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On the Computation of Extinction Time for Some Nonlinear Parabolic Equations

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Abstract

The phenomenon of extinction is an important property of solutions for many evolutionary equations. In this paper, a numerical simulation for computing the extinction time of nonnegative solutions for some nonlinear parabolic equations on general domains is presented. The solution algorithm utilizes the Donor-cell scheme in space and Euler's method in time. Finally, we will give some numerical experiments to illustrate our algorithm.

Keywords

1. Introduction

There is a large number of nonlinear partial differential equations of parabolic type whose solutions for given initial data become identically nulle in finite time T. Such a phenomenon is called extinction and T is called the extinction time. For certain problems, the extinction time can be computed explicitly, but in many cases one can only know the existence of extinction time.

Since the appearance of the pioneering work of Kalashnikov [1], extinction phenomenon in nonlinear parabolic equations has been studied extensively by many authors [2] [3]. Particular emphasis has been placed on the question as to the existence of extinction time [4]-[7].

Generally speaking, it is difficult to simulate extinction phenomenon accurately on general domains. Indeed, it is not at all clear if features of such a phenomenon as extinction can be well reflected in the discretized equation which approximates the original equation. In [8]-[11], some numerical schemes have been used to

study the extinction phenomenon of solutions for some nonlinear parabolic equations.

In this work, we propose a numerical algorithm for computing the extinction time for nonnegative solutions of some nonlinear parabolic equations. Our motivation is to reproduce the extinction phenomenon of some nonlinear parabolic equations on general domains.

This paper is organized as follows. In the next section, we present the problem model and some theoretical results. A discretization of this problem is derived in Section 3, while numerical experiments are reported in Section 4 and Section 5 is devoted to concluding remarks.

2. The Model Problem

In this work, we are concerned with the following initial-boundary value problem:

$$\frac{\partial u}{\partial t} = \Delta \phi(u) - F(u) \quad \text{in} \quad \Omega \times]0, \infty[, \tag{1}$$

$$u = 0 \quad \text{on} \quad \partial\Omega \times [0, \infty[, \tag{2}$$

$$u(0) = u_0 > 0 \quad \text{in} \quad \bar{\Omega} = \Omega \cup \partial\Omega, \tag{3}$$

where Ω is a $\mathbb{R}^2$ bounded domain with boundary $\partial\Omega$, u_0, ϕ and F, being given functions.

Furthermore, for any $t > 0$ and for all u defined in $\Omega \times]0, \infty[$, we will set $u(t)(x) = u(t, x)$, $x \in \Omega$.

Nonlinear parabolic equations of type (1) appear in various applications. In particular they are used to describe a phenomenon of thermal propagation in an absorptive medium where u stands for temperature [5]. In other applications, u is a concentration and the process is described as diffusion with absorption.

The problem of determining necessary and sufficient conditions on the functions ϕ and F which ensure the existence of an extinction time for solutions of (1)-(3) has been considered by several authors [1] [6] [7] [12].

In this section, we state the following result:

Theorem 1 *Assume that* $u \in C^1\left(0, \infty; L^2(\Omega)\right)$ *is a nonnegative solution of the problem* (1)-(3) *where* ϕ *and* F *are nondecreasing, nonnegative derivatives functions and if* $F(0) = 0$, *then*

$$\lim_{t \to \infty} \int_\Omega \int_0^{u(t)} F(s) \, ds = 0$$

Proof: First, let us set $G(u(t)) = \int_0^{u(t)} F(s) \, ds$. We have

$$\frac{d}{dt} \int_\Omega G(u) = \int_\Omega F(u) \frac{du}{dt}. \tag{i}$$

Multiplying Equation (1) by $F(u)$ and integrating over Ω it follows

$$\int_\Omega F(u) \frac{\partial u}{\partial t} = \int_\Omega F(u) \Delta \phi(u) - \int_\Omega (F(u))^2. \tag{ii}$$

