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Application of $\alpha\delta$-Closed Sets

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ABSTRACT

In this paper, we introduce the notion of $\alpha\delta$-US spaces. Also we study the concepts of $\alpha\delta$-convergence, sequentially $\alpha\delta$-compactness, sequentially $\alpha\delta$-continunity and sequentially $\alpha\delta$-sub-continuity and derive some of their properties.

Keywords: $\alpha\delta$-US Spaces; $\alpha\delta$-Convergence; Sequentially $\alpha\delta$-Compactness; Sequentially $\alpha\delta$-Continuity; Sequentially $\alpha\delta$-Sub-Continuity

1. Introduction

In 1967, A. Wilansky [1] introduced and studied the concept of US spaces. Also, the notion of $\alpha\delta$-closed sets of a topological space is discussed by R. Devi, V. Kokilavani and P. Basker [2,3]. The concept of slightly continuous functions is introduced and investigated by Erdal Ekici *et al.* [4]. In this paper, we define that a sequence $\{x_n\}$ in a space X is $\alpha\delta$-converges to a point $x \in X$ if $\{x_n\}$ is eventually in every $\alpha\delta$-open set containing x. Using this concept, we define the $\alpha\delta$-US space, Sequentially-$\alpha\delta$-continuous, Sequentially-Nearly-$\alpha\delta$-continuous, Sequentially-Sub-$\alpha\delta$-continuous and Sequentially-$\alpha\delta O$-compact of a topological space (X,τ).

2. Preliminaries

Throughout this paper, spaces X and Y always mean topological spaces. Let X be a topological space and A, a subset of X. The closure of A and the interior of A are denoted by $cl(A)$ and $\mathrm{int}(A)$, respectively. A subset A is said to be regular open (resp. regular closed) if $A=\mathrm{int}(cl(A))$ (resp. $A=cl(\mathrm{int}(A))$, the δ-interior [5] of a subset A of X is the union of all regular open sets of X contained in A and is denoted by $\mathrm{Int}_\delta(A)$. The subset A is called δ-open if $A=\mathrm{Int}_\delta(A)$, *i.e.*, a set is δ-open if it is the union of regular open sets. The complement of a δ-open set is called δ-closed.

Alternatively, a set $A \subset (X,\tau)$ is called δ-closed if $A = cl_\delta(A)$, where $cl_\delta(A)=\{x / x \in U \in \tau \Rightarrow \mathrm{int}(cl(A)) \cap A \neq \varphi\}$. The family of all δ-open (resp. δ-closed) sets in X is denoted by $\delta O(X)$ (resp. $\delta C(X)$). A subset A of X is called α-open [6] if $A \subset \mathrm{int}(cl(\mathrm{int}(A)))$ and the complement of a α-open are called α-closed. The intersection of all α-closed sets containing A is called the α-closure of A and is denoted by $\alpha cl(A)$, Dually, α-interior of A is defined to be the union of all α-open sets contained in A and is denoted by $\alpha\,\mathrm{int}(A)$.

We recall the following definition used in sequel.

Definition 2.1. A subset A of a space X is said to be

(a) An α-generalized closed [7] (αg-closed) set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-open in (X,τ).

(b) An $\alpha\delta$-closed [8] set if $cl_\delta(A) \subseteq U$ whenever $A \subseteq U$ and U is αg-open in (X,τ).

The complement of a $\alpha\delta$-closed set is said to be $\alpha\delta$-open. The intersection of all $\alpha\delta$-closed sets of X containing A is called $\alpha\delta$-closure of A and is denoted by $\alpha\delta_{Cl}(A)$. The union of all $\alpha\delta$-open sets of X contained in A is called $\alpha\delta$-interior of A and is denoted by $\alpha\delta_{Int}(A)$.

3. $\alpha\delta$-US Spaces

Definition 3.1. A sequence $\{x_n\}$ in a space X, $\alpha\delta$-converges to a point $x \in X$ if $\{x_n\}$ is eventually in every $\alpha\delta$-open set containing x.

Definition 3.2. A space X is said to be $\alpha\delta$-US if every sequence in X, $\alpha\delta$-converges to a point of X.

Definition 3.3. A space X is said to be

(a) $T_1^{\#\alpha\delta}$ if each pair of distinct points x and y in X there exists an $\alpha\delta$-open set U in X such that $x \in U$ and $y \notin U$ and a $\alpha\delta$-open set V in X such that $y \in V$ and $x \notin V$.

