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Availability Equivalence Factors of a General Repairable Parallel-Series System

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Abstract

The availability equivalence factors of a general repairable parallel-series system are discussed in this paper considering the availability function of the system. The system components are assumed to be repairable and independent but not identical. The life and repair times of the system components are exponentially distributed with different parameters. Two types of availability equivalent factors of the system are derived. The results derived in this paper generalize those given in the literature. A numerical example is introduced to illustrate how the idea of this work can be applied.

Keywords

Reliability Engineering, Duplication Methods, Repairable Systems

1. Introduction

In reliability analysis, there are two main methods to improve non-repairable system design. These methods are the reduction and redundancy methods [1]. In the reduction method, it is assumed that the system design can be improved by reducing the failure rate(s) of a set of system components by a factor ρ, $0<\rho<1$, [1]-[4]. The redundancy method assumes that the system can be improved by increasing its components [5].

There are more than one redundancy methods such as hot, warm, cold and cold with imperfect switch redundancy, named respectively as hot, warm, cold and cold with imperfect switch duplication methods [6]. The redundancy methods can be applied on repairable systems as well. In addition to the reduction method, the repairable system can be improved by increasing the repair rate of some of the system component(s) by a factor σ, $\sigma>1$, [7] [8].

Using the redundancy method may not be a practical solution for a system in which the minimum size and

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weight are overriding considerations: for example, in satellites or other space applications, in well-logging equipment, and in pacemakers and similar biomedical applications [9]. In such applications space or weight limitations may indicate an increase in component performance rather than redundancy. Then more emphasis must be placed on better design, manufacturing quality control and on controlling the operating environment. Therefore, the concept of reliability/availability equivalence takes place. In such concept, the design of the system that is improved according to reduction or increasing method should be equivalent to the design of the system improved according to one of redundancy methods. That is, in this concept, one may say that the performance of a system can be improved through an alternative design [10]. In this case, different system designs should be comparable based on a performance characteristic such as 1) the reliability function or mean time to failure in the case of no repairs or 2) the availability in the case of repairable systems.

The concept of comparing different designs is applied in the literature in order to: 1) improve the reliability of a non-repairable system [11]; 2) determine a representative service provider and create equivalent elements [12]; 3) derive the reliability equivalence factors of some non-repairable systems [2] and the references therein; and 4) derive the availability equivalence factors of a repairable system [7] [8].

The reliability equivalence concept applied on various non-repairable systems, [1] [2] [4] [13]-[17].

In this work, the reliability function and mean time to failure are used as characteristic measures to compare different system designs to derive reliability/mean time equivalence factors.

Repairable system indicates a system that can be repaired to operate normally in the event of any failure, such as automobiles, airplanes, computer network, manufacturing system, sewage systems, power plant or fire prevention system. Availability comprises "reliability" and "recovery part of unreliability after repair", indicating the probability that repairable systems, machines or components maintain the function at a specific moment [18]. It is generally expressed as the operable time over total time. Parallel-series system indicates sub-systems in which several components are connected in series, and then in parallel, or sub-systems that several components are connected in parallel, and then in series [19]. The reliability/availability of a parallel-series system has drawn continuous attention in both problem characteristics and solution methodologies [2], [19] and [20]. Recently, [7] [8] discussed the availability equivalence factors of a repairable series-parallel system with independent and identical (non-identical) components.

Our goal in this paper is to derive the availability equivalence factors of a repairable parallel-series system with independent and non-identical components. The availability function of the system will be used as a performance measure to compare different system designs of the original system and other improved systems in order to derive these factors.

The structure of this paper is organized as follows. Section 2 introduces the illustration of the parallel-series system and the system availability. Section 3 presents the availabilities of the systems improved according to five different methods that can be applied to improve the performance of the original system. In Section 4, two types of availability equivalence factors of the system are discussed. A numerical example is introduced in Section 5 to illustrate how the idea of this work can be applied. Finally, Section 6 is devoted to the conclusions, which handle the main results derived throughout this work.

2. A General Repairable Parallel-Series System

The system considered here consists of n subsystems connected in parallel, and with subsystem i consisting of m_i independent, repairable and nonidentical components connected in series for $i = 1, 2, \cdots, n.$ We refer to such system as a general repairable parallel-series system. **Figure 1** shows the diagram of that system.

Let T_{ij} and Y_{ij} be the lifetime and repair time, respectively, of component j in subsystem i, $1 \le i \le n$, $1 \le j \le m_i$. It is assumed that the life and repair times of component j in subsystem i, $1 \le i \le n$, $1 \le j \le m_i$, follow exponential distributions with failure rate λ_{ij} and repair rate μ_{ij}. Let N be the total number of the system components, that is $N = \sum_{i=1}^{n} m_i$.

