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Orbital Properties of Regular Chain

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Abstract

The strong Markov process had been obtained by Ray-Knight compacting; its orbit natures are discussed; the significance probability of kolmogorov forward and backward equations are explained.

Keywords

Regular Chain, Regular State, Transient State, Predictable, Kolmogorov Forward and Backward Equation

1. Introduction

General Markov chain only has locally strong Markov property, which is the main obstruction to solve the problem of Markov chain constructing [1] [2]. The papers construct a strong Markov chain corresponding to its transition function using Ray-Knight compact method [3] [4], which is named regular chain. The papers give an orbit construction of birth and death process [5] [6]. The papers solve the construction problem of two-sided birth and death process [3]-[11]. The papers prove that the appended points in the compacting and the points on the Martin entrance boundary are monogamy, under the condition of finite entrance boundary [12]-[14]. This paper makes a strong Markov process by Ray-Knight compacting, discusses its orbit nature and explains the significance probability of Kolmogorov forward and backward equations.

2. The Orbit Natures of Regular Chain

Assume $P(t)=\left(p_{ij}(t)\right)_{i,j\in E}$ is a honest transition function on $E=\{1,2,\cdots\}$, $Q=\left(q_{ij}\right)_{i,j\in E}$ is its density function, $R_{ij}(\lambda)$ is its resolvent, $\bar{E}$ is the Ray-Knight compacting of E, $\left(U^{\alpha}\right)_{\alpha>0}$ and $\left(P_t\right)_{t\ge 0}$ is the Ray resolvent and the semi-group correspondence, denote $D=\left\{x \middle| x\in\bar{E}, P_0(x,\cdot)=\delta_x(\cdot)\right\}$ as non-ramification point set, $E_R=\left\{x \middle| x\in\bar{E}, U^1(x,E)=1\right\}$, $E^+=E_R\cap D$, then E is Borel algebras on E^+, $X=\left(\Omega,F,F_t,X_t,\theta_t,P^x\right)$ is the

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regular chain of correspondence to $P(t)$. Denote $T_{fl}=\inf\{s|s\ge 0, X_s\ne X_0\}$ and $T_{re}=\inf\{s|s>0, X_s=X_0\}$ respectively as escape time and return time, by Blumenthal 0 - 1 law, for arbitrary $x\in E^+$, $P^x\{T_{fl}=0\}=0$ or 1, $P^x\{T_{re}=0\}$ or 1, if $P^x\{T_{fl}=\infty\}=1$, x is called absorption state, if $P^x\{T_{fl}>0\}=1$, x is called sojourn state, if $P^x\{T_{re}=0\}=1$, x is called regular state, if $P^x\{T_{re}=0\}=P^x\{T_{fl}=0\}=1$, x is called temporary state.

Theorem 1 Let $i\in E$, then

(1) i is a regular state,

(2) on P^i, the distribution of escape time T_{fl} is the exponential distribution of q_i,

(3) on P^i, $X_{T_{fl}}$ and T is mutual independent.

(4) if $0<q_i<\infty$, for arbitrary $j\in E, j\ne i$, $P^i\left\{X_{T_{fl}}=j\right\}=\dfrac{q_{ij}}{q_i}$.

Proof (1) Assume i is not a regular state, then $P^i\{T_{re}>0\}=1$. for arbitrary $t>0$, it is easy to check $\{X_t=i\}\subseteq\{T_{re}\le t\}$, and when $t\to 0$, $\{T_{re}\le t\}\to\{T_{re}=0\}$, thus

$$1=\lim_{t\downarrow 0}p_{ii}(t)=\lim_{t\downarrow 0}P^i\{X_t=i\}\le\lim_{t\downarrow 0}P^i\{T_{re}\le t\}=0,$$

this is a contradictory proposition.

(2) The proof is same as Theorem 5 in [15].