(b) $T_2^{\#\alpha\delta}$ if for each pair of distinct points x and y in X there exists an $\alpha\delta$-open sets U and V such

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that $U \cap V = \varphi$ and $x \in U$, $y \in V$.

Theorem 3.4. Every *αδ*-US-space is $T_1^{\#\alpha\delta}$.

Proof. Let X be an *αδ*-US-space and x, y be two distinct points of X. Consider the sequence $\{x_n\}$, where $x_n = x$ for any $n \in N$. Clearly $\{x_n\}$ *αδ*-converges to x. Since $x \neq y$ and X is *αδ*-US, $\{x_n\}$ does not *αδ*-converges to y, *i.e.*, there exists an *αδ*-open set U containing x but not y. Similarly, we obtain an *αδ*-open set V containing y but not x. Thus, X is $T_1^{\#\alpha\delta}$.

Theorem 3.5. Every $T_2^{\#\alpha\delta}$-space is *αδ*-US.

Proof. Let X be a $T_2^{\#\alpha\delta}$ space and $\{x_n\}$ a sequence in X. Assume that $\{x_n\}$ *αδ*-converges to two distinct points x and y. Then $\{x_n\}$ is eventually in every $T_2^{\#\alpha\delta}$ then $\{x_n\}$ is eventually in two disjoint *αδ*-open sets. This is a contradiction. Therefore, X is *αδ*-US.

Definition 3.6. A subset *A* of a space X is said to be

(a) Sequentially *αδ*-closed if every sequence in *A* *αδ*-converges to a point in *A*,

(b) Sequentially *αδO*-compact if every sequence in *A* has a subsequence which *αδ*-converges to a point in *A*.

Theorem 3.7. A space is *αδ*-US if and only if the diagonal set Δ is a sequentially *αδ*-closed subset of the product space $X \times X$.

Proof. Suppose that X is an *αδ*-US space and $\{(x_n, x_n)\}$ is a sequence in the diagonal Δ. It follows that $\{x_n\}$ is a sequence in X. Since X is *αδ*-US, the sequence $\{(x_n, x_n)\}$ *αδ*-converges to (x, x) which clearly belongs to Δ. Therefore, Δ is a sequentially *αδ*-closed subset of $X \times X$. Conversely, suppose that the diagonal Δ is a sequentially *αδ*-closed subset of $X \times X$. Assume that a sequence $\{x_n\}$ is *αδ*-converging to *x* and y. Then it follows that $\{(x_n, x_n)\}$ *αδ*-converges to (x, y). By hypothesis, since Δ is sequentially *αδ*-closed, we have $(x, y) \in \Delta$. Thus $x = y$. Therefore, X is *αδ*-US.

Theorem 3.8. If a space X is *αδ*-US and a subset *M* of *X* is sequentially $\alpha\delta O$-compact, then *M* is sequentially *αδ*-closed.

Proof. Assume that $\{x_n\}$ is any sequence in M which *αδ*-converges to a point $x \in X$. Since *M* is sequentially *αδO*-compact, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ *αδ*-converges to $m \in M$. Since X is *αδ*-US, we have $x = m$. This shows that *M* is sequentially *αδ*-closed.

Theorem 3.9. The product space of an arbitrary family of *αδ*-US topological space is an *αδ*-US topological space.

Proof. Let $\{x_\lambda : \lambda \in \Delta\}$ be a family of *αδ*-US topological spaces with the index set Δ. The product space of $\{x_\lambda : \lambda \in \Delta\}$ is denoted by $\prod X_\lambda$. Let $\{x_n(\lambda)\}$ be a sequence in $\prod X_\lambda$. Suppose that $\{x_n(\lambda)\}$ *αδ*-converges to two distinct points *x* and *y* in $\prod X_\lambda$. Then there exists a $\lambda_0 \in \Delta$ such that $x(\lambda_0) \neq y(\lambda_0)$. Then $\{x_n(\lambda_0)\}$ is a sequence in X_{λ_0}. Let V_{λ_0} be any *αδ*-open in X_{λ_0} containing $x(\lambda_0)$. Then $V = V_{\lambda_0} \times \prod_{\lambda \neq \lambda_0} X_\lambda$ is a *αδ*-open set of $\prod X_\lambda$ containing *x*. Therefore, $\{x_n(\lambda)\}$ is eventually in V. Thus $\{x_n(\lambda_0)\}$ is eventually in V_{λ_0} and it *αδ*-converges to $x(\lambda_0)$. Similarly, the sequence $\{x_n(\lambda_0)\}$ *αδ*-converges to $y(\lambda_0)$. This is a contradiction as X_{λ_0} is a *αδ*-US space. Therefore, the product space $\prod X_\lambda$ is *αδ*-US.

