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Coincidence and Common Fixed Point of Weakly Compatible Maps in Fuzzy Metric Space

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Abstract

The aim of this paper is to establish some new common fixed point theorems for generalized contractive maps in fuzzy metric space by using property (*E.A.*), common property (*E.A.*), $JCLR_{ST}$ property and CLR_S property. Our results improve and extend the results of Chauhan *et al.* [1] and Sedghi *et al.* [2] besides several known results. We also furnish an illustrative example in support of our results.

Keywords

***FM*-Space, Weakly Compatible Maps, CLR_g Property, Property (*E.A.*), The Common Property (*E.A.*), $JCLR_{ST}$ Property and Common Fixed Point**

1. Introduction

Aamri *et al*. [3] generalized the concept of non compatibility by defining the notion of property (*E.A.*) and proved common fixed point theorems under strict contractive conditions. Many authors have proved common fixed point theorems in different settings for different contractive conditions. For details, we refer to [4]-[13].

In 2005, Liu *et al.* [14] further improved it by common property (*E.A*) while proving common fixed point theorems under strict contractive conditions. Recently, Sintunavarat *et al.* [13], defined the notion of (CLR_g) property which is more general than (*E.A*) property.

Very recently, Manro *et al.* [15] introduced the notion of (CLR_S) property and Chauhan *et al.* [4] introduced the notion of ($JCLR_{ST}$) property.

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The aim of this paper is to establish some new common fixed point theorems for generalized contractive maps in fuzzy metric space by using property (*E.A.*), common property (*E.A.*), $JCLR_{ST}$ property and CLR_S property. Our results improve and extend the results of Chauhan *et al.* [1] and Sedghi *et al.* [2] besides several known results.

2. Preliminaries

Definition 2.1. [16] Let X be any set. A fuzzy set in X is a function with domain X and values in [0,1].

The concept of triangular norms (t-norms) is originally introduced by Menger [17] in study of statistical metric spaces.

Definition 2.2. [18] A binary operation*: $[0,1]\times[0,1]\to[0,1]$ is continuous t-norm if * satisfies the following conditions:

i) * is commutative and associative;

ii) * is continuous;

iii) $a * 1 = a$ for all $a\in[0,1]$;

iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in[0,1]$.

Examples of t-norms are:

$a * b = \min\{a,b\}$, $a * b = ab$ and $a * b = \max\{a+b-1, 0\}$ for all $a,b\in[0,1]$.

Definition 2.3. [6] A 3-tuple $(X,M,*)$ is a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions for all $x,y,z\in X$ and $s, t > 0$

i) $M(x, y, t) > 0$;

ii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$;

iii) $M(x,y,t)=M(y,x,t)$;

iv) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$;

v) $M(x,y,.):[0,\infty)\to[0,1]$ is continuous.

The function $M(x, y, t)$ denote the degree of nearness between x and y with respect to t.

In all that follows $(X,M,*)$ is a fuzzy metric space with the following property:

vi) $\lim\limits_{t\to\infty} M(x,y,t)=1$ for all $x,y\in X$ and $t > 0$.

Now we give some interesting examples of FM-spaces:

Example 2.1. Let (X, d) be a metric space. Define $a * b = a + b$, for all $a,b\in[0,1]$; $x,y\in X$ and $t > 0$. Define $M(x,y,t)=t/(t+d(x,y))$. Then $(X, M, *)$ is a FM-space.

Moreover, fuzzy metric M induced by a metric d is often referred to as the **Standard fuzzy metric**.

Definition 2.4. [6]**.** A sequence $\{x_n\}$ in fuzzy metric space $(X,M,*)$ is

i) convergent to a point $x\in X$ if

$$\lim_{n\to\infty} M(x_n,x,t)=1 \text{ for all } t>0,$$

ii) Cauchy sequence if

$$\lim_{n\to\infty} M(x_{n+p},x_n,t)=1$$

for all $t > 0$ and $p > 0$.

Definition 2.5. A pair of self maps (S, T) of a fuzzy metric space $(X,M,*)$ is

i) compatible [19] if

$$\lim_{n\to\infty} M(STx_n,TSx_n,t)=1$$

for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim\limits_{n\to\infty} Sx_n=\lim\limits_{n\to\infty} Tx_n=z$ for some $z\in X$.

ii) non-compatible if there exists at least one sequence $\{x_n\}$ in X such that $\lim\limits_{n\to\infty} Sx_n=\lim\limits_{n\to\infty} Tx_n=z$ for some $z\in X$ but either $\lim\limits_{n\to\infty} M(STx_n,TSx_n,t)\ne1$ or non-existent.

iii) weakly compatible [20] if S and T commute at coincidence points, that is, $STx = TSx$ whenever $Sx = Tx$.

iv) satisfy the property $(E.A)$ [3] if there exist a sequence $\{x_n\}$ in X such that $\lim\limits_{n\to\infty} Sx_n=\lim\limits_{n\to\infty} Tx_n=z$ for some $z\in X$.

v) satisfies the *common limit in the range of T property* (CLR_T) [13] if there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = Tz$ for some $z \in X$.

Definition 2.6. Two pairs of self maps (A, S) and (B, T) of fuzzy metric space $(X, M, *)$ is

i) satisfy the common property ($E.A$) [14] if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$ for some $z \in X$.

ii) satisfy the ($JCLR_{ST}$) property (with respect to maps S and T) [4] if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Sz = Tz$ for some $z \in X$.

iii) satisfy the (CLR_S) property (with respect to maps S) [15] if there exist two sequences $\{x_n\}$ and$\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Sz$

for some $z \in X$.