(3) If $q_i=0$ or ∞, then $P^i\{T_{fl}=\infty\}=1$ or $P^i\{T_{fl}=0\}=1$, the conclusion is true, if $0<q_i<\infty$, for arbitrary Borel subset $A\subset E^+$ and $t,s>0$,

$$\begin{aligned}P^i\left\{T_{fl}>t+s, X_{T_{fl}}\in A\right\}&=P^i\left\{T_{fl}>t, T_{fl}\circ\theta_t>s, X_{T_{fl}}\circ\theta_t\in A\right\}\\&=E^i\left\{P^{X_t}\left\{T_{fl}>s, X_{T_{fl}}\in A\right\};T_{fl}>t\right\}=P^i\left\{T_{fl}>t\right\}P^i\left\{T_{fl}>s, X_{T_{fl}}\in A\right\}.\end{aligned}$$

Let $s\to 0$, we have $P^i\left\{T_{fl}>t, X_{T_{fl}}\in A\right\}=P^i\left\{T_{fl}>t\right\}P^i\left\{X_{T_{fl}}\in A\right\}$.

then, on P^i, P^i, $X_{T_{fl}}$ and T is mutual independent.

(4) If $0<q_i<\infty$, for arbitrary $j\ne i,\lambda>0$, According to the strong Markov properties of X and (3), we can obtain that

$$\begin{aligned}R_{ij}(\lambda)&=\int_0^\infty e^{-\lambda}p_{ij}(t)dt=E^i\left[\int_0^\infty e^{-\lambda t}I_{\{j\}}(X_t)dt\right]=E^i\left[\int_{T_{fl}}^\infty e^{-\lambda t}I_{\{j\}}(X_t)dt\right]\\&=E^i\left[e^{-\lambda T_{fl}}\left[\int_0^\infty e^{-\lambda t}I_{\{j\}}(X_t)dt\right]\circ\theta_{T_{fl}}\right]=E^i\left[e^{-\lambda T_{fl}}E^{X_{T_{fl}}}\left[\int_0^\infty e^{-\lambda t}I_{\{j\}}(X_t)dt\right]\right]\\&=E^i\left[e^{-\lambda T_{fl}}\int_0^\infty e^{-\lambda t}P^{X_{T_{fl}}}[X_t=j]dt\right]=E^i\left[e^{-\lambda T_{fl}}\int_0^\infty e^{-\lambda t}P_t\left(X_{T_{fl}},\{j\}\right)dt\right]\\&=E^i\left[e^{-\lambda T_{fl}}U^\lambda\left(X_{T_{fl}},\{j\}\right)\right]=E^i\left[e^{-\lambda T_{fl}}\right]E^i\left[U^\lambda\left(X_{T_{fl}},\{j\}\right)\right]\end{aligned}$$

Give arbitrary $x\in E^+, x\ne j$, and continuous function $f(\cdot)$ on $\bar{E}$ with $f(x)=0, f(j)=1$,

$$0=f(x)=\lim_{\lambda\to\infty}\lambda U^\lambda f(x)\ge\lim_{\lambda\to\infty}\lambda U^\lambda(x,\{j\}),$$

but $\lim_{\lambda\to\infty}\lambda U^\lambda(j,\{j\})=\lim_{\lambda\to\infty}\lambda R_{jj}(\lambda)=1$, in addition,

$$\begin{aligned}q_{ij}&=\lim_{\lambda\to\infty}\lambda^2R_{ij}(\lambda)=\lim_{\lambda\to\infty}\lambda E^i\left[e^{-\lambda T_{fl}}\right]\lim_{\lambda\to\infty}\lambda E^i\left[U^\lambda\left(X_{T_{fl}},\{j\}\right)\right]\\&=\lim_{\lambda\to\infty}\frac{\lambda q_i}{\lambda+q_i}E^i\left[\lim_{\lambda\to\infty}U^\lambda\left(X_{T_{fl}},\{j\}\right)\right]=q_iP^i\left[X_{T_{fl}}=j\right],\end{aligned}$$

thus, $P^i\left\{X_{T_{fl}}=j\right\}=\dfrac{q_{ij}}{q_i}$.