4. Sequentially *αδO*-Compact Preserving Functions

Definition 4.1. A function $f : X \to Y$ is said to be

(a) Sequentially-*αδ*-continuous at $x \in X$ if the sequence $\{f(x_n)\}$ *αδ*-converges to $f(x)$ whenever a sequence $\{x_n\}$ *αδ*-converges to x. If f is sequentially *αδ*-continuous at each $x \in X$, then it is said to be sequentially *αδ*-continuous.

(b) Sequentially-Nearly-*αδ*-continuous, if for each sequence $\{x_n\}$ in X that *αδ*-converges to $x \in X$, there exists subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that the sequence $\{f(x_{n_k})\}$ *αδ*-converges to $\{f(x_n)\}$.

(c) Sequentially-Sub-*αδ*-continuous if for each point $x \in X$ and each sequence $\{x_n\}$ in *αδ*-converging to, there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and a point $y \in Y$ such that the sequence $\{f(x_{n_k})\}$ *αδ*-converges to y.

(d) Sequentially, *αδO*-compact preserving if the image $f(M)$ of every sequentially *αδO*-compact set M of X is a sequentially *αδO*-compact subset of Y.

Theorem 4.2. Let $f_1 : X \to Y$ and $f_2 : X \to Y$ be two sequentially *αδ*-continuous functions. If Y is *αδ*-US, then the set $E = \{x \in X : f_1(x) = f_2(x)\}$ is sequentially *αδ*-closed.

Proof. Suppose that Y is *αδ*-US and $\{x_n\}$ is any sequence in *E* that f_1-converges to $x \in X$. Since f_1 and f_2 are sequentially *αδ*-continuous functions, the sequence $\{f_1(x_n)\}$ (respectively, $\{f_2(x_n)\}$) converges to $f_1(x)$ (respectively, $f_2(x)$). Since $x_n \in E$ for each $n \in N$ and Y is *αδ*-US, $f_1(x) = f_2(x)$ and hence $x \in E$. This shows that E is sequentially *αδ*-closed.

Lemma 4.3. Every function $f : X \to Y$ is sequentially sub *αδ*-US *αδ*-US continuous if Y is sequentially *αδO*-compact.

Proof. Let $\{x_n\}$ be a sequence in X that *αδ*-US converges to $x \in X$. It follows that $\{f(x_n)\}$ is a sequence in Y. Since Y is sequentially *αδO*-compact, there exists a subsequence $\{f(x_{n_k})\}$ of $\{f(x_n)\}$ that $\alpha\delta$-converges to a point $y \in Y$. Therefore $f: X \to Y$ is sequentially sub $\alpha\delta$-continuous.

Theorem 4.4. Every sequentially nearly $\alpha\delta$-continuous function is sequentially $\alpha\delta O$-compact preserving.

Proof. Let $f: X \to Y$ be a sequentially nearly $\alpha\delta$-continuous function and M be any sequentially $\alpha\delta O$-compact subset of X. We will show that $f(M)$ is a sequentially $\alpha\delta O$-compact subset of Y. So, assume that $\{y_n\}$ is any sequence in $f(M)$. Then for each $n \in N$, there exists a point $x_n \in M$ such that $f(x_n) = y_n$. Now M is sequentially $\alpha\delta O$-compact, so there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ that $\alpha\delta$-converges to a point $x \in M$. Since f is sequentially nearly $\alpha\delta$-continuous, there exists a subsequence
$\{x_{n_k}(i)\}$ of $\{x_{n_k}\}$ such that $\{f(x_{n_k}(i))\}$ $\alpha\delta$-converges to $f(x)$. Therefore, there exists a subsequence
$\{y_{n_k}(i)\}$ of $\{y_n\}$ that $\alpha\delta$-converges to $f(x)$. This implies that $f(M)$ is a sequentially $\alpha\delta O$-compact set of Y.

Theorem 4.5. Every sequentially $\alpha\delta O$-compact preserving function is sequentially sub-$\alpha\delta$-continuous.