Lemma 2.1. [7] If for all $x, y \in X$, $t > 0$ and for a number $q \in (0,1)$, $M(x, y, qt) \ge M(x, y, t)$ then $x = y$.

3. Main Results

Let Φ be the set of all increasing and continuous functions $\phi : (0,1] \to (0,1]$ such that $\phi(t) > t$ for all $t \in (0,1]$.

Example 3.1: Let $\phi : (0,1] \to (0,1]$ defined by $\phi(t) = \sqrt{t}$ for all $t \in (0,1]$. Clearly, $\phi \in \Phi$.

Theorem 3.1: Let A, B, S and T be self mappings of a fuzzy metric space (X, M, *) satisfying the following:

(3.1)

$$M(Ax, By, t) \ge \phi\left(\min\left\{M(Sx, Ty, t), \sup_{t_1+t_2=\frac{2}{k}t} \min\{M(Ax, Sx, t_1), M(Ty, By, t_2)\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t} \max\{M(Ax, Ty, t_3), M(By, Sx, t_4)\}\right\}\right)$$

for all $x, y \in X$, $t > 0$ and for some $1 \le k < 2$, $\phi \in \Phi$;

(3.2) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$;

(3.3) pair (A,S) or (B,T) satisfies the property ($E.A$);

(3.4) the range of one of the maps A, B, S or T is a closed subset of X.

Then pairs (A, S) and (B, T) have coincidence point. Further if (A, S) and (B, T) be weakly compatible pairs of self maps of fuzzy metric space (X, M, *) then A, B, S and T have a unique common fixed point in X.

Proof: If the pair (B,T) satisfies the property ($E.A.$), then there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$.

Since, $B(X) \subseteq S(X)$, therefore, there exist a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$. Hence, $\lim_{n\to\infty} Sy_n = z$.

Also, since $A(X) \subseteq T(X)$, there exist a sequence $\{z_n\}$ in X such that $Tx_n = Az_n$. Hence, $\lim_{n\to\infty} Az_n = z$.

Suppose that $S(X)$ is a closed subset of X. Then $z = Su$ for some $u \in X$. Therefore, $\lim_{n\to\infty} Az_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z = Su$.

We first claim that $Au = z$.

If $Au \ne z$, then there exist $t_0 > 0$ such that

$$M\left(Au, z, \frac{2}{k}t_0\right) > M(Au, z, t_0). \quad (3.5)$$

The inequality (3.5) is always true when $Au \ne z$. To support our claim, we suppose on contrary that (3.5) is not true all $t > 0$, *i.e.*,

$$M\left(Au, z, \frac{2}{k}t\right) = M(Au, z, t). \quad (3.6)$$

Now, using equality (3.6) repeatedly, we get

$$M(Au,z,t)=M\left(Au,z,\frac{2}{k}t\right)=M\left(Au,z,\left(\frac{2}{k}\right)^2 t\right)=\cdots=M\left(Au,z,\left(\frac{2}{k}\right)^n t\right)\to 1$$

as $n\to\infty$. This gives, $M(Au,z,t)=1$ for all $t > 0$. Hence, $Au = z$, which gives contradiction.

Therefore, inequality (3.5) is always true for some $t_0 > 0$.

Using (3.1), take $x = u$, $y = y_n$, we get

$$M(Au,By_n,t_0)\ge\phi\left(\min\left\{M(Su,Ty_n,t_0),\sup_{t_1+t_2=\frac{2}{k}t_0}\min\{M(Au,Su,t_1),M(Ty_n,By_n,t_2)\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0}\max\{M(Au,Ty_n,t_3),M(By_n,Su,t_4)\}\right\}\right)$$

let $t_2=t_4=\varepsilon$ then $t_1=t_3=\frac{2}{k}t_0-\varepsilon$ where $\varepsilon\in\left(0,\frac{2}{k}t_0\right)$ and $n\to\infty$, we get

$$M(Au,z,t_0)\ge\phi\left(\min\left\{M(z,z,t_0),\min\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),M(z,z,\varepsilon)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),M(z,z,\varepsilon)\right\}\right\}\right)$$

$$M(Au,z,t_0)\ge\phi\left(\min\left\{1,\min\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),1\right\},\max\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),1\right\}\right\}\right)$$

$$M(Au,z,t_0)\ge\phi\left(M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right)\right)>M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right).$$

As $\varepsilon\to 0$, we get

$$M(Au,z,t_0)\ge M\left(Au,z,\frac{2}{k}t_0\right)$$

which gives contradiction, hence

$$Au = z.$$

Therefore, $Au = z = Su$ which shows that u is a coincidence point of the pair (A, S). As A and S are weakly compatible. Therefore, $ASu = SAu$ and then $AAu = ASu = SAu = SSu$.

On the other hand, since $A(X)\subseteq T(X)$, there exist v in X such that $Au = Tv$.

Now, we show that $Bv = z$.

If $Bv \neq z$, then again, as done above, there exist $t_0 > 0$ such that

$$M\left(Bv,z,\frac{2}{k}t_0\right)>M(Bv,z,t_0). \tag{3.7}$$

The inequality (3.7) is always true when $Bv \neq z$.