Special Cases: This system generalizes the following cases:

1) Repairable parallel-series system with identical components, when $\lambda_{ij} = \lambda$, $\mu_{ij} = \mu$, $j = 1, 2, \cdots, m_i$ and $i = 1, 2, \cdots, n.$

2) Repairable parallel system with non-identical components, when $m_i = 1$ and $i = 1, 2, \cdots, n.$

3) Repairable series system with non-identical components, when $n = 1$ and $j = 1, 2, \cdots, m.$

Let A_{ij}, be the availability of the component j in subsystem i and A_i be the availability of the subsystem i,

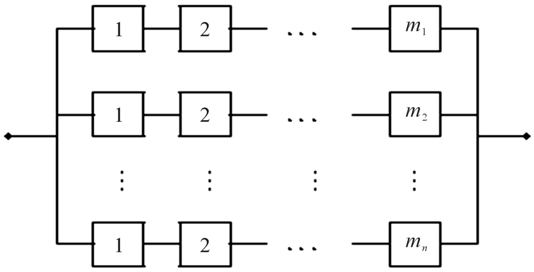

Figure 1. Parallel-series system structure.

$1 \le i \le n$, $1 \le j \le m_i$. One can easily derive A_{ij} and A_i, respectively, as, see [8]

$$A_{ij} = \frac{\mu_{ij}}{\mu_{ij} + \lambda_{ij}} = \frac{1}{1+\eta_{ij}}, \text{ where } \eta_{ij} = \frac{\lambda_{ij}}{\mu_{ij}}, \tag{1}$$

and

$$A_i = \prod_{j=1}^{m_i} A_{ij} = \prod_{j=1}^{m_i} \left(\frac{1}{1+\eta_{ij}}\right). \tag{2}$$

Therefore, the system availability, denoted A_S, can be derived as

$$A_s = 1 - \prod_{i=1}^{n} (1 - A_i) = 1 - \prod_{i=1}^{n} \left[1 - \prod_{j=1}^{m_i} \left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{3}$$

3. Different Designs of Improved System

The system can be improved according to one of the following three different methods:

1) Reduction method. In this method it is assumed that the component can be improved by reducing its failure rate by a factor ρ, $0 < \rho < 1$.

2) Increasing method. It is assumed in this method that the component can be improved by increasing its repair rate by a factor σ, $\sigma > 1$.

3) Standby redundancy method:

a) Hot duplication method: in this method we assume that the component is duplicated by an identical hot standby component.

b) Warm duplication method: in this method we assume that the component is duplicated by an identical warm standby component.

c) Cold duplication method: in this method we assume that the component is duplicated by an identical cold standby component.

In the following sections, we derive the availability of the system improved according to the methods mentioned above.

3.1. The Reduction Method

It is assumed in the reduction method that the system can be improved by reducing the failure rates of a set *R* components by a factor ρ, $0 < \rho < 1$. We assume that $R = \bigcup_{i=1}^{n} R_i$, where R_i is a set of the subsystem *i* components, $1 \le i \le n$. Also, we assume that $|R_i| = r_i$, $0 \le r_i \le m_i$, and $|R| = r = \sum_{i=1}^{n} r_i, (1 \le r \le N)$.

Let $A_{ij,\rho}$ be the availability of the component *j* in subsystem *i*, improved by reducing its failure rate λ_{ij} by the factor ρ. One can easily derive

$$A_{ij,\rho} = \frac{1}{1+\rho\eta_{ij}}, \text{ where } \eta_{ij} = \frac{\lambda_{ij}}{\mu_{ij}}. \tag{4}$$

Therefore, the availability of subsystem *i* improved by reducing the failure rates of a set R_i components by the factor ρ, denoted $A_{R_i,\rho}$, can be written as

$$A_{R_i,\rho} = \prod_{j\in R_i} A_{ij,\rho} \prod_{j\in \bar{R}_i} A_{ij} = \prod_{j\in R_i}\left(\frac{1}{1+\rho\eta_{ij}}\right)\prod_{j\in \bar{R}_i}\left(\frac{1}{1+\eta_{ij}}\right), \tag{5}$$

where $\bar{R}_i = M_i \setminus R_i$, M_i is the set of all subsystem *i* components, $M_i = \{1,2,\cdots,m_i\}$, $1\le i\le n$.

Finally, the availability of the system improved by reducing the failure rates of a set *R* components by the same factor ρ, denoted $A_{R,\rho}$, can be derived as

$$A_{R,\rho} = 1-\prod_{i=1}^{n}\left[1-\prod_{j\in R_i}\left(\frac{1}{1+\rho\eta_{ij}}\right)\prod_{j\in \bar{R}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{6}$$

3.2. The Increasing Method

It is assumed in the increasing method that the system can be improved by increasing the repair rates of a set *S* components by a factor σ, $\sigma > 1$. We assume that $S = \bigcup_{i=1}^{n} S_i$, where S_i is a set of the subsystem *i* components, $1\le i\le n$. Also, we assume that $|S_i| = s_i, 0\le s_i\le m_i$, and $|S| = s = \sum_{i=1}^{n} s_i$, $1\le s\le N$.

