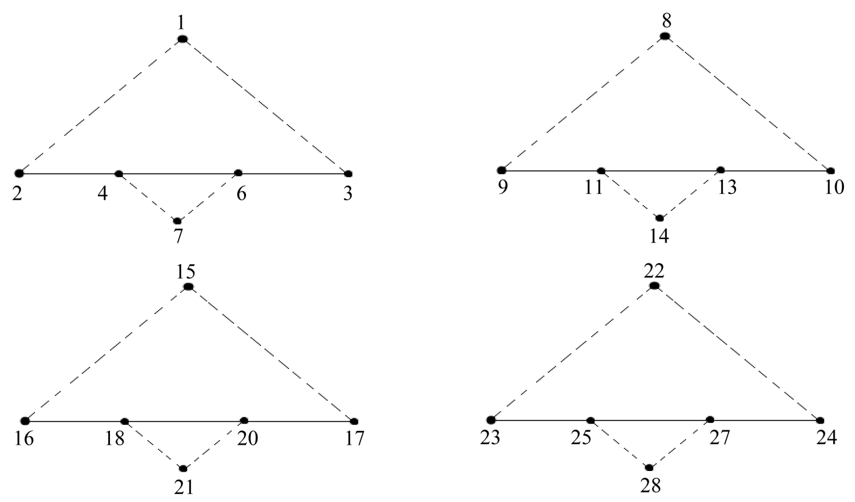

Figure 5. Root square mean labeling of $4DS(P_3)$.

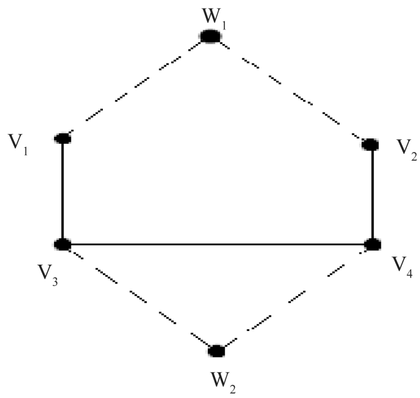

Figure 6. The graph $DS(P_2 \odot K_1)$.

$$f(v_1^i) = 7i-5, 1 \le i \le n$$

$$f(v_2^i) = 7i-4, 1 \le i \le n$$

$$f(v_3^i) = 7i-2, 1 \le i \le n$$

$$f(v_4^i) = 7i-1, 1 \le i \le n$$

$$f(w_1^i) = 7i-6, 1 \le i \le n$$

$$f(w_2^i) = 7i, 1 \le i \le n$$

Then the edges are labeled as

$$f(v_1^i v_3^i) = 7i-4, 1 \le i \le n$$

$$f(v_3^i v_4^i) = 7i-2, 1 \le i \le n$$

$$f(v_4^i v_2^i) = 7i-3, 1 \le i \le n$$

$$f(v_1^i w_1^i) = 7i-6, 1 \le i \le n$$

$$f(v_2^i w_1^i) = 7i-5, 1 \le i \le n$$

$$f(v_3^i w_2^i) = 7i-1, 1 \le i \le n$$

$$f(v_4^i w_2^i) = 7i, 1 \le i \le n$$

Then the edge labels are distinct and are from $\{1, 2, \cdots, q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.6: The labeling pattern of $4DS(P_2 \odot K_1)$ is shown in **Figure 7**.

Theorem 2.7: $nDS(P_2 \odot K_2)$ is a root square mean graph.

Proof: The graph $DS(P_2 \odot K_2)$ is shown in **Figure 8**.