Using (3.1), take $x = u$, $y = v$, we have

$$M(Au,Bv,t_0)\ge\phi\left(\min\left\{M(Su,Tv,t_0),\sup_{t_1+t_2=\frac{2}{k}t_0}\min\{M(Au,Su,t_1),M(Tv,Bv,t_2)\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0}\max\{M(Au,Tv,t_3),M(Bv,Su,t_4)\}\right\}\right)$$

let $t_1=t_3=\varepsilon$ then $t_2=t_4=\frac{2}{k}t_0-\varepsilon$ where $\varepsilon\in\left(0,\frac{2}{k}t_0\right)$, we get

$$M\left(z,Bv,t_0\right)\geq\phi\left(\min\left\{M\left(z,z,t_0\right),\min\left\{M\left(z,z,\varepsilon\right),M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(z,z,\varepsilon\right),M\left(Bv,z,\frac{2}{k}t_0-\varepsilon\right)\right\}\right\}\right)$$
$$M\left(z,Bv,t_0\right)\geq\phi\left(\min\left\{1,\min\left\{1,M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\},\max\left\{1,M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\}\right\}\right)$$
$$M\left(z,Bv,t_0\right)\geq\phi\left(M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right)>M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right).$$

As $\varepsilon\to 0$, we get

$$M\left(z,Bv,t_0\right)\geq M\left(z,Bv,\frac{2}{k}t_0\right)$$

which gives contradiction, hence $Bv = z$.

Therefore, $Bv = z = Au = Tv$ which shows that $Bv = Tv$, *i.e.*, v is a coincidence point of the pair (B, T). As B and T are weakly compatible, therefore, $BTv = TBv$ and hence, $BTv = TBv = TTv = BBv$.

Next, we show that $AAu = Au$, if not, then again as done above, there exist $t_0 > 0$ such that

$$M\left(AAu,Au,\frac{2}{k}t_0\right)>M\left(AAu,Au,t_0\right). \tag{3.8}$$

Using (3.1), take $x = Au$, $y = v$, we have

$$M\left(AAu,Bv,t_0\right)\geq\phi\left(\min\left\{M\left(SAu,Tv,t_0\right),\sup_{t_1+t_2=\frac{2}{k}t_0}\min\left\{M\left(AAu,SAu,t_1\right),M\left(Tv,Bv,t_2\right)\right\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0}\max\left\{M\left(AAu,Tv,t_3\right),M\left(Bv,SAu,t_4\right)\right\}\right\}\right)$$

let $t_2=t_4=\varepsilon$ then $t_1=t_3=\frac{2}{k}t_0-\varepsilon$ where $\varepsilon\in\left(0,\frac{2}{k}t_0\right)$, we get

$$M\left(AAu,Au,t_0\right)\geq\phi\left(\min\left\{M\left(AAu,Au,t_0\right),\min\left\{M\left(AAu,AAu,\frac{2}{k}t_0-\varepsilon\right),M\left(z,z,\varepsilon\right)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(AAu,Au,\frac{2}{k}t_0-\varepsilon\right),M\left(Au,AAu,\varepsilon\right)\right\}\right\}\right)$$
$$M\left(AAu,Au,t_0\right)\geq\phi\left(\min\left\{M\left(AAu,Au,t_0\right),1,\max\left\{M\left(AAu,Au,\frac{2}{k}t_0-\varepsilon\right),M\left(AAu,Au,\varepsilon\right)\right\}\right\}\right)$$
$$M\left(AAu,Au,t_0\right)\geq\phi\left(M\left(AAu,Au,\frac{2}{k}t_0-\varepsilon\right)\right)>M\left(AAu,Au,\frac{2}{k}t_0-\varepsilon\right).$$

As $\varepsilon\to 0$, we get $M\left(AAu,Au,t_0\right)\geq M\left(AAu,Au,\frac{2}{k}t_0\right)$ which gives contradiction, hence $AAu = Au$. Therefore, $AAu = Au = SAu$ and Au are a common fixed point of A and S. Similarly, we can prove that Bv is a common fixed point of B and T. As $Au = Bv$, we conclude that Au is a common fixed point of A, B, S and T.

The proof is similar when $T(X)$ is assumed to be a closed subset of X. The cases in which $A(X)$ or $B(X)$ is a closed subset of X are similar to the cases in which $T(X)$ or $S(X)$ respectively, is closed since

$$A\left(X\right)\subseteq T\left(X\right),\quad B\left(X\right)\subseteq S\left(X\right).$$

For uniqueness; let w be another fixed point of A, B, S and T. Then by (3.1), we have

$$M(Az,Bw,t_0) \geq \phi\left(\min\left\{M(Sz,Tw,t_0), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{M(Az,Sz,t_1), M(Tw,Bw,t_2)\},\right.\right.$$

$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0} \max\{M(Az,Tw,t_3), M(Bw,Sz,t_4)\}\right\}\right)$$

$$M(z,w,t_0) \geq \phi\left(\min\left\{M(z,w,t_0), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{M(z,z,t_1), M(w,w,t_2)\},\right.\right.$$

$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0} \max\{M(z,w,t_3), M(w,z,t_4)\}\right\}\right)$$

let $t_1 = t_3 = \varepsilon$ then $t_2 = t_4 = \frac{2}{k}t_0 - \varepsilon$ where $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$,

$$M(z,w,t_0) \geq \phi\left(\min\left\{M(z,w,t_0), \min\left\{M(z,z,\varepsilon), M\left(w,w,\frac{2}{k}t_0 - \varepsilon\right)\right\},\right.\right.$$

$$\left.\left.\max\left\{M(z,w,\varepsilon), M\left(w,z,\frac{2}{k}t_0 - \varepsilon\right)\right\}\right\}\right)$$

as $\varepsilon \to 0$, we get

$$M(z,w,t_0) \geq \phi\left(\min\left\{M(z,w,t_0), 1, M\left(w,z,\frac{2}{k}t_0\right)\right\}\right)$$

$$M(z,w,t_0) \geq \phi(M(w,z,t_0)) > M(w,z,t_0)$$

a contradiction, hence, $w = z$. It implies that A, B, S and T have unique common fixed point in X.