On one hand thanks to regularity of functions F, ϕ and u, we can write

$$\int_\Omega F(u) \Delta \phi(u) = \int_{\partial\Omega} F(u) \nabla \phi(u) \cdot n \, ds - \int_\Omega \nabla F(u) \cdot \nabla \phi(u)$$

where n is the unit outward to $\partial\Omega$, ds denotes an element of surface area, since u vanishes on $\partial\Omega$ and $F(0) = 0$, we have $\int_{\partial\Omega} F(u) \Delta \phi(u) \cdot n \, ds = 0$, hence

$$\int_\Omega F(u) \Delta \phi(u) = -\int_\Omega \nabla F(u) \cdot \nabla \phi(u). \tag{iii}$$

From (*ii*) and (*iii*), we deduce

$$\int_\Omega F(u) \frac{\partial u}{\partial t} = -\int_\Omega F'(u) \phi'(u) |\nabla u|^2 - \int_\Omega (F(u))^2$$

$$\leq -\int_\Omega (F(u))^2.$$

Since $F'(u) \geq 0$ and $\phi'(u) \geq 0$, and according to (*i*), we obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega}G(u) \leq -\int_{\Omega}\left(F(u)\right)^2. \qquad (iv)$$

On the other hand, multiplying Equation (1) by u yields

$$\int_{\Omega}u\frac{\partial u}{\partial t} = -\int_{\Omega}\phi'(u)|\nabla u|^2 - \int_{\Omega}uF(u) \leq 0,$$

which we rewrite as

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega}u^2 \leq 0,$$

then the application $t \mapsto \int_{\Omega}u^2$ is decreased in $\mathbb{R}^+$. It then follows

$$\int_{\Omega}\left(u(t)\right)^2 \leq \int_{\Omega}\left(u(0)\right)^2 = \int_{\Omega}u_0^2 \equiv M^2, \quad \text{for all } t > 0. \qquad (vi)$$

On the other hand, the increase of F implies that

$$G(u) = \int_0^u F(s)\,\mathrm{d}s \leq F(u) \times \int_0^u \mathrm{d}s = uF(u),$$

Thus

$$\int_{\Omega}G(u) \leq \int_{\Omega}uF(u) \leq \|u\|_{L^2(\Omega)} \times \|F(u)\|_{L^2(\Omega)}$$

and according to (*vi*) we obtain

$$\int_{\Omega}G(u) \leq |M|\|F(u)\|_{L^2(\Omega)},$$

This last inequality implies

$$-\frac{1}{M^2}\left(\int_{\Omega}G(u)\right)^2 \geq -\int_{\Omega}\left(F(u)\right)^2.$$

Then considering (*iv*), we deduced

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega}G(u) \leq -\int_{\Omega}\left[F(u)\right]^2 \leq -\frac{1}{M^2}\left(\int_{\Omega}G(u)\right)^2. \qquad (vii)$$

Setting $w(t) = \int_{\Omega}G(u(t))$, we obtain

$$w'(t) \leq -\frac{1}{M^2}w^2(t).$$

This gives after integrating

$$0 \leq w(t) \leq \frac{1}{\dfrac{1}{w(0)} + \dfrac{t}{M^2}}.$$

Knowing that $w(0) > 0$. it follows $w(t) \geq 0$. The passage to the limit allows us to write

$$0 \leq \lim_{t\to\infty}w(t) \leq \lim_{t\to\infty}\frac{1}{\dfrac{1}{w(0)} + \dfrac{t}{M^2}} = 0.$$

Finally,

$$\lim_{t\to\infty}w(t) = \lim_{t\to\infty}\int_{\Omega}\int_0^{u(t)}F(s)\,\mathrm{d}s = 0.$$

In addition to the assumption of increase of *F* in Theorem 2.1, if we assume that

$$F(s) \ge Cs^p, \forall s > 0, \forall p \ge 1, \tag{4}$$

C is a positive constant.

then the following result is easily shown.