Let $G = nDS(P_2 \odot K_2)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \cdots \cup V_n$ where $V_i = \{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, v_6^i, w_1^i, w_2^i, 1 \le i \le n\}$. Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ by

$$f(v_1^i) = 11i-5, 1 \le i \le n$$

$$f(v_2^i) = 11i-3, 1 \le i \le n$$

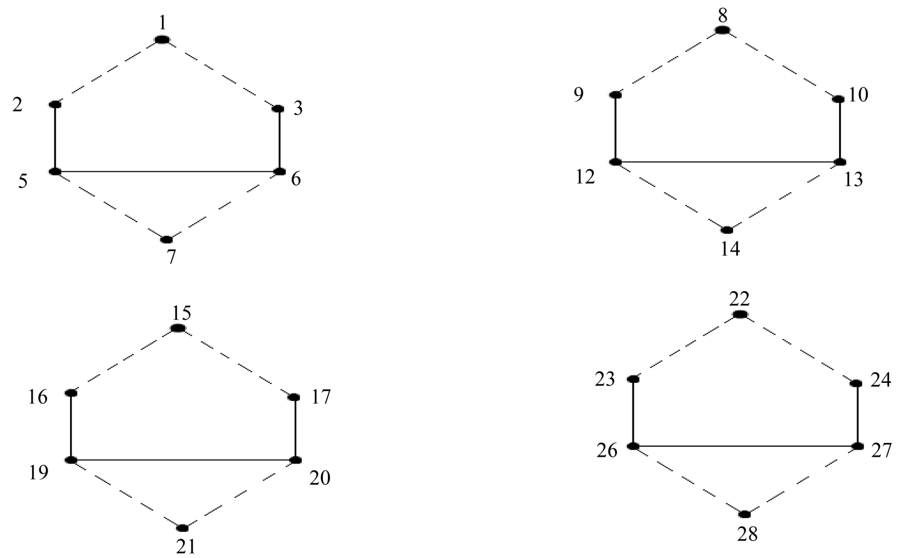

Figure 7. The labeling pattern of $4DS(P_2 \odot K_1)$.

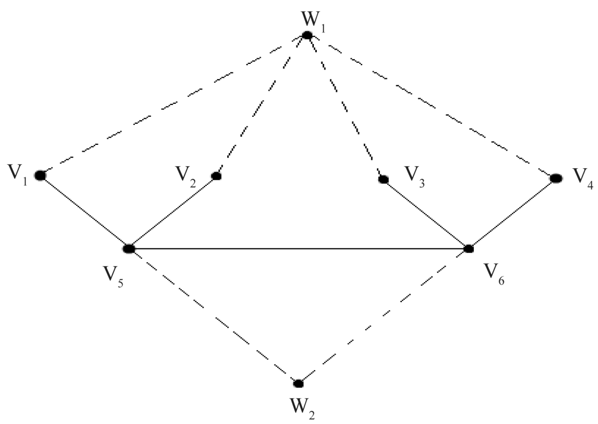

Figure 8. The graph $DS(P_2 \odot K_2)$.

$$f(v_3^i) = 11i - 2, 1 \le i \le n$$

$$f(v_4^i) = 11i - 1, 1 \le i \le n$$

$$f(v_5^i) = 11i - 9, 1 \le i \le n$$

$$f(v_6^i) = 11i - 7, 1 \le i \le n$$

$$f(w_1^i) = 11i, 1 \le i \le n$$

$$f(w_2^i) = 11i - 10, 1 \le i \le n$$

Then the edges are labeled as

$$f(v_5^i v_6^i) = 11i - 8, 1 \le i \le n$$

$$f(v_5^i v_1^i) = 11i - 7, 1 \le i \le n$$

$$f\left(v_5^i v_2^i\right) = 11i - 6, 1 \le i \le n$$

$$f\left(v_6^i v_3^i\right) = 11i - 5, 1 \le i \le n$$

$$f\left(v_6^i v_4^i\right) = 11i - 4, 1 \le i \le n$$

$$f\left(v_1^i w_1^i\right) = 11i - 3, 1 \le i \le n$$

$$f\left(v_2^i w_1^i\right) = 11i - 2, 1 \le i \le n$$

$$f\left(v_3^i w_1^i\right) = 11i - 1, 1 \le i \le n$$

$$f\left(v_4^i w_1^i\right) = 11i, 1 \le i \le n$$

$$f\left(v_5^i w_2^i\right) = 11i - 10, 1 \le i \le n$$

$$f\left(v_6^i w_2^i\right) = 11i - 9, 1 \le i \le n$$

Then the edge labels are distinct and are from $\{1, 2, \cdots, q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.8: The labeling pattern of $2DS\left(P_2 \odot K_2\right)$ is shown in **Figure 9**.