Hence the result.

Now we attempt to drop containment of subspaces by replacing property (*E.A.*) by a weaker condition common property (*E.A.*) in Theorem 3.1.

Theorem 3.2: Let A, B, S and T be self mappings of a fuzzy metric space (X, M, *) satisfying condition (3.1) of Theorem 3.1 and the following:

(3.9) the pair (A, S) and (B, T) share the common (*E.A.*) property;

(3.10) $S(X)$ and $T(X)$ are closed subsets of X.

Then the pairs (A, S) and (B, T) have a point of coincidence each. Moreover, A, B, S and T have a unique common fixed point provided both the pairs (A, S) and (B, T) are weakly compatible.

Proof: In view of (3.2), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = z$$

for some $z \in X$.

Since $S(X)$ is a closed subset of X, therefore, there exists a point u in X such that $z = Su$.

We claim that $Au = z$. If $Au \neq z$, then there exist $t_0 > 0$ such that

$$M\left(Au, z, \frac{2}{k}t_0\right) > M(Au, z, t_0). \tag{3.11}$$

The inequality (3.11) is always true when $Au \neq z$. To support our claim, we suppose on contrary that (3.11) is not true all $t > 0$, *i.e.*,

$$M\left(Au, z, \frac{2}{k}t\right) = M(Au, z, t). \tag{3.12}$$

Now, using equality (3.12) repeatedly, we get

$$M(Au,z,t)=M\left(Au,z,\frac{2}{k}t\right)=M\left(Au,z,\left(\frac{2}{k}\right)^2 t\right)$$
$$=\cdots=M\left(Au,z,\left(\frac{2}{k}\right)^n t\right)\to 1$$

as $n\to\infty$. This gives, $M(Au,z,t)=1$ for all $t > 0$. Hence, $Au = z$, which gives contradiction.

Therefore, inequality (3.11) is always true for some $t_0 > 0$. Using (3.1), take $x = u$, $y = y_n$, we get

$$M(Au,By_n,t_0)$$
$$\geq\phi\left(\min\left\{M(Su,Ty_n,t_0),\sup_{t_1+t_2=\frac{2}{k}t_0}\min\{M(Au,Su,t_1),M(Ty_n,By_n,t_2)\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0}\max\{M(Au,Ty_n,t_3),M(By_n,Su,t_4)\}\right\}\right)$$

let $t_2=t_4=\varepsilon$ then $t_1=t_3=\frac{2}{k}t_0-\varepsilon$ where $\varepsilon\in\left(0,\frac{2}{k}t_0\right)$ and $n\to\infty$, we get

$$M(Au,z,t_0)\geq\phi\left(\min\left\{M(z,z,t_0),\min\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),M(z,z,\varepsilon)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),M(z,z,\varepsilon)\right\}\right\}\right)$$
$$M(Au,z,t_0)\geq\phi\left(\min\left\{1,\min\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),1\right\},\max\left\{M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right),1\right\}\right\}\right)$$
$$M(Au,z,t_0)\geq\phi\left(M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right)\right)>M\left(Au,z,\frac{2}{k}t_0-\varepsilon\right).$$

As $\varepsilon\to 0$, we get $M(Au,z,t_0)\geq M\left(Au,z,\frac{2}{k}t_0\right)$ which gives contradiction, hence $Au = z$.

Therefore, $Au = z = Su$ which shows that u is a coincidence point of the pair (A, S).

Since $T(X)$ is also a closed subset of X, therefore $\lim_{n\to\infty}Ty_n = z$ in $T(X)$ and hence there exists v in X such that $Tv = z = Au = Su$. Now, we show that $Bv = z$.

If $Bv \neq z$, then again as done above, there exist $t_0 > 0$ such that

$$M\left(Bv,z,\frac{2}{k}t_0\right)>M(Bv,z,t_0).\tag{3.13}$$

The inequality (3.13) is always true when $Bv \neq z$.

Using (3.1), take $x = u$, $y = v$, we have

$$M(Au,Bv,t_0)$$
$$\geq\phi\left(\min\left\{M(Su,Tv,t_0),\sup_{t_1+t_2=\frac{2}{k}t_0}\min\{M(Au,Su,t_1),M(Tv,Bv,t_2)\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0}\max\{M(Au,Tv,t_3),M(Bv,Su,t_4)\}\right\}\right)$$

let $t_1=t_3=\varepsilon$ then $t_2=t_4=\frac{2}{k}t_0-\varepsilon$ where $\varepsilon\in\left(0,\frac{2}{k}t_0\right)$, we get

$$M\left(z,Bv,t_0\right)\geq\phi\left(\min\left\{M\left(z,z,t_0\right),\min\left\{M\left(z,z,\varepsilon\right),M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(z,z,\varepsilon\right),M\left(Bv,z,\frac{2}{k}t_0-\varepsilon\right)\right\}\right\}\right)$$
$$M\left(z,Bv,t_0\right)\geq\phi\left(\min\left\{1,\min\left\{1,M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\},\max\left\{1,M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right\}\right\}\right)$$
$$M\left(z,Bv,t_0\right)\geq\phi\left(M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right)\right)>M\left(z,Bv,\frac{2}{k}t_0-\varepsilon\right).$$

As $\varepsilon\to 0$, we get

$$M\left(z,Bv,t_0\right)\geq M\left(z,Bv,\frac{2}{k}t_0\right)$$

which gives contradiction, hence $Bv = z$.