Let $A_{ij,\sigma}$ be the availability of component *j* in subsystem *i* after increasing its repair rate μ_{ij} by the factor σ, $\sigma > 1$ and $A_{S_i,\sigma}$ be the availability of subsystem *i* which is improved by increasing the repair rates of a set S_i components by the same factor σ; and $A_{S,\sigma}$ be the availability of the system improved by increasing the repair rates of a set *S* components by the same factor σ. One can derive these availabilities in the following forms

$$A_{ij,\sigma} = \frac{\sigma\mu_{ij}}{\sigma\mu_{ij}+\lambda_{ij}} = \frac{\sigma}{\sigma+\eta_{ij}}, \tag{7}$$

$$A_{S_i,\sigma} = \prod_{j\in S_i} A_{ij,\sigma} \prod_{j\in \bar{S}_i} A_{ij} = \prod_{j\in S_i}\left(\frac{\sigma}{\sigma+\eta_{ij}}\right)\prod_{j\in \bar{S}_i}\left(\frac{1}{1+\eta_{ij}}\right), \tag{8}$$

$$A_{S,\sigma} = 1-\prod_{i=1}^{n}\left[1-\prod_{j\in S_i}\left(\frac{\sigma}{\sigma+\eta_{ij}}\right)\prod_{j\in \bar{S}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right], \tag{9}$$

where $\bar{S}_i = M \setminus S_i$, for $1\le i\le n$.

3.3. The Hot Duplication Method

It is assumed in the hot duplication method that the system can be improved by connecting every element in a set *B* components with an identical component in parallel. We assume that $B = \bigcup_{i=1}^{n} B_i$, where B_i is a set of the subsystem *i* components, $1\le i\le n$. Also, we assume that $|B_i| = h_i$, $0\le h_i\le m_i$, and $|B| = h = \sum_{i=1}^{n} h_i$, $1\le h\le N$.

Let $A_{B_i}^{H}$ be the availability of the subsystem *i* which is improved by improving a set $B_i \subseteq M_i$ components, $1\le i\le n$; and A_B^H be the availability of the system improved by improving a set *B* components according to the hot duplication method. One can derive

$$A_{B_i}^{H} = \prod_{j\in B_i}\left[1-\left(1-A_{ij}\right)^2\right]\prod_{j\in \bar{B}_i} A_{ij} = \prod_{j\in B_i}\left[1-\left(\frac{\eta_{ij}}{1+\eta_{ij}}\right)^2\right]\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right), \tag{10}$$

$$A_{B}^{H} = 1-\prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left[1-\left(\frac{\eta_{ij}}{1+\eta_{ij}}\right)^2\right]\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right], \tag{11}$$

where $\bar{B}_i = M_i \setminus B_i$, for $1\le i\le n$.

3.4. The Warm Duplication Method

We say that, a component *j* in subsystem *i* is warm duplicated if it is connected in parallel with a non-identical component, having a failure rate ν_{ij}, in parallel via a perfect switch. In the warm duplication method, it is assumed that the system can be improved when every component in a set B components is warm duplicated. We assume that $B=\bigcup_{i=1}^{n} B_i$, where B_i is a set of the subsystem i components, $1\le i\le n$. Also, we assume that $|B_i|=w_i$, $0\le w_i\le m_i$, and $|B|=w=\sum_{i=1}^{n} w_i, 1\le w\le N$.

Let A_{ij}^{W} be the availability of the component j in the subsystem i when it is improved according to the warm duplication method. Using Markov process, A_{ij}^{W} can be obtained as follows, see [21],

$$A_{ij}^{W}=\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+\frac{1}{2}\eta_{ij}^{2}+\frac{1}{2}\eta_{ij}\xi_{ij}}, \tag{12}$$

where $\xi_{ij}=\nu_{ij}/\mu_{ij}$, for $1\le j\le m_i$ and $1\le i\le n$.

Let $A_{B_i}^{W}$ be the availability of the subsystem i improved by improving B_i subsystem components according to the warm duplication method. Therefore, one can derive

$$A_{B_i}^{W}=\prod_{j\in B_i}\left(\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+\frac{1}{2}\eta_{ij}^{2}+\frac{1}{2}\eta_{ij}\xi_{ij}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right), \tag{13}$$

Finally, let A_{B}^{W} be the availability of the system improved by improving a set B components according to the warm duplication methods. Using Equation (13), we get

$$A_{B}^{W}=1-\prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left(\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+\frac{1}{2}\eta_{ij}^{2}+\frac{1}{2}\eta_{ij}\xi_{ij}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{14}$$

3.5. The Cold Duplication Method

It is assumed in the cold duplication method, that each component of set B components is connected in parallel with an identical component via a perfect switch. We assume that $B=\bigcup_{i=1}^{n} B_i$, where B_i is a set of the subsystem i components, $1\le i\le n$. Also, we assume that $|B_i|=c_i$, $0\le c_i\le m_i$, and $|B|=c=\sum_{i=1}^{n} c_i, 1\le c\le N$.