Remark 1 (3), (4) in the Theorem 1 are equivalence with the Theorem 6 in [15], but it require $q_j<\infty$, do not incloude $q_j = \infty$.

Remark 2 According to (2) in Theorem 1, $i \in E$ is a temporary state, if and only if i is a sojourn state of the regular chain $X = \left(\Omega, F, F_t, X_t, \theta_t, P^x\right)$.

Definition 1 Let $S_i(\omega) = \{s | X_s(\omega) = i\}$ is the constant set of i, the interval in $S_i(\omega)$ is called i-interval of X.

Theorem 2 If $q_i < \infty$, then for arbitrary $x \in E^+$, we can get a stopping time squence $a_1, b_1, a_2, b_2, \cdots$, with $a_k \le b_k$, when $\{a_k < \infty\}$, we have $a_k < b_k$, when $\{b_k < \infty\}$, we have $b_k < a_{k+1}$. And for arbitrary k,

$$S_i(\omega) = \bigcup_k \left[a_k(\omega), b_k(\omega)\right), P^x \text{ a.s.}.$$

For arbitrary $s < t$, denote $\xi_i(s,t)$ as the number of $[a_k, b_k)$ belong to $[s,t]$, we have

$$E^x\{\xi_i(s,t)\} \le q_i(t-s).$$

Proof Let

$$a_1 = \inf\{u | u \ge 0, X_u = i\}, b_1 = \inf\{u | u \ge a_1, X_u \ne i\},$$

$$a_{k+1} = \inf\{u | u \ge b_k, X_u = i\}, \quad b_{k+1} = \inf\{u | u \ge a_k, X_u \ne i\}, \quad k = 1, 2, \cdots,$$

where $a_k, b_k, k = 1, 2, \cdots$ are the stoping time of $\{F_t\}$. for arbitrary k, if $a_k < \infty$, since X is right continuity, $X_{a_k} = i$, and

$$\begin{aligned} P^x\left[a_k < \infty, b_k > a_k\right] &= P^x\left[a_k < \infty, T_{fl} \circ \theta a_k > 0\right] \\ &= E^x\left[P^x\left[T_{fl} \circ \theta a_k > 0 | F a_k\right], a_k < \infty\right] \\ &= E^x\left[P^i\left[T_{fl} > 0\right], a_k < \infty\right] = P^x\left[a_k < \infty\right] \end{aligned}$$

then we have almost sure $a_k < b_k$ on $\{a_k < \infty\}$.

Since X is strong Markov chain, and for arbitrary k,

$$\begin{aligned} & P^x\left[b_k < \infty, \exists \varepsilon > 0, X = i, \text{in} \left[b_k, b_k + \varepsilon\right)\right] \\ &= P^x\left[b_k < \infty, X_{b_k} = i, T_{fl} \circ \theta b_k > 0\right] \\ &= E^x\left[P^x\left[T_{fl} \circ \theta b_k > 0 | F b_k\right]; b_k < \infty, X_{b_k} = i\right] \\ &= E^x\left[P^i\left[T_{fl} > 0\right]; b_k < \infty, X_{b_k} = i\right] \\ &= P^x\left[X_{b_k} = i, b_k < \infty\right] \end{aligned}$$

then we have almost sure $X_{b_k} \ne i$ on $\{b_k < \infty\}$.

For arbitrary $0 < s < t$, obviously $\xi_i(s,t) = \xi_i(0, t-s) \circ \theta_s$, by Theorem 3.1 in [15]

$$E^x\{\xi_i(s,t)\} \le E^x\{\xi_i(0,t-s) \circ \theta_s\} = \sum_{k \in E} P^x\{X_s = k\} E^k\{\xi_i(0,t-s)\} \le q_i \cdot (t-s).$$

According to Fatou lemma, for arbitrary $t > 0$, $E^x\{\xi_i(0,t)\} \le q_i \cdot t$, then almost sure there are only finite $[a_k, b_k), k = 1, 2, \cdots$ in a finite interval, such that $\lim_{k \to \infty} a_k = \infty$, this means $S_i(\omega) = \bigcup_k \left[a_k(\omega), b_k(\omega)\right)$.