Proof. Suppose that $f: X \to Y$ is a sequentially $\alpha\delta O$-compact preserving function. Let x be any point of X and $\{x_n\}$ a sequence that $\alpha\delta$-converges to x. We denote the set $\{x_n : n \in N\}$ by A and put $M = A \bigcup \{x\}$. Since $\{x_n\}$ $\alpha\delta$-converges to x, M is sequentially $\alpha\delta O$-compact. By hypothesis, f is sequentially $\alpha\delta O$-compact subset of Y. Now in $f(M)$ there exists a subsequence $\{f(x_{n_k})\}$ of $\{f(x_n)\}$ that $\alpha\delta$-converges to a point $y \in f(M)$. This implies that f sequentially sub-$\alpha\delta$-continuous.

Theorem 4.6. A function $f: X \to Y$ is sequentially $\alpha\delta O$-compact preserving if and only if $f/M : M \to f(M)$ is sequentially sub-$\alpha\delta$-continuous for each sequentially $\alpha\delta O$-compact set M of X.

Proof. *Necessity*: Suppose that $f: X \to Y$ is a sequentially $\alpha\delta O$-compact preserving function. Then $f(M)$ is sequentially $\alpha\delta O$-compact in Y for each sequentially $\alpha\delta O$-compact subset M of X. Therefore, by Theorem 3.5 $f/M : M \to f(M)$ is sequentially sub-$\alpha\delta$-continuous.

Sufficiency: Let M be any sequentially $\alpha\delta O$-compact set of X. We will show that $f(M)$ is sequentially $\alpha\delta O$-compact subset of Y. Let $\{y_n\}$ be any sequence in $f(M)$. Then for each $n \in N$, there exists a point $x_n \in M$ such that $f(x_n) = y_n$. Since $\{x_n\}$ is a sequence in the sequentially $\alpha\delta O$-compact set M there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ that $\alpha\delta$-converges to a point in M. By hypothesis $f/M : M \to f(M)$ is sequentially sub-$\alpha\delta$-continuous, hence there exists a subsequence $\{y_{n_k}\}$ of $\{y_n\}$ that $\alpha\delta$-converges to $y \in f(M)$. This implies that $f(M)$ is sequentially $\alpha\delta O$-compact in Y.

Corollary 4.7. If a function $f: X \to Y$ is sequentially sub-$\alpha\delta$-continuous and $f(M)$ is sequentially $\alpha\delta$-closed in Y for each sequentially $\alpha\delta O$-compact set M of X, then f is sequentially $\alpha\delta O$-compact preserving.

Proof. It will be sufficient to show that $f/M : M \to f(M)$ is sequentially sub-$\alpha\delta$-continuous for each sequentially $\alpha\delta O$-compact set M of X and by Lemma 3.3. We have already done. So, let $\{x_n\}$ be any sequence in M that $\alpha\delta$-converges to a point $x \in M$. Then, since f is sequentially sub-$\alpha\delta$-continuous there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ and a point $y \in Y$ such that $\{f(x_{n_k})\}$ $\alpha\delta$-converges to y. Since $\{f(x_{n_k})\}$ is a sequence in the sequentially $\alpha\delta$-closed set $f(M)$ of Y, we obtain $y \in f(M)$. This implies that $f/M : M \to f(M)$ is sequentially sub $\alpha\delta$-continuous.

5. Slightly $\alpha\delta$-Continuous Functions

Definition 5.1. A function $f: X \to Y$ is said to be slightly $\alpha\delta$-continuous if for each $x \in X$ and for each $v \in CO(Y, f(x))$, there exists $U \in \alpha\delta O(X, x)$ such that $f(U) \subset V$, where $CO(Y, f(x))$ is the family of clopen sets containing $f(x)$ in a space Y.

Definition 5.2. Let $(D, \le)$ be a directed set A net $\{x_\lambda : \lambda \in D\}$ in X is said to be $\alpha\delta$-convergent to a point $x \in X$ if $\{x_\lambda\}_{\lambda \in D}$ is eventually in each $U \in \alpha\delta O(X, x)$.

Theorem 5.3. For a function $f: X \to Y$, the following are equivalent:

(a) f is slightly $\alpha\delta$-continuous.

(b) $f^{-1}(v) \in \alpha\delta O(X)$ for each $V \in CO(Y)$.

(c) $f^{-1}(v)$ *is* $\alpha\delta$-*cl*-open for each $V \in CO(Y)$.