Let A_{ij}^{C} is the availability of the component j in subsystem i when it is improved according to the cold duplication method; $A_{B_i}^{C}$ be the availability of subsystem i, which is improved according to cold duplication method; and A_{B}^{C} be the availability of the system improved by improving set B components according to the cold duplication method. Using Markov process theory, A_{ij}^{C} is, see [22],

$$A_{ij}^{C}=\frac{\mu_{ij}^{2}+\lambda_{ij}\mu_{ij}}{\mu_{ij}^{2}+\lambda_{ij}\mu_{ij}+\frac{1}{2}\lambda_{ij}^{2}}=\frac{1+\eta_{ij}}{1+\eta_{ij}+\frac{1}{2}\eta_{ij}^{2}}. \tag{15}$$

Using Equation (15) and the nature of the series subsystem i, one can derive

$$A_{B_i}^{C}=\prod_{j\in B_i}\left(\frac{1+\eta_{ij}}{1+\eta_{ij}+\frac{1}{2}\eta_{ij}^{2}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right). \tag{16}$$

Finally, using Equation (16) and the nature of the parallel connection of the subsystems, we get

$$A_{B}^{C}=1-\prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left(\frac{1+\eta_{ij}}{1+\eta_{ij}+\frac{1}{2}\eta_{ij}^{2}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{17}$$

4. Availability Equivalence Factors

In this section, we derive the availability equivalence factors of a repairable parallel-series system with independent, non-identical and repairable components. Two types of availability equivalence factors will be discussed. These two types are referred as availability equivalent reducing factor and availability equivalent increasing factor. Following the definition of reliability equivalence factors introduced in [1].

4.1. Availability Equivalence Reducing Factor

Availability equivalence reducing factor, in short AERF, referred as $\rho = \rho_{R,B}^{D}$, $D = H, W, C$ for hot, warm and cold, respectively, is defined as the factor ρ by which the failure rate of a set R components should be reduced in order to get equality of the availability of another better design which can be obtained from the original system by assuming hot, warm and cold duplications of a set B components. That is, $\rho = \rho_{R,B}^{D}$, for $D = H, W, C$, is the solution of the following equations in ρ,

$$A_{R,\rho} = A_B^D,\ D = H, W, C. \tag{18}$$

In what follows, we give the non-linear equations needed to be solved to get the three possible AERF's.

1) Hot availability equivalence reducing factor (HAERF): Substituting Equations (6) and (11) into Equation (18), $\rho = \rho_{R,B}^{H}$, is the solution of the following non-linear equation in ρ,

$$\prod_{i=1}^{n}\left[1-\prod_{j\in R_i}\left(\frac{1}{1+\rho\eta_{ij}}\right)\prod_{j\in \bar{R}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right] = \prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left[1-\left(\frac{\eta_{ij}}{1+\eta_{ij}}\right)^2\right]\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{19}$$

2) Warm availability equivalence reducing factor (WAERF): Substituting Equations (6) and (14) into Equation (18), $\rho = \rho_{R,B}^{W}$, is the solution of the following non-linear equation in ρ,

$$\prod_{i=1}^{n}\left[1-\prod_{j\in R_i}\left(\frac{1}{1+\rho\eta_{ij}}\right)\prod_{j\in \bar{R}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right] = \prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left(\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+\frac{1}{2}\eta_{ij}^2+\frac{1}{2}\eta_{ij}\xi_{ij}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{20}$$

3) Cold availability equivalence reducing factor (CAERF): Substituting Equations (6) and (17) into Equation (18), $\rho = \rho_{R,B}^{C}$, satisfies the following non-linear equation

$$\prod_{i=1}^{n}\left[1-\prod_{j\in R_i}\left(\frac{1}{1+\rho\eta_{ij}}\right)\prod_{j\in \bar{R}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right] = \prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left(\frac{1+\eta_{ij}}{1+\eta_{ij}+\frac{1}{2}\eta_{ij}^2}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{21}$$

Equations (19)-(21) have no closed solutions, therefore, a numerical technique method is needed to get their solutions.

4.2. Availability Equivalence Increasing Factor

Availability equivalence increasing factor, in short AEIF, referred as $\sigma = \sigma_{S,B}^{D}$, $D = H, W, C$ for hot, warm and cold, respectively, is defined as the factor σ by which the failure rate of a set S components should be reduced in order to get equality of the availability of another better design which can be obtained from the original system by assuming hot, warm and cold duplications of a set B components. That is, $\sigma = \sigma_{S,B}^{D}$, for $D = H, W, C$, is the solution of the following equations in σ.

$$A_{S,\sigma} = A_B^D,\ D = H, W, C. \tag{22}$$

In what follows, we give the non-linear equations needed to be solved to get the three possible AEIF's.