Theorem 3 If $q_i = \infty$, then

(1) Almost sure, $S_i(\omega)$ do not contain any interval,

(2) Almost sure, $S_i(\omega)$ is a dense set in itself.

Proof (1) Obviously, $S_i(\omega)$ is a optional set, denote $A_t(\omega) = \sup\{s | s < t, s \notin S_i(\omega)\}, t \ge 0, \omega \in \Omega$. (where we assume $\sup \varnothing = 0$), then $\{A_t\}$ is a monotone increasing left continuous process, and adapt in $\{F_t\}$, denote $B_t = \lim_{s \downarrow t} A_s$, thus $\{B_t\}$ is a optional right continuous process. Let

$$U = \{(\omega,t) | \exists \varepsilon > 0, \ni (t-\varepsilon, t+\varepsilon) \subseteq S_i(\omega)\}, \omega \in \Omega.,$$

It is easy to check that $U = \{(\omega,t) | B_t(\omega) < t\}$, thus U is a optional set adapt in $\{F_t\}$.

Assume D_U is debut time, If $P^x\{D_U < \infty\} > 0$, by Section Theorem, exists a stopping time T in $\{F_t\}$, such that $P^x\{T < \infty\} > 0$, and $(\omega, T(\omega)) \in U$ on $\{T < \infty\}$, by (2) in the theorem 1,

$$P^x[T<\infty] = P^x[T<\infty, X_T = i, \exists \varepsilon > 0, \ni X_. \equiv i \text{ on } (T-\varepsilon, T+\varepsilon)]$$
$$\le P^x[T<\infty, X_T = i, T_{fl} \circ \theta_T > 0] = E^x[P^i[T_{fl} > 0]; T < \infty] = 0$$

this is a contradictory proposition, thus $P^x\{D_U < \infty\} = 0$ and almost sure $S_i(\omega)$ do not contain any interval.

(2) The proof is similar to (1).

3. The Significance Probability of Kolmogorov Equations

Theorem 4 For arbitrary $i \in E$, $j \in E$ and $t \ge 0$

$$p'_{ij}(t) = -q_i p_{ij}(t) + \sum_{k \ne i} q_{ik} p_{kj}(t), \tag{1}$$

if and only if $P^i\{X_{T_{fl}} \in E\} = 1$.

Proof For arbitrary $\lambda > 0$,

$$\int_0^\infty e^{-\lambda t}\left[-q_i p_{ij}(t) + \sum_{k\ne i} q_{ik} p_{kj}(t)\right] = -q_i R_{ij}(\lambda) + \sum_{k\ne i} q_{ik} R_{kj}(\lambda)$$

$$\int_0^\infty e^{-\lambda t} p'_{ij}(t)\,dt = \int_0^\infty e^{-\lambda t} dp'_{ij}(t) = -\delta_{ij} + \lambda \int_0^\infty p_{ij}(t)\,dt = -\delta_{ij} + \lambda R_{ij}(\lambda)$$

then (1) and the following equation is equivalence.

$$(\lambda + q_i) R_{ij}(\lambda) - \sum_{k \ne i} q_{ik} R_{kj}(\lambda) = \delta_{ij}, \ \lambda > 0, j \in E \tag{2}$$