Theorem 2.9: $nDS\left(P_2 \odot \overline{K_3}\right)$ is a root square mean graph.

Proof: The graph $DS\left(P_2 \odot \overline{K_3}\right)$ is shown in **Figure 10**.

Let $G = nDS\left(P_2 \odot \overline{K_3}\right)$. Let the vertex set of G be $V = V_1 \cup V_2 \cup \cdots \cup V_n$ where $V_i = \left\{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, v_6^i, v_7^i, v_8^i, w_1^i, w_2^i, 1 \le i \le n\right\}$.

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ by

$$f\left(v_1^i\right) = 15i - 11, 1 \le i \le n$$

$$f\left(v_2^i\right) = 15i - 8, 1 \le i \le n$$

$$f\left(v_3^i\right) = 15i - 6, 1 \le i \le n$$

$$f\left(v_4^i\right) = 15i - 5, 1 \le i \le n$$

$$f\left(v_5^i\right) = 15i - 3, 1 \le i \le n$$

$$f\left(v_6^i\right) = 15i - 2, 1 \le i \le n$$

$$f\left(w_1^i\right) = 15i, 1 \le i \le n$$

$$f\left(w_2^i\right) = 15i - 14, 1 \le i \le n$$

$$f\left(v_7^i\right) = 15i - 13, 1 \le i \le n$$

$$f\left(v_8^i\right) = 15i - 12, 1 \le i \le n$$

Then the edges are labeled as

$$f\left(v_7^i v_1^i\right) = 15i - 11, 1 \le i \le n$$

$$f\left(v_7^i v_2^i\right) = 15i - 10, 1 \le i \le n$$

$$f\left(v_7^i v_3^i\right) = 15i - 9, 1 \le i \le n$$

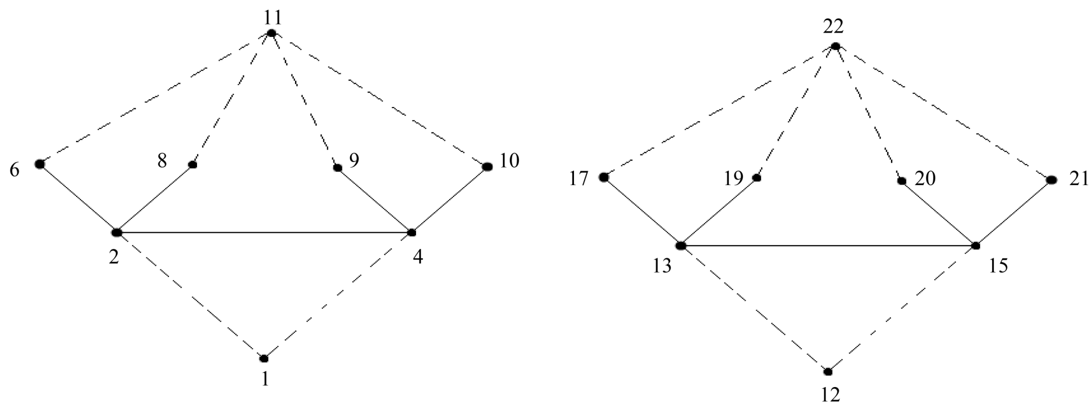

Figure 9. The labeling pattern of $2DS(P_2 \odot K_2)$.