1) Hot availability equivalence increasing factor (HAEIF): Substituting Equations (9) and (11) into Equation (22), $\sigma = \sigma_{S,B}^{H}$ is the solution of the following non-linear equation

$$\prod_{i=1}^{n}\left[1-\prod_{j\in S_i}\left(\frac{\sigma}{\sigma+\eta_{ij}}\right)\prod_{j\in \bar{S}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right] = \prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left[1-\left(\frac{\eta_{ij}}{1+\eta_{ij}}\right)^2\right]\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{23}$$

2) Warm availability equivalence increasing factor (WAEIF): Substituting Equations (9) and (14) into Equation (22), $\sigma = \sigma_{S,B}^{W}$ is the solution of the following equation in σ

$$\prod_{i=1}^{n}\left[1-\prod_{j\in S_i}\left(\frac{\sigma}{\sigma+\eta_{ij}}\right)\prod_{j\in \bar{S}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right] = \prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left(\frac{1+\eta_{ij}+\xi_{ij}}{1+\eta_{ij}+\xi_{ij}+\frac{1}{2}\eta_{ij}^2+\frac{1}{2}\eta_{ij}\xi_{ij}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \tag{24}$$

3) Cold availability equivalence increasing factor (CAEIF): Substituting Equations (9) and (17) into Equation (22), $\sigma = \sigma_{S,B}^{C}$ is the solution of the following equation in σ,

$$\prod_{i=1}^{n}\left[1-\prod_{j\in S_i}\left(\frac{\sigma}{\sigma+\eta_{ij}}\right)\prod_{j\in \bar{S}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]=\prod_{i=1}^{n}\left[1-\prod_{j\in B_i}\left(\frac{1+\eta_{ij}}{1+\eta_{ij}+\frac{1}{2}\eta_{ij}^{2}}\right)\prod_{j\in \bar{B}_i}\left(\frac{1}{1+\eta_{ij}}\right)\right]. \quad (25)$$

The above Equations (23)-(25) have no closed-form solutions in σ, so a numerical technique method to get the value of σ.

5. Numerical Results

To explain how one can utilize the previously obtained theoretical results, we introduce a numerical example. In such example, we calculate the two different availability equivalence factors of a general repairable parallel-series with n subsystems. Each subsystem consists of m_i, $i = 1, 2, \cdots, n$, non-identical components, under the following assumptions:

1) The parallel-series system has two subsystems, $n = 2$;

2) The subsystems have the components, $m_1 = 1$, $m_2 = 2$ then $N = m_1 + m_2 = 3$;

3) The values of the system parameters λ_{ij}, μ_{ij}, and $\nu_{ij}\left(i=1,2,\ j=1,\cdots,m_i\right)$ are presented in **Table 1**.

The objective is to improve the repairable parallel-series system by improving the performance of some components instead of increasing the number of these components.

We give the values of availability of the original system and of the design obtained using the duplication methods for the example considered in this section.

Table 2 shows the availability of the original and improved system obtained from the original system by applying hot, warm and cold duplications using all possible set B components, where $B = B_1 \cup B_2$ and ϕ is the empty set.

From the results shown in **Table 2**, one can easily see that:

1) $A_S < A_B^W < A_B^H < A_B^C$, for all possible set B components when $\lambda < \nu$;

2) $A_S < A_B^H < A_B^W < A_B^C$, for all possible set B components when $\lambda > \nu$;

Table 1. Set values of the system parameters.

i	j	$\lambda < \nu$			$\lambda > \nu$		
		λ	ν	μ	λ	ν	μ
1	1	0.10	0.12	1.1	0.12	0.1	1.1
2	1	0.11	0.13	1.2	0.13	0.11	1.2
	2	0.12	0.14	1.3	0.14	0.12	1.3

Table 2. The availability of the improved system, A_B^D, $D = H, W, C$.

$\lvert B\rvert$	$B = B_1 \cup B_2$	$\lambda < \nu$				$\lambda > \nu$			
		A_S	A_B^H	A_B^W	A_B^C	A_S	A_B^H	A_B^W	A_B^C
1	$B_1 = \{1\}$, $B_2 = \phi$	0.98655	0.99888	0.99879	0.99939	0.98176	0.99821	0.99833	0.99901
	$B_1 = \phi$, $B_2 = \{1\}$		0.99242	0.99238	0.99267		0.98959	0.98964	0.98997
	$B_1 = \phi$, $B_2 = \{2\}$		0.99246	0.99242	0.99271		0.98955	0.98960	0.98992
2	$B_1 = \{1\}$, $B_2 = \{1\}$		0.99937	0.99931	0.99967		0.99898	0.99905	0.99946
	$B_1 = \{1\}$, $B_2 = \{2\}$		0.99936	0.99932	0.99967		0.99897	0.99905	0.99945
	$B_1 = \phi$, $B_2 = \{1,2\}$		0.99882	0.99874	0.99936		0.99814	0.99825	0.99897
3	$B_1 = \{1\}$, $B_2 = \{1,2\}$		0.99990	0.99989	0.99997		0.99982	0.99984	0.99994

3) Improving the only one component in subsystem 1, according to the duplication method, provides a better design than that can be achieved by improving one component from the subsystem 2, according to the same method;

4) Duplicating two components, one from each subsystem, produces a better design than that can be obtained by duplicating the two components in subsystem 2, according to the same method; and

5) Cold duplicating all components in the system provides the best design, in the sense of having the highest availability.

We used Mathematica Program System to calculate all possible availability equivalence factors of the studied system. **Table 3** and **Table 4** give the hot, warm and cold ($D = H, W, C$) availability equivalence reducing factors, $\rho = \rho^{D}_{R,B}$, and the hot, warm and cold availability equivalence increasing factors, $\sigma = \sigma^{D}_{S,B}$, respectively, for all possible sets R, S and B.