Corollary 1 *Suppose that the assumptions of the Theorem* 2.1 *are satisfied, and if* (4) *holds for* $p \ge 1$, *then*

$$\lim_{t \to \infty} \|u(t)\|_{L^2(\Omega)} = 0. \tag{5}$$

Indeed, for all $u(t)$ solution of (1)-(3), it comes from the assumption (4) that

$$C \int_\Omega \int_0^{u(t)} s^p \mathrm{d}s \le \int_\Omega \int_0^{u(t)} F(s) \mathrm{d}s,$$

that gives

$$\int_\Omega u^{p+1}(t) \le \frac{p+1}{C} \int_\Omega \int_0^{u(t)} F(s) \mathrm{d}s.$$

As $p+1 \ge 2$, then

$$0 \le \int_\Omega u^2(t) \le \frac{p+1}{C} \int_\Omega \int_0^{u(t)} F(s) \mathrm{d}s.$$

So

$$0 \le \lim_{t \to \infty} \int_\Omega u^2(t) \le \frac{p+1}{C} \lim_{t \to \infty} \int_\Omega \int_0^{u(t)} F(s) \mathrm{d}s,$$

and as a consequence of Theorem 2.1

$$\lim_{t \to \infty} \|u(t)\|_{L^2(\Omega)} = 0. \qquad \square$$

In summary, under some assumptions we know that all nonnegative solutions of (1)-(3) have extinction time as $t \to \infty$. We want to determine whether extinction occurs in finite time for any given ϕ and F.

It is well known that, in general, there is no classical solution to this nonlinear parabolic equation for arbitrary choices of ϕ and F. However, there are some works dealing with approximation of extinction time for solutions of (1). For example, in [13] a numerical method to approximate the solutions of (1) has been developed in the case $N = 1$ and in [14] an algorithm based on splitting technique was derived to compute the extinction time for solutions on a rectangular domain.

In order to determine the extinction time for some ϕ and F, we will derive in the next section a numerical scheme based on Donor-cell scheme. Given a sufficiently small parameter $\epsilon > 0$, we would like to determine the positive real $T_\epsilon > 0$ such that a solution $u(t)$ of the problem (1)-(3) has to satisfy the above relation

$$\|u(t)\| \le \epsilon, \quad \forall t > T_\epsilon. \tag{6}$$

We shall call T_ϵ satisfying (6) as the ϵ-extinction time.

3. Discretization

3.1. Discretization of the Studied Domain

Let $\Omega \subset \mathbb{R}^2$ be a considered domain that we assume to be of irregular shape, we approximate Ω by a domain Ω_h whose boundary is specified by the set of boundary edges lying on gridlines. We imbed Ω_h in a rectangular domain $\Omega_R = [a,b] \times [c,d]$, $(a,b > 0)$ of smallest possible size. Given two nonzero integers $i_{\max}$ and $j_{\max}$, we set $\delta x = (b-a)/i_{\max}$ $\delta y = (d-c)/j_{\max}$ and we introduce on Ω_R a grid of step δx and δy in x and y direction respectively. The set of points (x_i, y_j) such that of $x_i = a + i\delta x$, $y_j = c + j\delta y$ defines the discretization of Ω_R into $i_{\max} \times j_{\max}$ cells (rectangular subdomains). For all $i = 1, \cdots, i_{\max}$, $j = 1, \cdots, j_{\max}$ cell (i,j) occupies the spatial region $[x_i, x_{i+1}] \times [y_j, y_{j+1}]$ and has center the point noted $X_{i,j} = (x_{i+1/2}, y_{i+1/2})$. The cells of Ω_R are then divided into *inner cell* (which lie completely in Ω), *external cell* (which lie in $\Omega_R \setminus \bar{\Omega}$) and *boundary* cells (which lie in a part of $\partial\Omega$). The problem model is then solved only in the inner

cells.