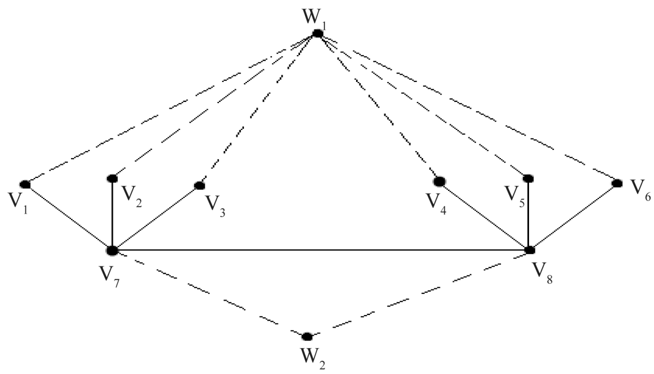

Figure 10. The graph $DS(P_2 \odot \overline{K_3})$.

$$f(v_8^i v_4^i) = 15i - 8, 1 \le i \le n$$

$$f(v_8^i v_5^i) = 15i - 7, 1 \le i \le n$$

$$f(v_8^i v_6^i) = 15i - 6, 1 \le i \le n$$

$$f(v_7^i w_2^i) = 15i - 14, 1 \le i \le n$$

$$f(v_8^i w_2^i) = 15i - 13, 1 \le i \le n$$

$$f(v_1^i w_1^i) = 15i - 5, 1 \le i \le n$$

$$f(v_2^i w_1^i) = 15i - 4, 1 \le i \le n$$

$$f(v_3^i w_1^i) = 15i - 3, 1 \le i \le n$$

$$f(v_4^i w_1^i) = 15i - 2, 1 \le i \le n$$

$$f(v_5^i w_1^i) = 15i - 1, 1 \le i \le n$$

$$f(v_6^i w_1^i) = 15i, 1 \le i \le n$$

$$f(v_7^i v_8^i) = 15i - 12, 1 \le i \le n$$

Then the edge labels are distinct and are from $\{1,2,\cdots,q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.10: The root square mean labeling of $2DS\left(P_2 \odot \overline{K_3}\right)$ is shown in **Figure 11**.

Theorem 2.11: $nDS(P_3 \odot K_1)$ is a root square mean graph.

Proof: The graph $DS(P_3 \odot K_1)$ is shown in **Figure 12**.

Let $G = nDS(P_3 \odot K_1)$. Let its vertex set be $V = V_1 \cup V_2 \cup \cdots \cup V_n$ where $V_i = \left\{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, v_6^i, w_1^i, w_2^i, 1 \le i \le n\right\}$.

Define a function $f: V(G) \to \{1,2,\cdots,q+1\}$ by

$$f\left(v_1^i\right) = 10i-3, 1 \le i \le n$$

$$f\left(v_2^i\right) = 10i-2, 1 \le i \le n$$

$$f\left(v_3^i\right) = 10i-1, 1 \le i \le n$$

$$f\left(v_4^i\right) = 10i-8, 1 \le i \le n$$

$$f\left(v_5^i\right) = 10i-5, 1 \le i \le n$$

$$f\left(v_6^i\right) = 10i-6, 1 \le i \le n$$

$$f\left(w_1^i\right) = 10i, 1 \le i \le n$$

$$f\left(w_2^i\right) = 10i-9, 1 \le i \le n$$

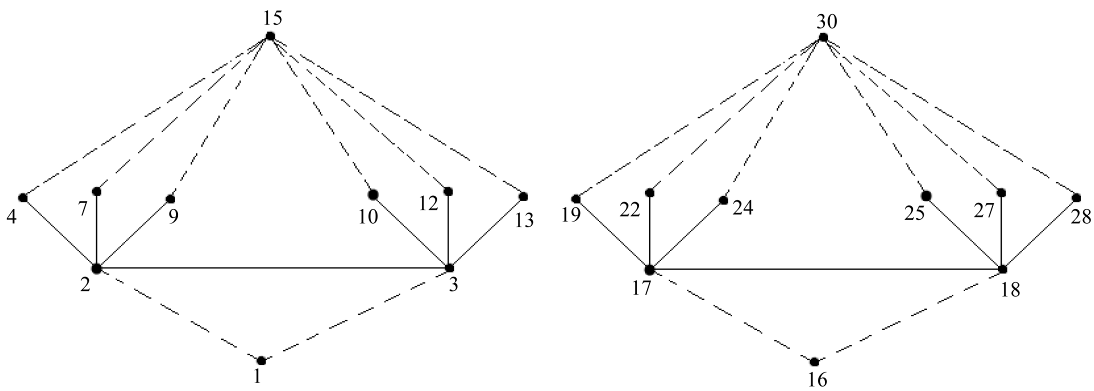

Figure 11. The root square mean labeling of $2DS\left(P_2 \odot \overline{K_3}\right)$.