From the results presented in **Table 3**, **Table 4**, we can immediately conclude that:

Table 3. The AERF ($\rho^{D}_{R,B}$, $D = H, W, C$) for different R, B, when $\lambda < \nu$.

$\|R\|$	$R = R_1 \cup R_2$	$\|B\|=1$			$\|B\|=2$			$\|B\|=3$
		$B_1=\{1\}$, $B_2=\phi$	$B_1=\phi$, $B_2=\{1\}$	$B_1=\phi$, $B_2=\{2\}$	$B_1=\{1\}$, $B_2=\{1\}$	$B_1=\{1\}$, $B_2=\{2\}$	$B_1=\phi$, $B_2=\{1,2\}$	$B_1=\{1\}$, $B_2=\{1,2\}$
				$\rho^{H}_{R,B}$				
	$R_1=\{1\}$, $R_2=\phi$	0.07692	0.54214	0.53933	0.04323	0.04301	0.08092	0.00670
1	$R_1=\phi$, $R_2=\{1\}$	NA	0.07746	0.07202	NA	NA	NA	NA
	$R_1=\phi$, $R_2=\{2\}$	NA	0.08333	0.07792	NA	NA	NA	NA
	$R_1=\{1\}$, $R_2=\{1\}$	0.13082	0.64953	0.64706	0.07696	0.07660	0.13692	0.01264
2	$R_1=\{1\}$, $R_2=\{2\}$	0.13139	0.65015	0.64767	0.07735	0.07699	0.13751	0.01272
	$R_1=\phi$, $R_2=\{1,2\}$	0.07384	0.53094	0.52811	0.04144	0.04123	0.07769	0.00641
3	$R_1=\{1\}$, $R_2=\{1,2\}$	0.26678	0.72971	0.72775	0.19877	0.19826	0.27378	0.07731
				$\rho^{W}_{R,B}$				
	$R_1=\{1\}$, $R_2=\phi$	0.08333	0.54518	0.54214	0.04707	0.04682	0.08677	0.00777
1	$R_1=\phi$, $R_2=\{1\}$	NA	0.08333	0.07746	NA	NA	NA	NA
	$R_1=\phi$, $R_2=\{2\}$	NA	0.08916	0.08333	NA	NA	NA	NA
	$R_1=\{1\}$, $R_2=\{1\}$	0.14058	0.65219	0.64953	0.08333	0.08292	0.14577	0.01464
2	$R_1=\{1\}$, $R_2=\{2\}$	0.14118	0.65280	0.65015	0.08375	0.08333	0.14638	0.01473
	$R_1=\phi$, $R_2=\{1,2\}$	0.08002	0.53398	0.53094	0.04513	0.04489	0.08333	0.00744
3	$R_1=\{1\}$, $R_2=\{1,2\}$	0.27793	0.73182	0.72971	0.20758	0.20702	0.28375	0.08333
				$\rho^{C}_{R,B}$				
	$R_1=\{1\}$, $R_2=\phi$	0.04167	0.52376	0.52071	0.02269	0.02256	0.04394	0.00198
1	$R_1=\phi$, $R_2=\{1\}$	NA	0.04198	0.03612	NA	NA	NA	NA
	$R_1=\phi$, $R_2=\{2\}$	NA	0.04808	0.04225	NA	NA	NA	NA
	$R_1=\{1\}$, $R_2=\{1\}$	0.07435	0.63329	0.63058	0.04168	0.04146	0.07814	0.00377
2	$R_1=\{1\}$, $R_2=\{2\}$	0.07473	0.63393	0.63122	0.04191	0.04169	0.07854	0.00380
	$R_1=\phi$, $R_2=\{1,2\}$	0.03994	0.51251	0.50946	0.02173	0.02161	0.04212	0.00190
3	$R_1=\{1\}$, $R_2=\{1,2\}$	0.19508	0.71682	0.71466	0.14323	0.14283	0.20043	0.04189

Table 4. The AEIF ($\sigma^D_{S,B}$, $D = H, W, C$) for different S, B, when $\lambda < \nu$.