A matrix of size $i_{\max} \times j_{\max}$ gives a description of the discretized domain. For example, consider three sets of indices I, B et E corresponding to the inner, boundary and external cells, we then admit to define the following matrix

$$C(i,j) = \begin{cases} 1, & \text{if } (i,j) \in I, \\ -1, & \text{if } (i,j) \in B, \\ 0, & \text{if } (i,j) \in E. \end{cases} \tag{7}$$

the matrix to identify cell types.

The idea of this numerical treatment of general domains has been suggested by Griebel *et al*. in [15]. An example of this numerical treatment is illustrated in **Figure 1** and its matrix representative is given by the following (8).

$$C = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & 0 \\
0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & 0 \\
0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & -1 & 0 & 0 & 0 \\
0 & -1 & 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & -1 & 1 & 1 & 1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} \tag{8}$$

3.2. Spatial Discretization

First of all, let us give an approximation of the diffusion operator at the point $X_{i,j}$ which we rewrite as

$$\left[\Delta\phi(u)\right]_{X_{i,j}} = \left[\frac{\partial}{\partial x}\left(\phi'(u)\frac{\partial u}{\partial x}\right)\right]_{X_{i,j}} + \left[\frac{\partial}{\partial y}\left(\phi'(u)\frac{\partial u}{\partial y}\right)\right]_{X_{i,j}} \tag{9}$$
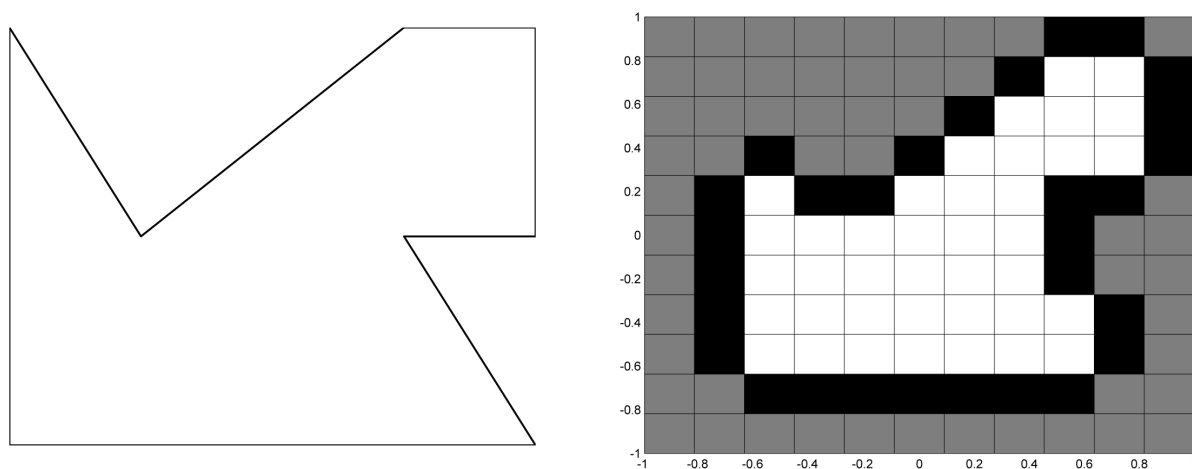

Figure 1. An example of the discretization of a non rectangular domain into cells.

Let $V_{i,j}$ be the approximation of $\left[\phi'(u)\dfrac{\partial u}{\partial x}\right]$ at the cell center $X_{i,j}$. In the following we do apply a discretization that is similar to the one of Donor-cell scheme where the expression $\left[\phi'(u)\dfrac{\partial u}{\partial x}\right]$ is approached by a progressive finite differences scheme and $\left[\dfrac{\partial}{\partial x}\left(\phi'(u)\dfrac{\partial u}{\partial x}\right)\right]$ by a central finite differences scheme.