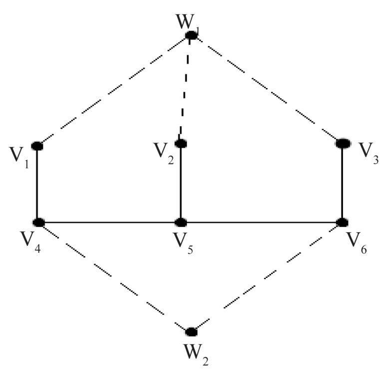

Figure 12. The graph $DS(P_3 \odot K_1)$.

Then the edges are labeled as

$$f\left(v_4^i v_1^i\right)=10i-5, 1\le i\le n$$

$$f\left(v_5^i v_2^i\right)=10i-4, 1\le i\le n$$

$$f\left(v_6^i v_3^i\right)=10i-3, 1\le i\le n$$

$$f\left(v_1^i w_1^i\right)=10i-2, 1\le i\le n$$

$$f\left(v_2^i w_1^i\right)=10i-1, 1\le i\le n$$

$$f\left(v_3^i w_1^i\right)=10i, 1\le i\le n$$

$$f\left(v_4^i w_2^i\right)=10i-9, 1\le i\le n$$

$$f\left(v_6^i w_2^i\right)=10i-8, 1\le i\le n$$

Then the edge labels are distinct and are from $\{1,2,\cdots,q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.12: The labeling pattern of $3DS\left(P_3 \odot K_1\right)$ is shown in **Figure 13**.

Theorem 2.13: $nDS\left(K_{1,3}\right)$ is a root square mean graph.

Proof: The graph $DS\left(K_{1,3}\right)$ is shown in **Figure 14**.

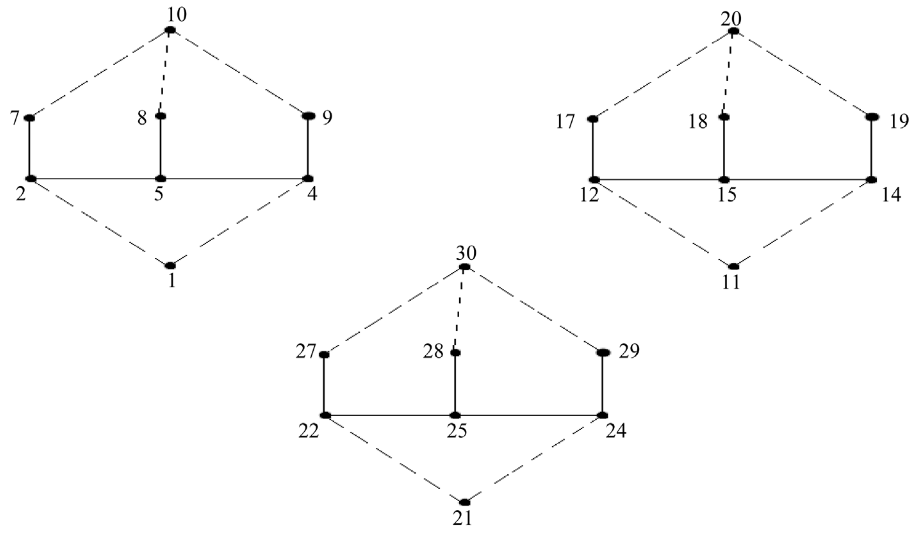

Figure 13. The labeling pattern of $3DS\left(P_3 \odot K_1\right)$.