According to Theorem 1, we have

$$\begin{aligned}
R_{ij}(\lambda) &= E^i\left[\int_0^\infty e^{-\lambda t} I_{\{j\}}(X_t)\,dt\right] \\
&= E^i\left[\int_0^{T_{fl}} e^{-\lambda t}\delta_{ij}\,dt\right] + E^i\left[\int_{T_{fl}}^\infty e^{-\lambda t} I_{\{j\}}(X_t)\,dt\right] \\
&= \frac{1}{\lambda} E^i\left[1 - e^{-\lambda T_{fl}}\right]\cdot\delta_{ij} + E^i\left[e^{-\lambda T_{fl}}\cdot\left[\int_0^\infty e^{-\lambda t} I_{\{j\}}(X_t)\,dt\right]\cdot\theta_{T_{fl}}\right] \\
&= \frac{1}{\lambda} E^i\left[1 - e^{-\lambda T_{fl}}\right]\cdot\delta_{ij} + E^i\left[e^{-\lambda t T_{fl}} U^\lambda\left(X_{T_{fl}}, \{j\}\right)\right] \\
&\ge \frac{1}{\lambda} E^i\left[1 - e^{-\lambda T_{fl}}\right]\cdot\delta_{ij} + E^i\left[e^{-\lambda t T_{fl}}\right] E^i\left[U^\lambda\left(X_{T_{fl}}, \{j\}\right); X_{T_{fl}} \in E\right] \\
&= \frac{\delta_{ij}}{\lambda + q_i} + \frac{q_{ij}}{\lambda + q_i}\sum_{k\ne j}\frac{q_{ik}}{q_i} R_{kj}(\lambda) = \frac{\delta_{ij} + \sum_{k\ne j} q_{ik} R_{kj}}{\lambda + q_i}
\end{aligned}$$

and the necessary and sufficient condition of equality is $P^i\{X_{T_{fl}} \in E\} = 1$.

For arbitrary $i \in E, q_i < \infty$, let $\left[a_k^{(i)}, b_k^{(i)}\right)$ is the first k i-interval of $S_i(\omega)$,

$$S_\infty^-(\omega) = \{t | t > 0, \text{for arbitrary } \varepsilon > 0, (t-\varepsilon, t) \text{ have infinite jumps}\}$$

Corollary 1 The following conditions are equivalence [16] [17].

(1) The backward equation of Kolmogorov is true,
(2) For arbitrary $i \in E, P^i\left\{X_{T_\beta} \in E\right\} = 1$,
(3) Density matrix Q is conservative,
(4) Almost sure, for all $i \in E$ and $\left[a_k^{(i)}, b_k^{(i)}\right)$, we have $X_{b_k^{(i)}} \in E$.

Theorem 5 For arbitrary $i, j \in E$

$$p_{ij}'(t) = -p_{ij}(t) q_j + \sum_{k \neq j} p_{ik}(t) q_{kj}, \forall t \geq 0 \tag{3}$$

if and only if for all j-interval $\left[a_l^{(j)}, b_l^{(j)}\right)$ almost sure $a_k^{(i)} \notin S_\infty^-$.

Proof (1) Asumme $t_1 < t_2, k \in E, k \neq j, n \in \mathrm{N}$,

$$U = \left\{\exists\, l, t_1 < a_l^{(j)} < t_2 < b_l^{(j)}, a_l^{(j)} \notin S_\infty^-, X_{a_l^{(j)}-} = k\right\}, s_m^n = t_1 + \frac{m}{n}(t_2 - t_1),$$

$$U_m^n = \left\{X_{s_m^n} = k, X_\cdot \equiv j \text{ on } \left[s_m^n + \frac{t_2 - t_1}{n}, t_2\right]\right\}, m = 0,1,2,\cdots,n-1,\ U^n = \bigcup_{m=0}^{n-1} U_m^n.$$