$\lvert S\rvert$	$S = S_1 \cup S_2$	$\lvert B\rvert = 1$: $B_1 = \{1\}$, $B_2 = \phi$	$\lvert B\rvert = 1$: $B_1 = \phi$, $B_2 = \{1\}$	$\lvert B\rvert = 1$: $B_1 = \phi$, $B_2 = \{2\}$	$\lvert B\rvert = 2$: $B_1 = \{1\}$, $B_2 = \phi$	$\lvert B\rvert = 2$: $B_1 = \phi$, $B_2 = \{1\}$	$\lvert B\rvert = 2$: $B_1 = \phi$, $B_2 = \{2\}$	$\lvert B\rvert = 3$: $B_1 = \{1\}$, $B_2 = \phi$
				$\sigma^H_{S,B}$				
	$S_1 = \{1\}$, $S_2 = \phi$	13.0000	1.8445	1.8542	23.1343	23.2500	12.3580	149.2960
1	$S_1 = \phi$, $S_2 = \{1\}$	NA	12.9091	13.8852	NA	NA	NA	NA
	$S_1 = \phi$, $S_2 = \{2\}$	N	12.0000	12.8333	N	N	N	N
	$S_1 = \{1\}$, $S_2 = \{1\}$	7.6442	1.5396	1.5455	12.9940	13.0549	7.3034	79.1162
2	$S_1 = \{1\}$, $S_2 = \{2\}$	7.6110	1.5381	1.5440	12.9283	12.9888	7.2722	78.6321
	$S_1 = \phi$, $S_2 = \{1,2\}$	NA	NA	NA	NA	NA	NA	NA
3	$S_1 = \{1\}$, $S_2 = \{1,2\}$	3.7485	1.3704	1.3741	5.0310	5.0438	3.6525	12.9354
				$\sigma^W_{S,B}$				
	$S_1 = \{1\}$, $S_2 = \phi$	12.0000	1.8343	1.8445	21.2464	21.3602	11.5245	128.6470
1	$S_1 = \phi$, $S_2 = \{1\}$	NA	12.0000	12.9091	NA	NA	NA	NA
	$S_1 = \phi$, $S_2 = \{2\}$	NA	11.2152	12.0000	NA	NA	NA	NA
	$S_1 = \{1\}$, $S_2 = \{1\}$	7.1132	1.5333	1.5396	12.0000	12.0600	6.8604	68.3015
2	$S_1 = \{1\}$, $S_2 = \{2\}$	7.0831	1.5319	1.5381	11.9404	12.0000	6.8318	67.8861
	$S_1 = \phi$, $S_2 = \{1,2\}$	NA	NA	NA	NA	NA	NA	NA
3	$S_1 = \{1\}$, $S_2 = \{1,2\}$	3.5980	1.3665	1.3704	4.8174	4.8305	3.5242	12.0000
				$\sigma^C_{S,B}$				
	$S_1 = \{1\}$, $S_2 = \phi$	24.0000	1.9093	1.9205	44.0802	44.3267	22.7607	504.5480
1	$S_1 = \phi$, $S_2 = \{1\}$	NA	23.8182	27.6840	NA	NA	NA	NA
	$S_1 = \phi$, $S_2 = \{2\}$	NA	20.7991	23.6667	NA	NA	NA	NA
	$S_1 = \{1\}$, $S_2 = \{1\}$	13.4495	1.5791	1.5859	23.9930	24.1223	12.7973	265.1530
2	$S_1 = \{1\}$, $S_2 = \{2\}$	13.3809	1.5775	1.5842	23.8585	23.9870	12.7328	263.4860
	$S_1 = \phi$, $S_2 = \{1,2\}$	NA	NA	NA	NA	NA	NA	NA
3	$S_1 = \{1\}$, $S_2 = \{1,2\}$	5.1260	1.3951	1.3993	6.9816	7.0014	4.9894	23.8707

1) Hot duplication of the only one component in subsystem 1, $B_1 = \{1\}$ and $B_2 = \phi$ increases the system availability from $A_S = 0.98655$ to $A^H_B = 0.99888$, $B = B_1 \cup B_2$, see **Table 2**. The improved system with A^H_B = 0.99888 can be achieved by performing one of the following:

a) Reducing the failure rate(s) of (see **Table 3**): i) the only component in subsystem 1, $R = R_1 \cup R_2$, where $R_1 = \{1\}$ and $R_2 = \phi$, by the HAERF $\rho^H_{R,B} = 0.07692$, ii) the only component in subsystem 1 and the first component in subsystem 2, $R_1 = \{1\}$, $R_2 = \{1\}$, by the HAERF $\rho^H_{R,B} = 0.13082$, iii) the only component in subsystem 1 and the second component in subsystem 2, $R_1 = \{1\}$, $R_2 = \{2\}$, by the HAERF $\rho^H_{R,B} = 0.13139$, iv) the two components in subsystem 2, $R_1 = \phi$ and $R_2 = \{1,2\}$, by the HAERF $\rho^H_{R,B} = 0.07384$, v) all the three components, $R_1 = \{1\}$, $R_2 = \{1,2\}$, by the HAERF $\rho^H_{R,B} = 0.26678$.

b) Increasing the repair rate(s) of (see **Table 4**): i) the only component in subsystem 1, $S = S_1 \cup S_2$, where $S_1 = \{1\}$ and $S_2 = \phi$, by the HAEIF $\sigma^H_{S,B} = 13.0000$, ii) the only component in subsystem 1 and first component in subsystem 2, $S_1 = \{1\}$, $S_2 = \{1\}$, by the HAEIF $\sigma^H_{S,B} = 7.6442$, iii) the only component in subsystem 1 and second component in subsystem 2, $S_1 = \{1\}$, $S_2 = \{2\}$, by the HAEIF $\sigma^H_{S,B} = 7.6110$, iv) all the three components, $S_1 = \{1\}$, $S_2 = \{1,2\}$, by the HAEIF $\sigma^H_{S,B} = 3.7485$.