(d) for each $x \in X$ and for each net $\{x_\lambda\}_{\lambda \in D}$ in X.

Proof. $(a) \Rightarrow (b)$. Let $V \in CO(Y)$ and let $x \in f^{-1}(V)$, then $(x) \in V$. Since f is slightly $\alpha\delta$-continuous, there is a $U \in \alpha\delta O(X, x)$ such that $(U) \subset V$. Thus $f^{-1}(U) = \bigcup_x \{U : x \in f^{-1}(V)\}$, that is $f^{-1}(U)$ is a union of $\alpha\delta$-open sets. Hence $f^{-1}(U) \in \alpha\delta O(X)$.

$(b) \Rightarrow (c)$. Let $V \in CO(Y)$, then $(Y - V) \in CO(X)$.

By hypothesis $f^{-1}(Y - V) = X - f^{-1}(V) \in \alpha\delta O(X)$. Thus $f^{-1}(U)$ is $\alpha\delta$-closed.

$(c) \Rightarrow (d)$. Let $\{x_\lambda\}_{\lambda \in D}$ be a net in X $\alpha\delta$-converging to x and let $V\in CO(Y,f(x))$. There is thus a $U\in\alpha\delta O(X,x)$ such that $(U)\subset V$. There is thus a $\lambda_0\in D$ such that $\lambda_0\le\lambda$ implies $x_\lambda\in U$ since $\{x_\lambda\}_{\lambda\in D}$ is *αδ*-convergent to x. Thus $f(x_\lambda)\in f(U)\subset V$ for all λ. Thus $\{f(x_\lambda)\}_{\lambda\in D}$ is *αδ*-convergent to $f(x)$.

$(d)\Rightarrow(a)$ Suppose that f is not slightly *αδ*-continuous at a point $x\in X$, then there exists a $V\in CO(Y,f(x))$ such that $f(U)$ does not contained in V for each $U\in\alpha\delta O(X,x)$. So $f(U)\cap(Y-V)\ne\varphi$ and thus $U\cap f^{-1}(Y-V)\ne\varphi$ for each $U\in\alpha\delta O(X,x)$, since $\alpha\delta O(X,x)$ is directed by set inclusion C, there exists a selection function x_U from $\alpha\delta O(X,x)$ into X for each $U\in\alpha\delta O(X,x)$. Thus $\{x_U\}_U\in\alpha\delta O(X,x)$ is a net in X *αδ*-converging to x. Since $X_U\in U\cap f^{-1}(Y-V)=U-f^{-1}(V)$ and so $f(x_U)\notin V$, for each U, $\{f(x_U)\}_U\in\alpha\delta O(X,x)$ is not eventually in $V\in CO(Y,f(x))$, which is a contradiction. Hence (a) holds.

Theorem 5.4. If $f:X\to Y$ is slightly *αδ*-continuous and $g:Y\to Z$ is slightly continuous, then their composition $g\circ f$ is slightly *αδ*-continuous.

Proof. Let $V\in CO(Z)$, then $g^{-1}(V)\in CO(Y)$. Since f is slightly *αδ*-continuous, $f^{-1}(g^{-1}(V))=(g\circ f)^{-1}(V)\in\alpha\delta O(X)$. Thus $g\circ f$ is Slightly *αδ*-continuous.

Theorem 5.5. The following are equivalent for a function $f:X\to Y$:

(a) f is slightly *αδ*-continuous,

(b) for each $x\in X$ and for each $V\in CO(Y,f(x))$, there exists *αδ-cl*-open set U such that $f(U)\subset U$,

(c) for each closed set F of Y, $f^{-1}(F)$ is *αδ*-closed,

(d) $f(cl(A))\subset\alpha\delta_{cl}(f(A))$ for each $A\subset X$ and

(e) $cl(f^{-1}(B))\subset f^{-1}(\alpha\delta_{cl}(B))$ for each $B\subset Y$.

Proof. $(a)\Rightarrow(b)$ Let $x\in X$ and $V\in CO(Y,f(x))$ by Theorem 4.3. $f^{-1}(V)$ is clopen. Put $U=f^{-1}(V)$, then $x\in U$ and $f(U)\subset V$.

$(b)\Rightarrow(c)$ is obvious.

$(c)\Rightarrow(d)$ since $\alpha\delta_{cl}(f(A))$ is the smallest *αδ*-closed set containing $f(A)$, hence by (c), we have (d).