Therefore, $Bv = z = Tv$ which shows that v is a coincidence point of the pair (B, T).

Since the pairs (A, S) and (B, T) are weakly compatible and $Au = Su$, $Bv = Tv$, therefore, $Az = ASu = SAu = Sz$, $Bz = BTv = TBv = Tz$.

If $Az \neq z$, then again as done above, there exist $t_0 > 0$ such that

$$M\left(Az,z,\frac{2}{k}t_0\right)>M\left(Az,z,t_0\right). \tag{3.14}$$

Using (3.1), take $x = z$, $y = v$, we have

$$M\left(Az,Bv,t_0\right)\geq\phi\left(\min\left\{M\left(Sz,Tv,t_0\right),\sup_{t_1+t_2=\frac{2}{k}t_0}\min\left\{M\left(Az,Sz,t_1\right),M\left(Tv,Bv,t_2\right)\right\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0}\max\left\{M\left(Az,Tv,t_3\right),M\left(Bv,Sz,t_4\right)\right\}\right\}\right)$$

let $t_2 = t_4 = \varepsilon$ then $t_1 = t_3 = \frac{2}{k}t_0-\varepsilon$ where $\varepsilon\in\left(0,\frac{2}{k}t_0\right)$, we get

$$M\left(Az,z,t_0\right)\geq\phi\left(\min\left\{M\left(z,z,t_0\right),\min\left\{M\left(Az,z,\frac{2}{k}t_0-\varepsilon\right),M\left(z,z,\varepsilon\right)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(Az,z,\frac{2}{k}t_0-\varepsilon\right),M\left(z,z,\varepsilon\right)\right\}\right\}\right)$$
$$M\left(Az,z,t_0\right)\geq\phi\left(\min\left\{1,\min\left\{M\left(Az,z,\frac{2}{k}t_0-\varepsilon\right),1\right\},\max\left\{M\left(Az,z,\frac{2}{k}t_0-\varepsilon\right),1\right\}\right\}\right)$$
$$M\left(Az,z,t_0\right)\geq\phi\left(M\left(Az,z,\frac{2}{k}t_0-\varepsilon\right)\right)>M\left(Az,z,\frac{2}{k}t_0-\varepsilon\right).$$

As $\varepsilon\to 0$, we get

$$M\left(Az,z,t_0\right)\geq M\left(Az,z,\frac{2}{k}t_0\right)$$

which gives contradiction, hence $Az = z$.

Therefore, $Az = z = Sz$.

Similarly, one can prove that $Bz = Tz = z$. Hence, $Az = Bz = Sz = Tz$, and z is common fixed point of A, B, S and T.

Uniqueness easily follows by the use of inequality (3.1).

Hence the result.

Now we attempt to drop containment of subspaces by using weaker condition $JCLR_{ST}$ property in Theorem 3.2.

Theorem 3.3: Let *A*, *B*, *S* and *T* be four selfmaps in fuzzy metric space $(X, M, *)$ satisfying condition (3.1) of Theorem 3.1 and (3.15) (*A*, *S*) and (*B*, *T*) shares the $JCLR_{ST}$ property.

Then pairs (*A*, *S*) and (*B*, *T*) have coincidence point. Further if (*A*, *S*) and (*B*, *T*) be weakly compatible pair of self maps of *X* then *A*, *B*, *S* and *T* have a unique common fixed point in *X*.

Proof: The pairs (*A*, *S*) and (*B*, *T*) satisfy the ($JCLR_{ST}$) property, then there exist two sequences $\{x_n\}$ and $\{y_n\}$ in *X* such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = Su = Tu$ for some $u \in X$.

Firstly, we claim that $Tu = Bu$. Suppose not, then there exist $t_0 > 0$ such that

$$M\left(Tu, Bu, \frac{2}{k}t_0\right) > M\left(Tu, Bu, t_0\right). \tag{3.16}$$

The inequality (3.16) is always true when $Tu \neq Bu$. To support our claim, we suppose on contrary that (3.16) is not true all $t > 0$, *i.e.*,

$$M\left(Tu, Bu, \frac{2}{k}t\right) = M\left(Tu, Bu, t\right) \tag{3.17}$$

Now, using equality (3.17) repeatedly, we get

$$M\left(Tu, Bu, t\right) = M\left(Tu, Bu, \frac{2}{k}t\right) = M\left(Tu, Bu, \left(\frac{2}{k}\right)^2 t\right) = \cdots = M\left(Tu, Bu, \left(\frac{2}{k}\right)^n t\right) \to 1$$

as $n \to \infty$. This gives, $M\left(Tu, Bu, t\right) = 1$ for all $t > 0$. Hence, $Tu = Bu$, which gives contradiction.

Therefore, inequality (3.16) is always true for some $t_0 > 0$.

