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Non-Traveling Wave Solutions for the (2+1)-Dimensional Breaking Soliton System

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ABSTRACT

In this work, starting from the (G'/G)-expansion method and a variable separation method, a new non-traveling wave general solutions of the (2+1)-dimensional breaking soliton system are derived. By selecting appropriately the arbitrary functions in the solutions, special soliton-structure excitations and evolutions are studied.

Keywords: (G'/G)-Expansion Method; Variable Separation Approach; Breaking Soliton System; Non-Traveling Wave Solution

1. Introduction

Modern soliton theory is widely applied in many natural sciences [1-4] such as chemistry, biology, mathematics, communication, and particularly in almost all branches of physics like fluid dynamics, plasma physics, field theory, optics, and condensed matter physics, etc. [5-8]. In order to find new exact solutions of nonlinear equations, much methods have been proposed, such as the Lie group method of infinitesimal transformations, the nonclassical Lie group method, the Clarkson and Kruskal direct method (CK) [9,10], the conditional similarity reduction method [11-13] and the improved mapping approach, etc. [14-23].

Recently, the (G'/G)-expansion method was proposed to obtain the new exact solutions of the nonlinear evolution equations [24]. Subsequently the powerful (G'/G)-expansion method has been widely used by many differential such as in [25-30]. However, the previous works have mainly concentrated on obtaining new exact traveling wave solutions for the nonlinear evolution equations.

In this paper, by using the (G'/G)-expansion method, we construct non-traveling wave solutions with arbitrary functions in the (2+1)-dimensional breaking soliton system

$$u_t - bu_{xxy} + 4b(uv)_x = 0, \tag{1}$$

$$v_x - u_y = 0, \tag{2}$$

where b is an arbitrary constant, the system (1)-(2) was used to describes the (2+1)-dimensional interaction of Riemann wave propagated along the y-axis with long wave propagated along the x-axis and it seems to have been investigated extensively where overlapping solutions have been derived [31]. In the past, we have obtained the Annihilation solitons and chaotic solitons by the improved mapping approach [32]. Since the detailed physical background of the breaking soliton system has been given in [31], we neglect the corresponding description.

2. The (*G*′/*G*)-Expansion Method and Non-Traveling Wave Solutions to the (2+1)-Dimensional Breaking Soliton System

Before starting to apply the (G'/G)-expansion method, we will give a simple description of the method. For doing this, suppose that a (2+1)-dimensional nonlinear equation, say in three independent variables x, y and t, is given by

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xy}, u_{xx}, u_{yy}, \cdots) = 0. \tag{3}$$

The fundamental idea of the (G'/G)-expansion method is that the solutions of equation (3) can be expressed by a polynomial in (G'/G) as follows [24,29,30]:

$$u = \sum_{i=1}^{m} a_i \left(\frac{G'}{G}\right)^i + a_0, \tag{4}$$

where $G = G(\xi)$, $\xi = \alpha x + \beta y + \omega t$ is traveling wave transformation, and $\alpha, \beta, \omega, a_i (i = 1, 2, \cdots, m)$ are constants to be determined later, G satisfies the second order LODE as follow:

$$G'' + \lambda G' + \mu G = 0. \tag{5}$$

In order to construct the non-traveling wave solutions with arbitrary function $\xi(x,y,t)$ for the (2+1)-dimensional breaking soliton system (1)-(2), we suppose its solutions can be express as follow:

$$u=\sum_{i=0}^{m}a_i\left(\frac{G'}{G}\right)^i, \tag{6}$$

$$v=\sum_{j=0}^{n}b_j\left(\frac{G'}{G}\right)^j, \tag{7}$$

where $a_i\left(i=0,1,2,\cdots,m\right)$, $b_j\left(j=0,1,2,\cdots,n\right)$ are the functions of x,y,t to be determined later, $\xi=\xi(x,y,t)$ is the arbitrary function of x, y, t, and $G(\xi)$ satisfies the second order LODE (5).

Applying the homogenous balance principle, we obtain $m=n=2$. Thus (6)-(7) can be converted into

$$u=a_0+a_1\left(\frac{G'}{G}\right)+a_2\left(\frac{G'}{G}\right)^2, \tag{8}$$

$$v=b_0+b_1\left(\frac{G'}{G}\right)+b_2\left(\frac{G'}{G}\right)^2, \tag{9}$$

For simplifying the computation, we seek for the variable separation solutions of the breaking soliton system (2) by taking $\xi(x,y,t)=\gamma(x)+\eta(y+ct)$.