Obviously $U_0^n, U_1^n, \cdots, U_n^{n-1}$ are not intersection. It is easy to check if there are infinite n to make $\omega \in U^n$, then $\omega \in U$, and if $\omega \in U$, then existing N, when $n > N$, we have $\omega \in U^n$, thus that $\lim_{n\to\infty} U^n = U$, and

$$P^i\{U\} = \lim_{n\to\infty} P^i\left\{U^n\right\} = \lim_{n\to\infty} \sum_{m=0}^{n-1} P^i\left\{U_m^n\right\} = \lim_{n\to\infty} \sum_{m=0}^{n-1} p_{ik}\left(s_m^n\right) p_{kj}\left(\frac{t_2 - t_1}{n}\right) \mathrm{e}^{-q_j\left(t_2 - s_m^n - \frac{t_2 - t_1}{n}\right)} = \int_{t_1}^{t_2} p_{ik}(s) q_{kj} \mathrm{e}^{-q_i(t-s)} \mathrm{d}s.$$

(2) For arbitrary $t_1 < t_2$, by (1),

$$\begin{aligned}
p_{ij}(t_2) &= P^i\left[\exists\, l, a_l^{(j)} < t_2 < b_l^{(j)}\right] \\
&= P^i\left[\exists\, l, a_l^{(j)} \leq t_1 < t_2 < b_l^{(j)}\right] + P^i\left[\exists\, l, t_1 < a_l^{(j)} < t_2 < b_l^{(j)}\right] \\
&\geq P_{ij}(t_1) \mathrm{e}^{-q_i(t_2 - t_1)} + P^i\left[\exists\, l, t_1 < a_l^{(j)} < t_2 < b_l^{(j)}, a_l^{(j)} \notin S_\infty^-\right] \\
&= P_{ij}(t_1) \mathrm{e}^{-q_i(t_2 - t_1)} + \sum_{k \neq j} P^i\left[\exists\, l, t_1 < a_l^{(j)} < t_2 < b_l^{(j)}, a_l^{(j)} \notin S_\infty^-, X_{a_l^{(j)}-} = k\right] \\
&= P_{ij}(t_1) \mathrm{e}^{-q_i(t_2 - t_1)} + \sum_{k \neq j} \int_{t_1}^{t_2} p_{ik}(s) q_{kj} \mathrm{e}^{-q_i(t-s)} \mathrm{d}s
\end{aligned} \tag{4}$$

and the necessary and sufficient condition of equality is $P^i\left\{\forall l, a_l^{(j)} \notin S_\infty^-\right\} = 1$.

Thus we get the equation

$$\frac{p_{ij}(t_2) - p_{ij}(t_1) \mathrm{e}^{-q_i(t_2 - t_1)}}{t_2 - t_1} = \frac{\sum_{k \neq j} \int_{t_1}^{t_2} p_{ik}(s) q_{kj} \mathrm{e}^{-q_i(t-s)} \mathrm{d}s}{t_2 - t_1}, \tag{5}$$

let t_2 go to t_1 in Equation (5), we can obtain Equation (3).

Corollary 2 The Kolmogorov forward equations are true if and only if for all $i \in E$ and i-interval $\left[a_k^{(i)}, b_k^{(i)}\right)$, almost sure $a_k^{(i)} \notin S_\infty^-$.

Remark 3 Equation(3) is equivalent to

$$R_{ij}(\lambda) \cdot \left(\lambda + q_j\right) - \sum_{k \neq j} R_{ik}(\lambda) q_{kj} = \delta_{ij}, \forall \lambda > 0. \tag{6}$$

Remark 4 If $P(t)$ contains some transient state, then Equation (1) is true if and only if

$$P^i\left[\forall\, l, \exists k \in E, \left\{s_n\right\}_{n=1}^\infty, \ni s_n \uparrow a_l^{(j)}, X_{s_n} = k\right] = 1$$

Remark 5 Under the condition of $P^i\left\{\forall l, X_{a_l^{(j)}-} \in E\right\}=1$, Equation (1) is not probably true. for the example in Remark 1, the Ray-Knight compaction of E under the resolvent $R_{ij}(\lambda)$ is E, thus, the corresponding regular chain meets the equation $P^i\left[\forall\, l,\, X_{a_l^{i(\,)}-} \in E\right]=1$, but according to Corollary 2, Doob process does not satisfy Kolmogorov forward equation, then $R_{ij}(\lambda)$ also does not satisfy forward equation.