Furthermore, we set $\phi'_{i,j} = \phi'\left(U_{i,j}\right)$ where $U_{i,j} \approx u\left(X_{i,j}\right)$ and we note

$$\Phi = \left(\phi'_{0,0}, \cdots, \phi'_{i_{max}-1,0}, \phi'_{0,1}, \cdots, \phi'_{i_{max}-1,1}, \cdots, \phi'_{0,j_{max}-1}, \cdots, \phi'_{i_{max}-1,j_{max}-1}\right)^{\mathrm{T}} \tag{10}$$

If $\underline{V}_x$ denotes the vector of components $V_{i,j}$ then, one can write

$$\underline{V}_x = \operatorname{diag}\left(\Phi\right) \times D_x^p \underline{U} \tag{11}$$

where $\operatorname{diag}\left(\Phi\right)$ the diagonal matrix whose diagonal is the vector Φ and D_x^p denotes forward differentiation matrix the x-direction.

On the other hand, denoting by $W_{i,j}$ the approached value of $\left[\dfrac{\partial}{\partial x}\left(\phi'(u)\dfrac{\partial u}{\partial x}\right)\right]$ at the cell center $X_{i,j}$ and noting by

$$\underline{W}_x = \left(W_{0,0}, \cdots, W_{i_{max}-1,0}, W_{0,1}, \cdots, W_{i_{max}-1,1}, \cdots, W_{0,j_{max}-1}, \cdots, W_{i_{max}-1,j_{max}-1}\right)^{\mathrm{T}}$$

the vector of the value of $\left[\dfrac{\partial}{\partial x}\left(\phi'(u)\dfrac{\partial u}{\partial x}\right)\right]$ at point $X_{i,j}$, it follows through the central difference scheme, the relation

$$\underline{W}_x = D_x^c \underline{V}_x \tag{12}$$

which is written by the mean of the Equation (12) as

$$\underline{W}_x = D_x^c \times \operatorname{diag}\left(\Phi\right) \times D_x^p \underline{U}, \tag{13}$$

where D_x^c denotes central differentiation matrix in the x-direction.

Similarly, given $\underline{W}_y$, the vector of the approached values of $\left[\dfrac{\partial}{\partial y}\left(\phi'(u)\dfrac{\partial u}{\partial y}\right)\right]$ at point $X_{i,j}$, we obtain

$$\underline{W}_y = D_y^c \times \operatorname{diag}\left(\Phi\right) \times D_y^p \underline{U}, \tag{14}$$

where D_y^p denotes forward differentiation matrix in the y-direction.

From Equations (13) and (14), we deduce the approximation of the operator $u \mapsto \Delta\phi(u)$ at cell center $X_{i,j}$, in matrix form:

$$L\left(\underline{U}\right) = \left(D_x^c \times \operatorname{diag}\left(\Phi\right) \times D_x^p + D_y^c \times \operatorname{diag}\left(\Phi\right) \times D_y^p\right)\underline{U} \tag{15}$$

where $L\left(\underline{U}\right)$ is the vector of value of $\Delta\phi(u)$ at points $X_{i,j}$. Thus, we have defined, an approximation operator L to approach the operator $u \mapsto \Delta\phi(u)$.

However, it should be noted that Φ is a vector dependent of $\underline{U}$.

Considering lexicographic numerotation, we note by $\tilde{U}(t)$ the vector of the values of u in points $X_{i,j} \in \Omega$ at time t. Knowing that $u \equiv 0$ at points $X_{i,j} \in \Omega_R \setminus \overline{\Omega}$, and the differentiation matrices $D_x^c, D_y^c, D_x^p, D_y^p$ to approach the derivative on the set of the points $X_{i,j}$ we can replace respectively by the matrices which are obtained by deleting the rows and columns corresponding to the indices of the points $X_{i,j}$ of $\Omega_R \setminus \overline{\Omega}$.