Substituting (8)-(9) into the system (1)-(2), collecting all terms with the same power of G'/G together. Then setting each coefficient of the polynomials to zero, we can derive a set of over-determined partial differential equation for $a_0,a_1,a_2,b_0,b_1,b_2,\gamma$ and η.

$$\left(\frac{G'}{G}\right)^5: 3\gamma'^2\eta_y+2b_2\gamma'=0, \tag{10}$$

$$\begin{aligned}\left(\frac{G'}{G}\right)^4&: -6a_1\gamma'^2\eta_y-54\lambda a_2\gamma'^2\eta_y+6a_2\gamma''\eta_y+4a_2b_{2x}-16\lambda a_2b_2\gamma'+12a_{2x}\gamma'\eta_y\\&-12a_1b_2\gamma'+4a_{2x}b_2+6a_{2y}\gamma'^2-12a_2b_1\gamma'=0,\end{aligned} \tag{11}$$

$$\begin{aligned}\left(\frac{G'}{G}\right)^3&: 4bb_2a_{1x}+4ba_2b_{1x}+4ba_1b_{2x}+2ba_{1y}\gamma'^2-2ba_{2xx}\eta_y+2ba_1\gamma''\eta_y-2a_2\eta_t-12b\lambda a_1\gamma'^2\eta_y\\&-16b\mu a_2b_2\gamma'-40b\mu a_2\gamma'^2\eta_y-38b\lambda^2a_2\gamma'^2\eta_y-12b\lambda a_1b_2\gamma'+20b\lambda a_{2x}\gamma'\eta_y-12b\lambda a_2b_1\gamma'\\&-8ba_0b_2\gamma'-8ba_1b_1\gamma'-8ba_2b_0\gamma'+4ba_{1x}\gamma'\eta_y+10b\lambda a_2\gamma'^2\eta_y-4ba_{2xy}\gamma'+4ba_{2x}b_1=0,\end{aligned} \tag{12}$$

$$a_2\eta_y-b_2\gamma'=0, \tag{13}$$

$$\begin{aligned}\left(\frac{G'}{G}\right)^2&: 4ba_1b_{1x}-2\lambda a_2\eta_t-4ba_0b_1\gamma'-2b\lambda a_{2xx}\eta_y-52b\lambda\mu a_2\gamma'^2\eta_y-12b\mu a_2b_1\gamma'+6b\lambda a_{1x}\gamma'\eta_y\\&-a_1\eta_t-8b\lambda a_1b_1\gamma'-12b\mu a_1b_2\gamma'-8b\mu a_1\gamma'^2\eta_y-7b\lambda^2a_1\gamma'^2\eta_y-8b\lambda^3a_2\gamma'^2\eta_y-8b\lambda a_2b_0\gamma'\\&+4ba_{2x}b_0+16b\mu a_{2x}\gamma'\eta_y-4ba_1b_0\gamma'-8b\lambda a_0b_2\gamma'+8b\mu a_2\gamma''\eta_y+3b\lambda a_1\gamma''\eta_y+4b\lambda^2a_2\gamma''\eta_y\\&+4ba_{0x}b_2+8b\lambda^2a_{2x}\gamma'\eta_y+4ba_{1x}b_1-ba_{1xx}\eta_y+4ba_0b_{2x}+4ba_2b_{0x}=0,\end{aligned} \tag{14}$$

$$2\lambda a_2\eta_y+b_{2x}-2\lambda b_2\gamma'+a_1\eta_y-b_1\gamma'=0, \tag{15}$$

$$\begin{aligned}\left(\frac{G'}{G}\right)^1&: 12b\lambda\mu a_{2x}\gamma'\eta_y-b\lambda a_{1xx}\eta_y-16b\mu^2a_2\gamma'^2\eta_y-b\lambda^3a_1\gamma'^2\eta_y-8b\lambda\mu a_1\gamma'^2\eta_y-8b\mu a_1b_1\gamma'+2b\lambda^2a_{1x}\gamma'\eta_y\\&-4b\lambda a_1b_0\gamma'-8b\mu a_2b_0\gamma'-14b\lambda^2\mu a_2\gamma'^2\eta_y-4b\lambda a_0b_1\gamma'-8b\mu a_0b_2\gamma'+2b\mu a_1\gamma''\eta_y+6b\lambda\mu a_2\gamma''\eta_y+b\lambda^2\gamma''\eta_y\\&+4ba_0b_{1x}+4ba_{0x}b_1+4ba_1b_{0x}-2b\mu a_{2xx}\eta_y+a_{1t}-\lambda a_1\eta_t-2\mu a_2\eta_t+4b\mu a_{1x}\gamma'\eta_y+4ba_{1x}b_0=0,\end{aligned} \tag{16}$$