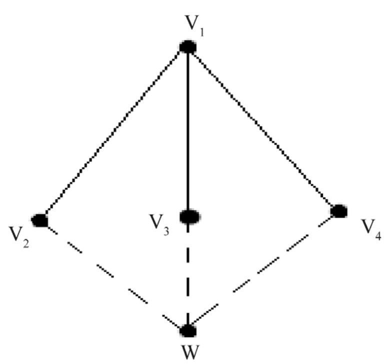

Figure 14. The graph $DS\left(K_{1,3}\right)$.

Let $G = nDS(K_{1,3})$. Let its vertex set be $V = V_1 \cup V_2 \cup \cdots \cup V_n$ where $V_i = \{v_1^i, v_2^i, v_3^i, v_4^i, w^i, 1 \le i \le n\}$.

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ by

$$f(v_1^i) = 6i - 5, 1 \le i \le n$$

$$f(v_2^i) = 6i - 4, 1 \le i \le n$$

$$f(v_3^i) = 6i - 2, 1 \le i \le n$$

$$f(v_4^i) = 6i - 1, 1 \le i \le n$$

$$f(w^i) = 6i, 1 \le i \le n$$

Then the edges are labeled as

$$f(v_1^i v_2^i) = 6i - 5, 1 \le i \le n$$

$$f(v_1^i v_3^i) = 6i - 4, 1 \le i \le n$$

$$f(v_1^i v_4^i) = 6i - 3, 1 \le i \le n$$

$$f(v_2^i w^i) = 6i - 2, 1 \le i \le n$$

$$f(v_3^i w^i) = 6i - 1, 1 \le i \le n$$

$$f(v_4^i w^i) = 6i, 1 \le i \le n$$

Then the edge labels are distinct and are from $\{1, 2, \cdots, q\}$. Hence by definition 1.1, G is a root square mean graph.

Example 2.14: The labeling pattern of $4DS(K_{1,3})$ is shown in **Figure 15**.

Theorem 2.15: $nDS(C_3 \hat{O} K_{1,2})$ is a root square mean graph.

Proof: The graph $DS(C_3 \hat{O} K_{1,2})$ is shown in **Figure 16**.

Let $G = nDS(C_3 \hat{O} K_{1,2})$. Let its vertex set be $V = V_1 \cup V_2 \cup \cdots \cup V_n$

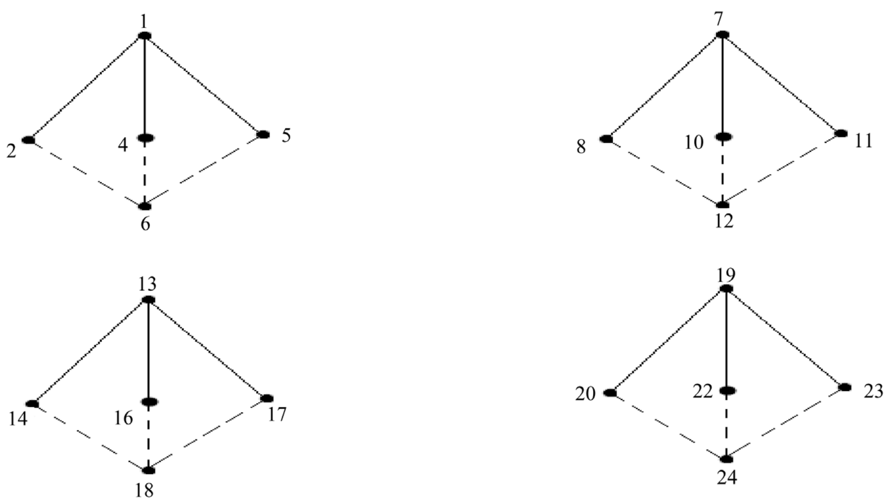

Figure 15. The labeling pattern of $4DS(K_{1,3})$.