2) Warm duplication of the only component in subsystem 1, $B_1 = \{1\}$ and $B_2 = \phi$, increases the system availability from $A_S = 0.98655$ to $A^W_B = 0.99879$, $B = B_1 \cup B_2$ see **Table 2**. The improved system with $A^W_B =$ 0.99879, can be achieved by performing one of the following:

a) Reducing the failure rate(s) of (see **Table 3**): i) the only component in subsystem 1, $R = R_1 \cup R_2$ where $R_1 = \{1\}$ and $R_2 = \phi$, by the WAERF $\rho^W_{R,B} = 0.08333$, ii) the only component in subsystem 1 and the first component of subsystem 2, $R_1 = \{1\}$, $R_2 = \{1\}$, by the WAERF $\rho^W_{R,B} = 0.14058$, iii) the only component in subsystem 1 and the second component of subsystem 2, $R_1 = \{1\}$, $R_2 = \{1\}$, by the WAERF $\rho^W_{R,B} = 0.14118$, iv) the two components in subsystem 2, $R_1 = \phi$, $R_2 = \{1,2\}$, by the WAERF $\rho^W_{R,B} = 0.08002$, v) all three components, $R_1 = \{1\}$, $R_2 = \{1,2\}$, by the WAERF $\rho^W_{R,B} = 0.27793$.

b) Increasing the repair rate(s) of (see **Table 4**): i) the only component in subsystem 1, $S = S_1 \cup S_2$ where $S_1 = \{1\}$ and $S_2 = \phi$, by the WAEIF $\sigma^W_{S,B} = 12.0000$, ii) the only component in subsystem 1 and first component of subsystem 2, $S_1 = \{1\}$, $S_2 = \{1\}$, by the WAEIF $\sigma^W_{S,B} = 7.1132$, iii) the only component in subsystem 1 and second component of subsystem 2, $S_1 = \{1\}$, $S_2 = \{2\}$, by the WAEIF $\sigma^W_{S,B} = 7.0831$, iv) all three components, $S_1 = \{1\}$, $S_2 = \{1,2\}$, by the WAEIF $\sigma^W_{S,B} = 3.5980$.

3) Cold duplication of the only component in subsystem 1, $B_1 = \{1\}$ and $B_2 = \phi$, increases the system availability from $A_S = 0.98655$ to $A^C_B = 0.99939$, see **Table 2**. The improved system with $A^C_B = 0.99939$, can be achieved by performing one of the following:

a) Reducing the failure rate(s) of (see **Table 3**): i) the only component in subsystem 1, $R = R_1 \cup R_2$ where $R_1 = \{1\}$ and $R_2 = \phi$ by the CAERF $\rho^C_{R,B} = 0.04167$, ii) the only component in subsystem 1 and first component of subsystem 2, $R_1 = \{1\}$, $R_2 = \{1\}$, by the CAERF $\rho^C_{R,B} = 0.07435$, iii) the only component in subsystem 1 and second component of subsystem 2, $R_1 = \{1\}$, $R_2 = \{2\}$, by the CAERF $\rho^C_{R,B} = 0.07473$, iv) the two components in subsystem 2, $R_1 = \phi$, $R_2 = \{1,2\}$, by the CAERF $\rho^C_{R,B} = 0.03994$, v) all three components, $R_1 = \{1\}$, $R_2 = \{1,2\}$, by the CAERF $\rho^C_{R,B} = 0.19508$.

b) Increasing the repair rate(s) of (see **Table 4**): i) the only component in subsystem 1, $S = S_1 \cup S_2$ where $S_1 = \{1\}$ and $S_2 = \phi$ by the CAEIF $\sigma^C_{S,B} = 24.0000$, ii) the only component in subsystem 1 and first component of subsystem 2, $S_1 = \{1\}$, $S_2 = \{1\}$, by the CAEIF $\sigma^C_{S,B} = 13.4495$, iii) the only component in subsystem 1 and second component of subsystem 2, $S_1 = \{1\}$, $S_2 = \{2\}$, by the CAEIF $\sigma^C_{S,B} = 13.3809$, iv) all three components, $S_1 = \{1\}$, $S_2 = \{1,2\}$, by the CAEIF $\sigma^C_{S,B} = 5.1260$.

4) In the same manner, we can illustrate the rest of results shown in **Table 3** and **Table 4**.

5) The notation NA, means that there is no possible equivalence between the two improved systems that can be achieved by reducing (increasing) the failure (repair) rates of the set $R(S)$ of system components and that can be achieved by duplicating elements of set *B* of system components.

6. Conclusions

This paper discusses the availability equivalence factors of a general repairable parallel-series system with independent but non-identical components. The system studied here generalizes several well-known systems such as a repairable parallel-series system with independent and identical components; repairable series and repairable parallel systems with independent and non-identical or identical components. We derived two types of the availability equivalence factors of the system. We presented a numerical example to illustrate how the theoretical results derived in the paper can be applied.

Indeed there are several possible extensions of this work. As an example, the case of a general repairable parallel-series system with non-constant failure rates can be studied.

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