Using (3.1), take $x = x_n$, $y = u$, we get

$$M\left(Ax_n, Bu, t_0\right) \geq \phi\left(\min\left\{M\left(Sx_n, Tu, t_0\right), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{M\left(Ax_n, Sx_n, t_1\right), M\left(Tu, Bu, t_2\right)\right\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{M\left(Ax_n, Tu, t_3\right), M\left(Bu, Sx_n, t_4\right)\right\}\right\}\right)$$

let $t_1 = t_3 = \varepsilon$ then $t_2 = t_4 = \frac{2}{k}t_0 - \varepsilon$ where $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$ and $n \to \infty$, we get

$$M\left(Tu, Bu, t_0\right) \geq \phi\left(\min\left\{M\left(Tu, Tu, t_0\right), \min\left\{M\left(Tu, Tu, \varepsilon\right), M\left(Tu, Bu, \frac{2}{k}t_0 - \varepsilon\right)\right\},\right.\right.$$
$$\left.\left.\max\left\{M\left(Tu, Tu, \varepsilon\right), M\left(Bu, Tu, \frac{2}{k}t_0 - \varepsilon\right)\right\}\right\}\right)$$

$$M\left(Tu, Bu, t_0\right) \geq \phi\left(\min\left\{1, \min\left\{1, M\left(Tu, Bu, \frac{2}{k}t_0 - \varepsilon\right)\right\}, \max\left\{1, M\left(Tu, Bu, \frac{2}{k}t_0 - \varepsilon\right)\right\}\right\}\right)$$

$$M\left(Tu, Bu, t_0\right) \geq \phi\left(M\left(Tu, Bu, \frac{2}{k}t_0 - \varepsilon\right)\right) > M\left(Tu, Bu, \frac{2}{k}t_0 - \varepsilon\right)$$

As $\varepsilon \to 0.$, we get

$$M\left(Tu, Bu, t_0\right) \geq M\left(Tu, Bu, \frac{2}{k}t_0\right)$$

which gives contradiction, hence $Tu = Bu$.

Next, we show that $Au = Tu$. Suppose not, then again as done above, there exist $t_0 > 0$ such that

$$M\left(Au, Tu, \frac{2}{k}t_0\right) > M\left(Au, Tu, t_0\right). \tag{3.18}$$

Using (3.1), take $x = u$, $y = y_n$, we get

$$\begin{aligned} &M\left(Au, By_n, t_0\right) \\ &\geq \phi\left(\min\left\{M\left(Su, Ty_n, t_0\right), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{M\left(Au, Su, t_1\right), M\left(Ty_n, By_n, t_2\right)\right\},\right.\right. \\ &\qquad \left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{M\left(Au, Ty_n, t_3\right), M\left(By_n, Su, t_4\right)\right\}\right\}\right) \end{aligned}$$

let $t_2 = t_4 = \varepsilon$ then $t_1 = t_3 = \frac{2}{k}t_0 - \varepsilon$ where $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$ and $n \to \infty$, we get

$$\begin{aligned} M\left(Au, Tu, t_0\right) &\geq \phi\left(\min\left\{M\left(Su, Su, t_0\right), \min\left\{M\left(Au, Tu, \frac{2}{k}t_0 - \varepsilon\right), M\left(Tu, Tu, \varepsilon\right)\right\},\right.\right. \\ &\qquad \left.\left.\max\left\{M\left(Au, Tu, \frac{2}{k}t_0 - \varepsilon\right), M(Tu, Tu, \varepsilon)\right\}\right\}\right) \end{aligned}$$

$$M\left(Au, Tu, t_0\right) \geq \phi\left(\min\left\{1, \min\left\{M\left(Au, Tu, \frac{2}{k}t_0 - \varepsilon\right), 1\right\}, \max\left\{M\left(Au, Tu, \frac{2}{k}t_0 - \varepsilon\right), 1\right\}\right\}\right)$$

$$M\left(Au, Tu, t_0\right) \geq \phi\left(M\left(Au, Tu, \frac{2}{k}t_0 - \varepsilon\right)\right) > M\left(Au, Tu, \frac{2}{k}t_0 - \varepsilon\right)$$

As $\varepsilon \to 0$, we get $M\left(Au, Tu, t_0\right) \geq M\left(Au, Tu, \frac{2}{k}t_0\right)$ which gives contradiction, hence $Au = Tu$. Hence, $Au = Bu = Su = Tu = z$ (say). Since the pair (A, S) is weakly compatible, $ASu = SAu$ and then $Az = Sz$. Similarly, as the pair (B, T) is weakly compatible, $BTu = TBu$ and then $Tz = Bz$.

Next, we claim that $Az = z$, suppose not. Then by (3.1), take $x = z$, $y = u$, we get

$$\begin{aligned} &M\left(Az, Bu, t_0\right) \\ &\geq \phi\left(\min\left\{M\left(Sz, Tu, t_0\right), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\left\{M\left(Az, Sz, t_1\right), M\left(Tu, Bu, t_2\right)\right\},\right.\right. \\ &\qquad \left.\left.\sup_{t_3+t_4=\frac{2}{k}t_0} \max\left\{M\left(Az, Tu, t_3\right), M\left(Bu, Sz, t_4\right)\right\}\right\}\right) \end{aligned}$$

let $t_1 = t_3 = \varepsilon$ then $t_2 = t_4 = \frac{2}{k}t_0 - \varepsilon$ where $\varepsilon \in \left(0, \frac{2}{k}t_0\right)$,

$$\begin{aligned} M\left(Az, z, t_0\right) &\geq \phi\left(\min\left\{M\left(Az, z, t_0\right), \min\left\{M\left(Az, Az, \varepsilon\right), M\left(z, z, \frac{2}{k}t_0 - \varepsilon\right)\right\},\right.\right. \\ &\qquad \left.\left.\max\left\{M\left(Az, z, \varepsilon\right), M\left(z, Az, \frac{2}{k}t_0 - \varepsilon\right)\right\}\right\}\right) \end{aligned}$$

$$M\left(Az, z, t_0\right) \geq \phi\left(\min\left\{M\left(Az, z, t_0\right), 1, \max\left\{M\left(Az, z, \varepsilon\right), M\left(z, Az, \frac{2}{k}t_0 - \varepsilon\right)\right\}\right\}\right)$$