Given the Equation (15), the discrete system approaching the problem (1)-(3) is rewritten by

$$\frac{\mathrm{d}\tilde{U}}{\mathrm{d}t} = \tilde{D} \times \operatorname{diag}\left(\tilde{\Phi}\right)\tilde{U} - \underline{F}\left(\tilde{U}\right), \tag{16}$$

where we have set

$$\tilde{D} = \tilde{D}_x^c \times \tilde{D}_x^p + \tilde{D}_y^c \times \tilde{D}_y^p$$

and where $\operatorname{diag}(\tilde{\Phi})$ is the matrix obtained of $\operatorname{diag}(\Phi)$ by deleting the rows and columns corresponding to the indices of the points $X_{i,j}$ of $\Omega_R \setminus \bar{\Omega}$ and $\underline{F}(\tilde{U})$ is the vector of values of $F(u(X_{i,j}))$, $X_{i,j} \in \Omega$. Furthermore, the initial condition is written

$$\tilde{U}(0) = \tilde{U}_0 \tag{17}$$

where $\tilde{U}_0$ is the vector obtained from U_0 by deleting the elements corresponding to the indices of the points $X_{i,j}$ of $\Omega_R \setminus \bar{\Omega}$

3.3. Temporal Discretization

For a time step fixed δt, we consider the sequence $(t_n)_{n \geq 0}$ defined by $t_{n+1} = t_n + \delta t$ et $t_0 = 0$. Then, we denote $\tilde{U}^n$ the approximation at time t_n of vector $\tilde{U}(t)$ solution of (16)-(17). Using the explicit Euler method, the semi-discrete scheme is written

$$\begin{cases} \tilde{U}^{n+1} = \tilde{U}^n + \delta t \tilde{D} \times \operatorname{diag}(\tilde{\Phi}^n) \tilde{U}^n - \delta t \underline{F}(\tilde{U}^n), \\ \tilde{U}^0 = \tilde{U}(0), \end{cases} \tag{18}$$

where we set $\operatorname{diag}(\tilde{\Phi}^n) = \operatorname{diag}(\tilde{\Phi}(\tilde{U}^n))$.

It should be noticed that if the time step δt is chosen to be little enough, and F satisfied the growth condition (4),

$$\lim_{n \to \infty} \left\| \tilde{U}^n \right\| = 0 \tag{19}$$

Thus the extinction time is obtained using simple itrations process until the stopping criterion

$$\left\| \tilde{U}^n \right\| \leq \epsilon \tag{20}$$

is satisfied. Here ϵ is the given tolerance number. The sequence of computations to be performed is summarized as follows

Algorithm 3.1
1. Read $i_{\max}, j_{\max}, \delta t, \epsilon$ and *itermax*.
2. Compute δx and δy.
3. Define Ω_R (rectangular) and Ω (non rectangular) such $\Omega \subset \Omega_R$.
4. Compute the matrix $D_x^c, D_y^c, D_x^p, D_y^p, \tilde{D}$ and $\operatorname{diag}(\Phi)$.
5. Set $t = 0$, $n = 0$.
6. Assign initial value to $\tilde{U}$.
7. Set $\tilde{U}\text{old} = \tilde{U}_0$.
8. **While** $\left\| \tilde{U}\text{old} \right\| < \epsilon$ and $n < \text{itermax}$, do.
9. Compute $\tilde{U}\text{new}$ according to (18).
10. Set $\tilde{U}\text{old} = \tilde{U}\text{new}$.
11. $t = t + \delta t$, $n = n+1$.
End while

4. Numerical Experiments

Let Ω be a bounded domain in $\mathbb{R}^2$. Consider the initial value problem

$$\frac{\partial u}{\partial t} = \Delta u - u^q, \quad \text{in} \quad \Omega \times]0, \infty[\tag{21}$$

$$u = 0 \quad \text{on} \quad \partial\Omega \times [0, \infty[\tag{22}$$

$$u(0) = u_0 > 0 \quad \text{in} \quad \bar{\Omega} = \Omega \cup \partial\Omega. \tag{23}$$

where u_0 is the continuous nonnegative function in $\bar{\Omega}$, vanishing on $\partial\Omega$, and $q > 0$.

