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Traveling Wave Solutions and Kind Wave Excitations for the (2 + 1)-Dimensional Dissipative Zabolotskaya-Khokhlov Equation

Xiajie Liu, Chunliang Mei, Songhua Ma
College of Science, Lishui University, Lishui, China
Email: msh6209@aliyun.com

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ABSTRACT

In this work, with the help of the symbolic computation system Maple and the Riccati mapping approach and a linear variable separation approach, a new family of traveling wave solutions of the (2 + 1)-dimensional dissipative Zabolotskaya-Khokhlov equation (DZK) is derived. Based on the derived solitary wave solution, some novel kind wave excitations are investigated.

Keywords: Mapping Approach; Dissipative Zabolotskaya-Khokhlov Equation; Traveling Wave Solution; Kind Wave Excitation

1. Introduction

In nonlinear science, soliton theory plays an essential role and has been applied in almost all the natural sciences, especially in all the physical branches such as fluid physics, condensed matter, biophysics, plasma physics, nonlinear optics, quantum field theory, and particle physics, etc. [1-5]. How to find exact solutions of nonlinear partial differential equations (PDEs) plays an important role in the research of nonlinear physical phenomena. So it is always an interesting topic to search for meaningful solutions for PDEs. In order to find some new exact solutions, a wealth of effective methods have been set up, for instance, the bilinear method, the standard Painlevé truncated expansion, the method of "coalescence of eigenvalue" or "wavenumbers", the homogenous balance method, the homotopy-perturbation method, the hyperbolic function method, the Jacobian elliptic method, the (G'/G)-expansion method, the variable separation method, and the mapping equation method [6-15], etc. Among these methods, the mapping equation approach is one of the most effectively straightforward algebraic methods to construct exact solutions of NPDE [16-19]. In this paper, via the Riccati mapping equation we find some new exact solutions of the (2 + 1)-dimensional dissipative Zabolotskaya-Khokhlov equation (DZK). Based on the derived solution, we obtain some kind wave excitations of the equation.

2. New Traveling Wave Solutions of the DZK Equation

The (2 + 1)-dimensional dissipative Zabolotskaya-Khokhlov equation is

$$U_{xt} + U_x^2 + UU_{xx} - U_{xxx} + U_{yy} = 0 \quad (1)$$

In Ref. [20], some new exact solutions and time solitons have been discussed by (G'/G)-expansion method. As is well known, to search for the solitary wave solutions for a nonlinear physical model, we can apply different approaches. One of the most efficient methods of finding soliton excitations of a physical model is the so-called mapping approach. The basic ideal of the algorithm is as follows. For a given nonlinear partial differential equation (NPDE) with the independent variables $x = (x_0 = t, x_1, x_2, \cdots, x_m)$ and the dependent variable u, in the form

$$P\left(u, u_t, u_{xi}, u_{x_i x_j}, \cdots\right) = 0, \quad (2)$$

where P is in general a polynomial function of its arguments, and the subscripts denote the partial derivatives, the solution can be assumed to be in the form

$$u=\sum_{i=0}^{n}\left\{A_i\phi^i\left(q\right)\right\} \tag{3}$$

with

$$\phi'=\sigma+\phi^2, \tag{4}$$

where σ is a constant and the prime denotes the different-tiation with respect to q. To determine U explicitly, one may substitute (3) and (4) into the given NPDE and collect coefficients of polynomials of Φ, then eliminate each coefficient to derive a set of partial differential equations of A_i, and q, and solve the system of partial differential equations to obtain A_i, and q. Finally, as (4) is known to possess the solutions

$$\phi=\begin{cases}-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}q\right), & \sigma<0,\\ -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}q\right), & \sigma<0,\\ \sqrt{\sigma}\tan\left(\sqrt{\sigma}q\right), & \sigma>0,\\ -\sqrt{\sigma}\cot\left(\sqrt{\sigma}q\right), & \sigma>0,\\ \dfrac{-1}{q}, & \sigma=0,\end{cases} \tag{5}$$

Substituting A_i, q and (5) into (3), one obtains the exact solutions to the given NPDE. Now we apply the mapping approach to (1). By the balancing procedure, the ansatz (3) becomes

$$U=A_0+A_1\phi\left(q\right) \tag{6}$$

with

$$q=lx+my+nt, \tag{7}$$

where A_0, A_1, l, m, n are arbitrary constants. Substituting (6), (7) and (4) into (1) and collecting coefficients of polynomials of ϕ, then setting each coefficient to zero, we have

$$A_0=-\frac{\ln+m^2}{l^2}, A_1=2l \tag{8}$$

Based on the solutions of (4), one thus obtains following exact solutions of Equation (1):

$$U_1=-\frac{\ln+m^2}{l^2}-\frac{2l^3\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\left(lx+my+nt\right)\right)}{l^2},\sigma<0 \tag{9}$$

$$U_2=-\frac{\ln+m^2}{l^2}-\frac{2l^3\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\left(lx+my+nt\right)\right)}{l^2},\sigma<0, \tag{10}$$

$$U_3=-\frac{\ln+m^2}{l^2}-\frac{2l^3\sqrt{\sigma}\tan\left(\sqrt{\sigma}\left(lx+my+nt\right)\right)}{l^2},\sigma>0, \tag{11}$$

$$U_4=-\frac{\ln+m^2}{l^2}-\frac{2l^3\sqrt{\sigma}\cot\left(\sqrt{\sigma}\left(lx+my+nt\right)\right)}{l^2}\sigma>0, \tag{12}$$

$$U_5=-\frac{l^2nx+lmny+\ln^2t+m^2lx}{\left(lx+my+nt\right)l^2}-\frac{m^3y+m^2nt+2l^3}{\left(lx+my+nt\right)l^2},\sigma=0. \tag{13}$$