As $\varepsilon \to 0$, we get

$$M(Az,z,t_0) \geq \phi\left(\min\left\{M(Az,z,t_0),1,\max\left\{M(Az,z,0),M\left(z,Az,\frac{2}{k}t_0\right)\right\}\right\}\right)$$

$$M(Az,z,t_0) \geq \phi\left(\min\left\{M(Az,z,t_0),1,M\left(z,Az,\frac{2}{k}t_0\right)\right\}\right)$$

$$M(Az,z,t_0) \geq \phi\left(M(z,Az,t_0)\right) > M(z,Az,t_0)$$

a contradiction, hence, $Az = Bz = z$. Therefore, z is a common fixed point of A and B. Similarly, we prove that $Sz = Tz = z$ by taking $x = u$, $y = z$ in (3.1). Therefore, we conclude that $z = Az = Bz = Sz = Tz$ this implies that A, B, S and T have common fixed point in X.

Uniqueness easily follows by the use of inequality (3.1).

Next we attempt to drop closedness of range of maps and relax containment of two subspaces to one subspace by replacing property (*E.A.*) by a weaker condition CLR_S property in Theorem 3.1.

Theorem 3.4: Let A, B, S and T be four selfmaps fuzzy metric space (X, M, *) satisfying condition (3.1) of Theorem 3.1 and

(3.19) (A, S) and (B, T) shares the CLR_S property (CLR_T property)

(3.20) $A(X) \subset T(X)$ $\left(B(X) \subset S(X)\right)$.

Then pairs (A, S) and (B, T) have coincidence point. Further if (A, S) and (B, T) be weakly compatible pair of self maps of X then A, B, S and T have a unique common fixed point in X.

Proof: Proof of this theorem easily follows on same lines of Theorem 3.2.

On taking $A = B$ and $S = T$ in Theorem 3.1 then we get the following interesting result which is improved version of Theorem 1 of Sedghi *et al.* [2].

Corollary 3.1: Let A and S be self mappings of a fuzzy metric space (X, M, *) satisfying the following:

(3.21)

$$M(Ax,Ay,t) \geq \phi\left(\min\left\{M(Sx,Sy,t),\sup_{t_1+t_2=\frac{2}{k}t}\min\left\{M(Ax,Sx,t_1),M(Sy,Ay,t_2)\right\},\right.\right.$$
$$\left.\left.\sup_{t_3+t_4=\frac{2}{k}t}\max\left\{M(Ax,Sy,t_3),M(Ay,Sx,t_4)\right\}\right\}\right)$$

for all $x,y \in X$, $t > 0$ and for some $1 \leq k < 2$, $\phi \in \Phi$;

(3.22) $A(X) \subseteq S(X)$;

(3.23) pair (A, S) satisfies the property (*E.A*)

(3.24) $A(X)$ or $S(X)$ is a closed subset of X.

Then pair (A, S) has a coincidence point. Further, if pair A and S be weakly compatible self maps of fuzzy metric space (X, M, *), then A and S have a unique common fixed point in X.

On taking $A = B$ and $S = T$ in Theorem 3.4 then we get the following interesting result which is improved version of Theorem 3.3 of Chauhan *et al*. [3].

Corollary 3.2: Let A and S be self mappings of a fuzzy metric space (X, M, *) satisfying the following:

(3.25) (A, S) satisfies the CLR_S property.

Then pair (A, S) has a coincidence point. Further if pair A and S be weakly compatible self maps of X then A and S has a unique common fixed point in X.

Finally, we conclude this paper by furnishing example to demonstrate Theorem 3.3 besides exhibiting its superiority over earlier relevant results.

Example 3.2. Let $(X,M,*)$ be a fuzzy metric space where $a * b = a.\ b$ for all $a,b \in [0,1]$ and $X = [3, 19)$. Let $\phi:(0,1] \to (0,1]$ be defined as $\phi(t) = \sqrt{t}$ for all $t \in (0,1]$, Clearly, $\phi \in \Phi$. Define A, B, S and T by

$$A(x) = \begin{cases} 1 & x \in \{1\} \cup (3,15) \\ x+11 & x \in (1,3] \end{cases}, \quad B(x) = \begin{cases} 1 & x \in \{1\} \cup (3,15) \\ x+5 & x \in (1,3] \end{cases},$$

$$S(x)=\begin{cases}1 & x=1\\ 6 & x\in(1,3]\\ \dfrac{x+1}{4} & x\in(3,15)\end{cases}$$

and

$$T(x)=\begin{cases}1 & x=1\\ 11 & x\in(1,3]\\ x-2 & x\in(3,15)\end{cases}.$$

Take $\{x_n\}=\{y_n\}=\left\{3+\frac{1}{n}\right\}$, clearly

$$\lim_{n\to\infty}Ax_n=\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}By_n=\lim_{n\to\infty}Ty_n=1=T1=S1$$

for some $1\in X$.

Thus, (A, S) and (B,T) satisfies $JCLR_{ST}$ property.

Also, $AX=\{1\}\cup(12,14]$, $BX=\{1\}\cup(6,8]$, $SX=[1,4)\cup\{6\}$, TX = (1, 13) and condition (3.1) is satisfied by maps A, B, S and T. Thus, the maps A, B, S and T satisfy all conditions of Theorem 3.3. Hence, A, B, S and T have a unique common fixed point x = 1.Moreover it should be noted that AX, BX, SX and TX are not closed subsets of X. Also, $AX\not\subset TX$ and $BX\not\subset SX$. Also, A, B, S and T are all discontinuous maps at fixed point x = 1.