Equation (21) models heat propagation in medium where the solution u stands for temperature.

For our numerical experiments we have consider **Figure 2** to be our studied domain and we have use discretization parameters $\delta t = 10^{-5}$, and $i_{\max} = j_{\max} = 50$.

We would like to numerically estimate the extinction time for solutions of problem (21)-(23) with the initial condition given by

$$u_0(x, y) = 1 - (x^2 + y^2)$$

First, for fixed accurate value $\epsilon_0 > 0$, we estimate the $\mathbb{R}^N$-euclidian norm of the sequence $\tilde{U}^n$ solution of the numerical scheme (18) for various values of parameters n. **Table 1** and **Figure 3** clearly show that the approximation extinction time can be given by

$$T_{\epsilon_0} = \lim_{n \to \infty} T_{\epsilon_0}^n \tag{24}$$

We can see in **Table 1** that this value is approximated by

$$T_{\epsilon_0} = 0.61 \tag{25}$$

Also, the extinction process is illustrated by **Figure 4** where we can appreciate the numerical solution extinct in a finite time.

5. Concluding Remarks

In this paper, a numerical algorithm based on Donor-cell scheme was proposed in order to compute the extinction time for nonnegative solutions of some nonlinear parabolic equations on general domains. We have verified
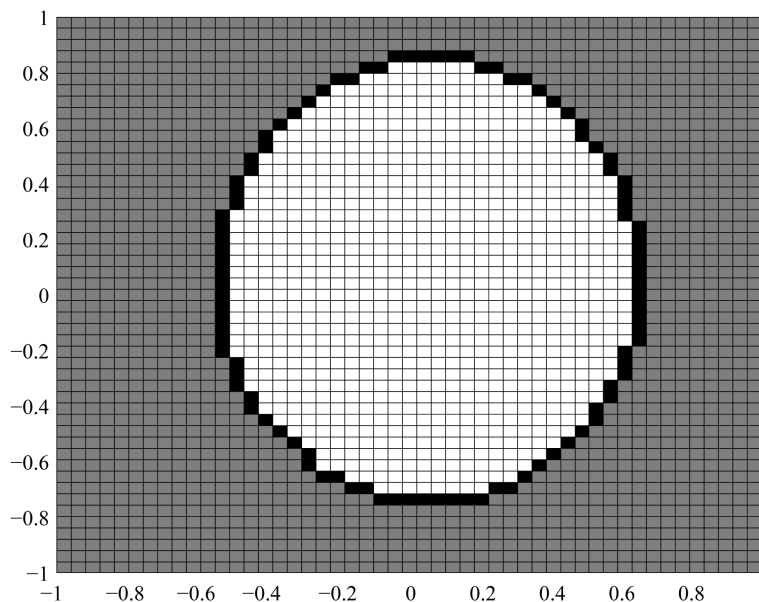

Figure 2. Discretization of studied domain $\Omega \subset [-1,1] \times [-1,1]$ into cells.

Table 1. Numerical extinction time relatively to time iteration parameter n.