3. Kind Wave Excitations of DZK Equation

In the following discussion, we merely analyze kind wave excitations of DZK equation. According to the solution $U_2\left(10\right)$, when we set the parameters $l=10,\ m=-10,\ n=-10,=-10$ at time $t=0$, we can obtain a kind wave excitation of the physical quantity U_2 presented in **Figure 1**.

Furthermore, According to the solution U_2, when we set the parameters $l=20,\ m=20,\ n=20$ and $\sigma=-10$ at times a) $t=-6$, b) $t=-3$, c) $t=0$, d) $t=3$, e) $t=6$, we can obtain the time evolution of a kind wave presented in **Figure 2**. From **Figure 2**, one finds that the kind wave moves in the same direction and the amplitude, velocity, and wave shape of the kind wave do not undergo any change with time.

4. Summary and Discussion

In summary, with the help of a Riccati mapping method and a linear variable separation method, we find some new exact solutions of the (2 + 1)-dimensional dissipative Zabolotskaya-Khokhlov equation. Based on the de-

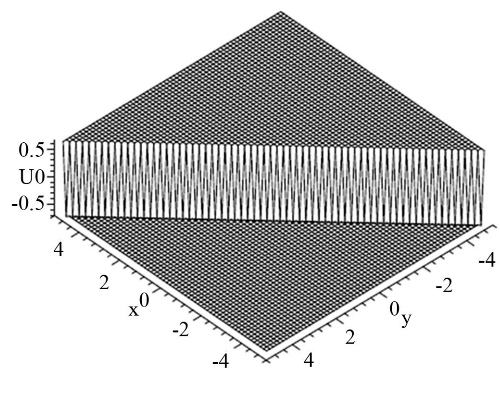

Figure 1. Plot of the kind wave structure for the physical quantity U_2.

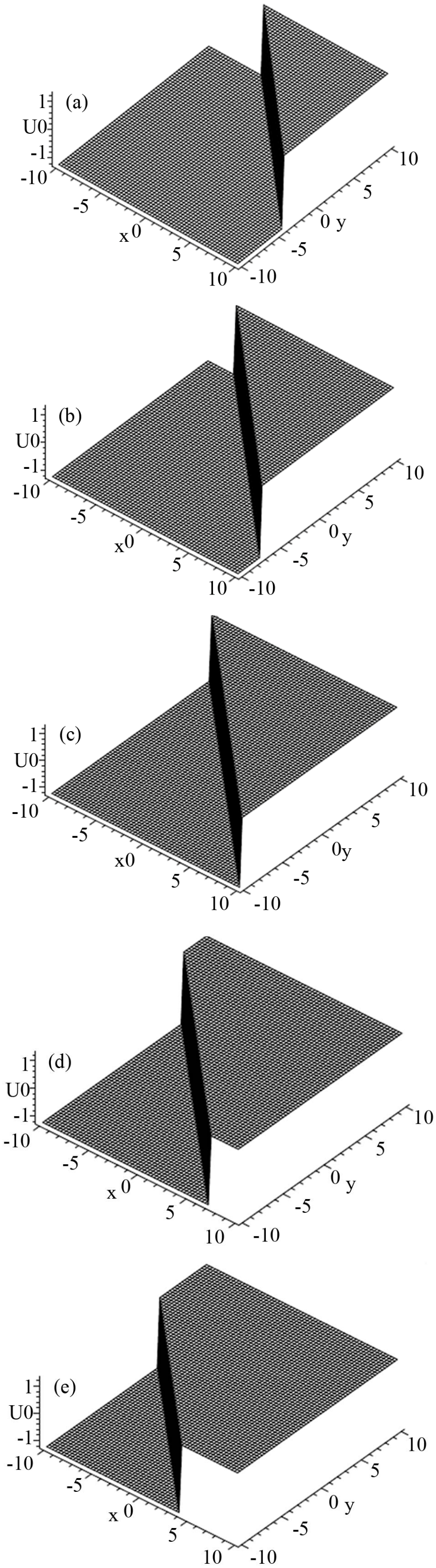

Figure 2. Plot of the time evolution of a kind wave for the physical quantity U_2.

rived solution U_2, we obtained the kind wave solution and studied the time evolution of a kind wave, which are different from the ones of the previous work. Because of wide applications of the DZK equation in physics, more properties are worthy to be studied such as its Lax pair, symmetry reduction, bilinear form, and Darboux transformation, etc. All these properties are worthy of studying further.

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