$(d)\Rightarrow(e)$ for each $V\subset Y$, $f(cl(f^{-1}(B)))\subset\alpha\delta_{cl}(f(f^{-1}(B)))\subset\alpha\delta_{cl}(B)$. Hence $f(cl(f^{-1}(B)))\subset\alpha\delta_{cl}(B)$ $\Rightarrow cl(f^{-1}(B))\subset f^{-1}(\alpha\delta_{cl}(B))$.

$(e)\Rightarrow(a)$ Let $V\in CO(Y)$. then $(Y-V)\in CO(X)$, by (e), we have $cl(f^{-1}(Y-V))\subset f^{-1}(\alpha\delta_{cl}(Y-V))=f^{-1}(Y-V)$, since every closed set is *αδ*-closed, thus $f^{-1}(Y-V)=X-f^{-1}(V)$ is closed and thus *αδ*-closed, thus $f^{-1}(V)\in\alpha\delta O(X)$ and f is slightly *αδ*-continuous.

Theorem 5.6. If $f:X\to Y$ is a slightly *αδ*-continuous injection and Y is clopen T_1, then X is $T_1^{\#\alpha\delta}$.

Proof. Suppose that Y is clopen T_1. For any distinct points x and y in X, there exist $V,W\in CO(Y)$ such that $f(x)\in V, f(y)\notin V, f(x)\notin W$ and $f(y)\in W$. Since f is slightly *αδ*-continuous, $f^{-1}(V)$ and $f^{-1}(W)$ are *αδ*-open subsets of X such that $x\in f^{-1}(V), y\notin f^{-1}(V), x\notin f^{-1}(W)$ and $y\in f^{-1}(W)$. This shows that X is $T_1^{\#\alpha\delta}$.

Theorem 5.7. *If* $f:X\to Y$ is a slightly *αδ*-continuous surjection and Y is clopen T_2, then X is $T_2^{\#\alpha\delta}$.

Proof. For any pair of distinct points x and y in X, there exist disjoint clopen sets U and V in Y such that $f(x)\in U$ and $f(y)\in V$. Since *f* is slightly *αδ*-continuous, $f^{-1}(U)$ and $f^{-1}(V)$ are *αδ*-open in X containing x and y respectively. Therefore $f^{-1}(U)\cap f^{-1}(V)=\varphi$ because $U\cap V=\varphi$. This shows that X is $T_2^{\#\alpha\delta}$.

Definition 5.8. A space is called *αδ*-regular if for each *αδ*-closed set F and each point $x\notin F$, there exist disjoint open sets U and V such that $F\subset U$ and $x\in V$.

Definition 5.9. A space is said to be *αδ*-normal if for every pair of disjoint *αδ*-closed subsets F_1 and F_2 of X, there exist disjoint open sets U and V such that $F_1\subset U$ and $F_2\subset V$.

Theorem 5.10. If *f* is slightly *αδ*-continuous injective open function from an *αδ*-regular space X onto a space then Y is clopen regular.

Proof. Let *F* be clopen set in Y and be $y\notin F$, take $y=f(x)$. Since *f* is slightly *αδ*-continuous, $f^{-1}(F)$ is a *αδ*-closed set, take $G=f^{-1}(F)$, we have $x\notin G$. Since X is *αδ*-regular, there exist disjoint open sets U and V such that $G\subset U$ and $\in V$. We obtain that $F=f(G)\subset f(U)$ and $y=f(x)\in f(V)$ such that *f*(*U*) and *f*(*V*) are disjoint open sets. This shows that Y is clopen regular.

Theorem 5.11. If f is slightly *αδ*-continuous injective open function from a *αδ*-normal space X onto a space Y, then Y is *cl*-open normal.

Proof. Let F_1 and F_2 be disjoint *cl*-open subsets of Y Since f is slightly *αδ*-continuous, $f^{-1}(F_1)$ and $f^{-1}(F_2)$ are *αδ*-closed sets. Take $U = f^{-1}(F_1)$ and $V = f^{-1}(F_2)$. We have $U \cap V = \varphi$. Since X is *αδ*-regular, there exist disjoint open sets *A* and *B* such that $U \subset A$ and $V \subset B$. We obtain that $F_1 = f(U) \subset f(A)$ and $F_2 = f(V) \subset f(B)$ such that $f(A)$ and $f(B)$ are disjoint open sets. Thus, *Y* is *cl*-open normal.

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