If $P(t)=\left(p_{ij}(t)\right)_{i,j\in E}, t\geq 0$ is an non-honest transition function with total stability, then we can construct a honest transition function $\overline{P}(t)=\left(\overline{p}_{ij}(t)\right)_{i,j\in E_\Delta}, t\geq 0$ on $E_\Delta = E\cup\{\Delta\}$, such that

$$p_{\Delta\Delta}(t)=1,\ p_{\Delta i}(t)=0,\ p_{i\Delta}=1-\sum_{k\in E} p_{ik}(t),\ \forall i\in E, \tag{7}$$

where the density matrix of $\overline{P}(t)$ is $\overline{Q}=\left(\overline{q}_{ij}(t)\right)_{i,j\in E_\Delta}$, such that

$$\overline{q_\Delta}=0,\ \overline{q_{\Delta j}}=0,\ \overline{q_{ij}}=q_{ij},\ \forall i,j\in E \tag{8}$$

the resolvent of $\overline{P}(t)$ is $\overline{R}_{ij}(\lambda)$, then

$$\overline{R_{\Delta\Delta}}(\lambda)=\frac{1}{\lambda},\ \overline{R_{\Delta j}}(\lambda)=0,\ \overline{R_{ij}}(\lambda)=R_{ij}(\lambda),\ \forall i,j\in E \tag{9}$$

Assume $X=\left(\Omega,F,F_t,X_t,\theta_t,P^x\right)$ is a regular chain corresponding to $\overline{P}(t)$. For $\overline{q}_\Delta(t)=0$, by Theorem 1, Δ is a absorption state, this is $p^\Delta\left\{X_t=\Delta,\forall t\geq 0\right\}=1$.

Set $\xi=\inf\left\{s\,|\,X_s=\Delta\right\}, X^\xi=\left\{X_t\,|\,t<\xi\right\}$, obviously X^ξ is a killing Markov process, for arbitrary $i,j\in E$, $\lambda>0$, and $R_{ij}(\lambda)=\overline{R_{ij}}(\lambda)=\int_0^\infty \mathrm{e}^{-\lambda t}P^i\left\{X_t=j\right\}\mathrm{d}t=\int_0^\infty \mathrm{e}^{-\lambda t}P^i\left\{X_t=j,t<\xi\right\}\mathrm{d}t$, we known the transition function of X^ζ is $P(t)$.

For arbitrary $i,j\in E$, since $\overline{p_{\Delta j}}=0, \overline{p_{ij}}(t)=p_{ij}(t)$, then for arbitrary $t\geq 0$ the following equations are Equivalence.

$$\overline{p'_{ij}}(t)=-\overline{p_{ij}}(t)\overline{q_j}+\sum_{k\in E_\Delta,k\neq j}\overline{p_{ik}}(t)\overline{q_{kj}},$$

$$p'_{ij}(t)=-p_{ij}(t)q_j+\sum_{k\in E,k\neq j}p_{ik}(t)q_{kj}.$$

It is easy to get:

Proposition 1 Assume $P(t)=\left(p_{ij}(t)\right)_{i,j\in E}$ is an non-honest transition function with total stability, X^ξ is corresponding Markov process with killing, then $P(t)$ satisfy Kolmogorov backward equation if and only if almost sure for all $i\in E$ and i-interval, $X_{b_k^{(i)}}\in E_\Delta$.

Proposition 2 Assume $P(t)=\left(p_{ij}(t)\right)_{i,j\in E}$ is an non-honest transition function with total stability, X^ξ is corresponding Markov process with killing, then $P(t)$ satisfy Kolmogorov forward equation if and only if almost sure for all $i\in E$ and i-interval, $a_k^{(i)}\notin S_\infty^-$.

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