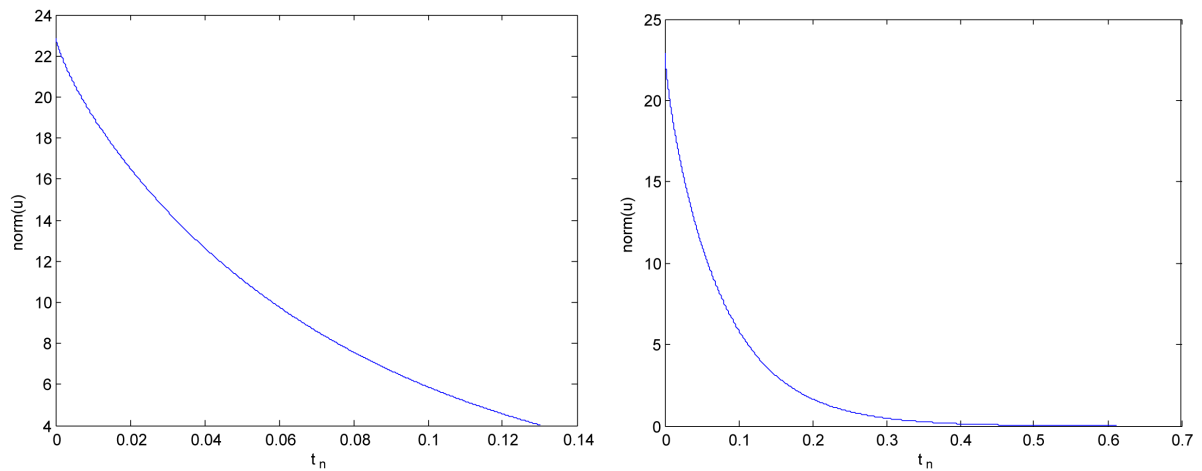| n	13,000	16,000	21,000	31,000	61,000	70,000	100,000	>500,000
T	0.13	0.16	0.21	0.31	0.61	0.6123	0.6123	0.6123

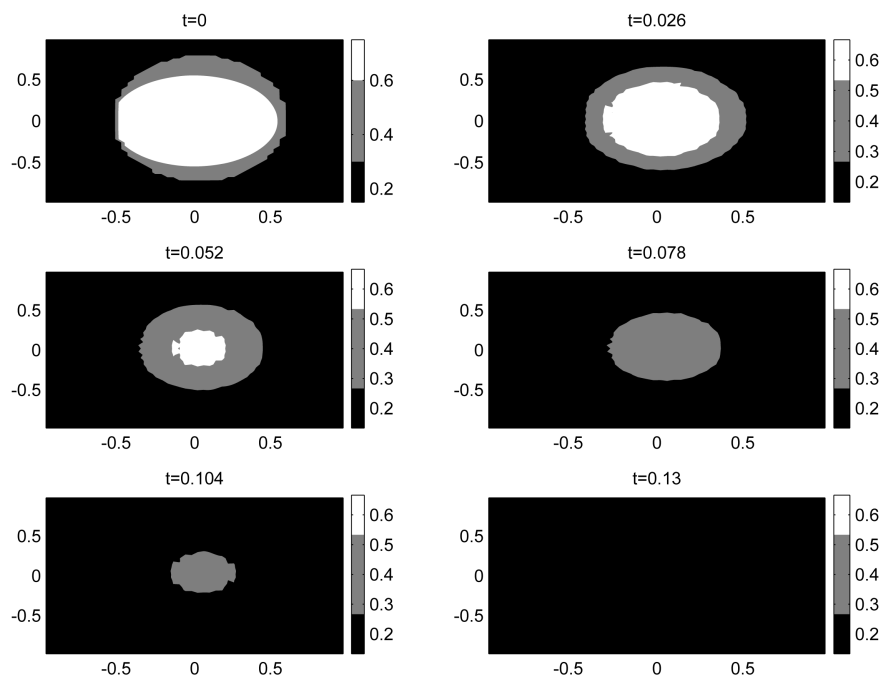**Figure 3.** Variation norm of the numerical solution.

Figure 4. Extinction phenomenon of the numerical solution.

experimentally for a class of nonlinear parabolic equations that the numerical algorithm is efficient for computing the extinction time of solutions.

In the works to come, it will be better to apply the numerical algorithm to study, for example, moving boundary problems and extinction problems in environment.

Acknowledgements

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