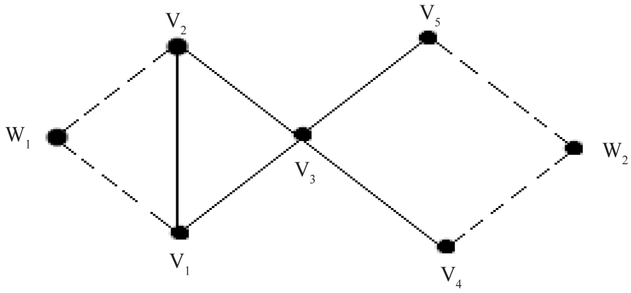

Figure 16. The graph $DS\left(C_3\hat{O}K_{1,2}\right)$.

where $V_i = \left\{v_1^i, v_2^i, v_3^i, v_4^i, v_5^i, w_1^i, w_2^i, 1 \le i \le n\right\}$.

Define a function $f : V(G) \to \{1, 2, \cdots, q+1\}$ by

$$f\left(v_1^i\right) = 9i - 7, 1 \le i \le n$$

$$f\left(v_2^i\right) = 9i - 5, 1 \le i \le n$$

$$f\left(v_3^i\right) = 9i - 4, 1 \le i \le n$$

$$f\left(v_4^i\right) = 9i - 2, 1 \le i \le n$$

$$f\left(v_5^i\right) = 9i - 1, 1 \le i \le n$$

$$f\left(w_1^i\right) = 9i - 8, 1 \le i \le n$$

$$f\left(w_2^i\right) = 9i, 1 \le i \le n$$

Then the edges are labeled as

$$f\left(w_1^i v_1^i\right) = 9i - 8, 1 \le i \le n$$

$$f\left(w_1^i v_2^i\right) = 9i - 7, 1 \le i \le n$$

$$f\left(v_1^i v_2^i\right) = 9i - 6, 1 \le i \le n$$

$$f\left(v_1^i v_3^i\right) = 9i - 5, 1 \le i \le n$$

$$f\left(v_2^i v_3^i\right) = 9i - 4, 1 \le i \le n$$

$$f\left(v_3^i v_4^i\right) = 9i - 3, 1 \le i \le n$$

$$f\left(v_3^i v_5^i\right) = 9i - 2, 1 \le i \le n$$

$$f\left(v_4^i w_2^i\right) = 9i - 1, 1 \le i \le n$$

$$f\left(v_5^i w_2^i\right) = 9i, 1 \le i \le n$$

Then the edge labels are distinct and are from $\{1, 2, \cdots, q\}$. Hence by definition 1.1, G is a root square mean graph.

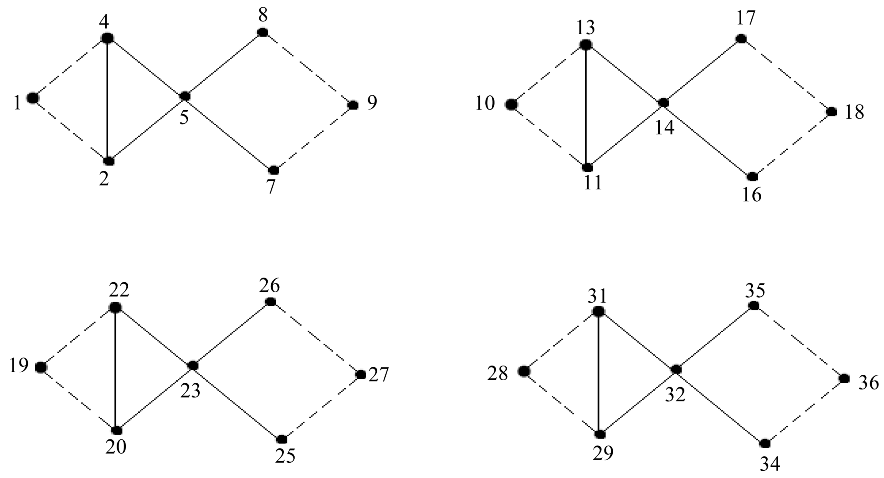

Figure 17. The root square mean labeling of $4DS(C_3 \hat{O} K_{1,2})$.

Example 2.16: The root square mean labeling of $4DS(C_3 \hat{O} K_{1,2})$ is given in **Figure 17**.

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