Definition 3.1 [21] Two families of self mappings $\{A_i\}_{i=1}^{m}$ and $\{B_j\}_{j=1}^{n}$ are said to be pairwise commuting if

i) $A_iA_j=A_jA_i$, $i,j\in\{1,2,3,\cdots,m\}$,
ii) $B_iB_j=B_jB_i$, $i,j\in\{1,2,3,\cdots,n\}$,
iii) $A_iB_j=B_jA_i$, $i\in\{1,2,3,\cdots,n\}$, $j\in\{1,2,3,\cdots,n\}$.

As an application of Theorem 3.2, we prove a common fixed point theorem for four finite families of mappings on fuzzy metric spaces. While proving our result, we utilize Definition 3.1 which is a natural extension of commutativity condition to two finite families.

Theorem 3.4: Let $\{A_1,A_2,\cdots,A_m\}$, $\{B_1,B_2,\cdots,B_n\}$, $\{S_1,S_2,\cdots,S_p\}$ and $\{T_1,T_2,\cdots,T_q\}$ be four finite families of self mappings of a fuzzy metric space (X, M, *) such that $A=A_1\cdot A_2\cdots A_m$, $B=B_1\cdot B_2\cdots B_n$, $S=S_1\cdot S_2\cdots S_p$ and $T=T_1\cdot T_2\cdots T_q$ satisfying the conditions (3.1), (3.9), (3.10) and (3.26) the pairs of families $(\{A_i\},\{S_k\})$ and $(\{B_r\},\{T_t\})$ commute pairwise.

Then the pairs (A,S) and (B,T) have a point of coincidence each. Moreover, $\{A_i\}_{i=1}^{m},\{S_k\}_{k=1}^{p}$, $\{B_r\}_{r=1}^{n}$ and $\{T_t\}_{t=1}^{q}$ have a unique common fixed point.

Proof: By using (3.26), we first show that AS = SA as

$$\begin{aligned}AS&=(A_1A_2\cdots A_m)(S_1S_2\cdots S_p)=(A_1A_2\cdots A_{m-1})(A_mS_1S_2\cdots S_p)=(A_1A_2\cdots A_{m-1})(S_1S_2\cdots S_pA_m)\\&=(A_1A_2\cdots A_{m-2})(A_{m-1}S_1S_2\cdots S_pA_m)=(A_1A_2\cdots A_{m-2})(S_1S_2\cdots S_pA_{m-1}A_m)\\&=\cdots=A_1(S_1S_2\cdots S_pA_2\cdots A_m)=(S_1S_2\cdots S_p)(A_1A_2\cdots A_m)=SA\end{aligned}.$$

Similarly one can prove that BT = TB. And hence, obviously the pair (A, S) is compatible and (B, T) is weakly compatible. Now using Theorem 3.1, we conclude that A, S, B and T have a unique common fixed point in X, say z.

Now, one needs to prove that z remains the fixed point of all the component mappings.

For this consider

$$\begin{aligned}A(A_iz)&=((A_1A_2\cdots A_m)A_i)z=(A_1A_2\cdots A_{m-1})(A_mA_i)z=(A_1A_2\cdots A_{m-1})(A_iA_m)z\\&=(A_1A_2\cdots A_{m-2})(A_{m-1}A_iA_m)z=(A_1A_2\cdots A_{m-2})(A_iA_{m-1}A_m)z\\&=\cdots=A_1(A_iA_2\cdots A_m)z=(A_1A_i)(A_2\cdots A_m)z\\&=(A_iA_1)(A_2\cdots A_m)z=A_i(A_1A_2\cdots A_m)z=A_iAz=A_iz\end{aligned}.$$

Similarly, one can prove that

$$A(S_k z) = S_k(Az) = S_k z,$$

$$S(S_k z) = S_k(Sz) = S_k z,$$

$$S(A_i z) = A_i(Sz) = A_i z,$$

$$B(B_r z) = B_r(Bz) = B_r z,$$

$$B(T_t z) = T_t(Bz) = T_t z,$$

$$T(T_t z) = T_t(Tz) = T_t z$$

and

$$T(B_r z) = B_r(Tz) = B_r z,$$

which show that (for all i, r, k and t) $A_i z$ and $S_k z$ are other fixed point of the pair (A, S) whereas $B_r z$ and $T_t z$ are other fixed points of the pair (B, T). As A, B, S and T have a unique common fixed point, so, we get

$$z = A_i z = S_k z = B_r z = T_t z,$$

for all $i = 1,2,\cdots,m$, $k = 1,2,\cdots,p$, $r = 1,2,\cdots,n$, $t = 1,2,\cdots,q$. which shows that z is a unique common fixed point of $\{A_i\}_{i=1}^{m}, \{S_k\}_{k=1}^{p}$, $\{B_r\}_{r=1}^{n}$ and $\{T_t\}_{t=1}^{q}$.

Remark 3.2: Theorem 3.4 is a slight but partial generalization of Theorem 3.2 as the commutativity requirements in this theorem are slightly stronger as compared to Theorem 3.1.

Remark 3.3. From the above results, it is asserted that for the existence of common fixed point of two pairs of self maps in fuzzy metric spaces satisfying $JCLR_{ST}$ property the following conditions are never required:

a) the containment of ranges amongst the involved maps;
b) the completeness of the whole space/subspace;
c) the closedness of space/subspaces;
d) continuity requirement amongst the involved maps.

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