$$2\mu a_2\eta_y-2\mu b_2\gamma'+\lambda a_1\eta_y+b_{1x}-\lambda b_1\gamma'=0, \tag{17}$$

$$\begin{aligned}\left(\frac{G'}{G}\right)^0&: 2b\lambda\mu a_{1x}\gamma'\eta_y-6b\lambda\mu^2a_2\gamma'^2\eta_y+4b\mu^2a_{2x}\gamma'\eta_y-4b\mu a_0b_1\gamma'-\mu a_1\eta_t+b\lambda\mu a_1\gamma''\eta_y-b\mu a_{1xx}\eta_y\\&+a_{0t}+4ba_{0x}b_0+2b\mu^2a_2\gamma''\eta_y+ba_{0xxy}-4b\mu a_1b_0\gamma'+4ba_0b_{0x}-2b\mu^2a_1\gamma'^2\eta_y-b\lambda^2\mu a_1\gamma'^2\eta_y=0,\end{aligned} \tag{18}$$

$$\mu a_1\eta_y - a_{0y} - \mu b_1\gamma' + b_{0x} = 0. \quad (19)$$

Solving the Equations (10)-(19) yields

$$a_0 = \frac{3b\lambda\gamma'\gamma'' - b\lambda^2\gamma'^3 - 2b\mu\gamma'^3 - b\gamma''' - c\gamma'}{4b\gamma'},$$
$$a_1 = \frac{3}{2}\left(\gamma'' - \lambda\gamma'^2\right), \quad a_2 = -\frac{3}{2}\gamma'^2, \quad (20)$$
$$b_0 = -\frac{3}{2}\mu\gamma'\eta_y, \quad b_1 = -\frac{3}{2}\lambda\gamma'\eta_y, \quad b_2 = -\frac{3}{2}\gamma'\eta_y.$$

Substituting (20) and the general solutions of Equation (5) into (6)-(7), we can obtain the general non-traveling wave solutions for the (2+1)-dimensional breaking soliton system (1)-(2).

Case 1. When $\lambda^2 - 4\mu > 0$, by the general solutions of Equation (5) we can derive

$$\frac{G'}{G} = -\frac{\lambda}{2} + \Delta 1\frac{C_1\cosh\Delta 1(\gamma+\eta) + C_2\sinh\Delta 1(\gamma+\eta)}{C_1\sinh\Delta 1(\gamma+\eta) + C_2\cosh\Delta 1(\gamma+\eta)}. \quad (21)$$

Thus, the hyperbolic solutions of the system (1)-(2) are expressed as follows:

$$u_1 = \frac{2b\gamma'^3\Delta_1^2 - b\gamma''' - c\gamma'}{4b\gamma'} + \frac{3}{2}\gamma''\Delta_1\frac{C_1\cosh\Delta_1(\gamma+\eta) + C_2\sinh\Delta_1(\gamma+\eta)}{C_1\sinh\Delta_1(\gamma+\eta) + C_2\cosh\Delta_1(\gamma+\eta)}$$
$$-\frac{3}{2}\gamma'^2\Delta_1^2\left[\frac{C_1\cosh\Delta_1(\gamma+\eta) + C_2\sinh\Delta_1(\gamma+\eta)}{C_1\sinh\Delta_1(\gamma+\eta) + C_2\cosh\Delta_1(\gamma+\eta)}\right]^2, \quad (22)$$

$$v_1 = \frac{3}{2}\gamma'\eta_y\Delta_1^2\left\{1 - \left[\frac{C_1\cosh\Delta_1(\gamma+\eta) + C_2\sinh\Delta_1(\gamma+\eta)}{C_1\sinh\Delta_1(\gamma+\eta) + C_2\cosh\Delta_1(\gamma+\eta)}\right]^2\right\}, \quad (23)$$

where $\Delta_1 = \frac{\sqrt{\lambda^2 - 4\mu}}{2}$, $\gamma' \neq 0$, γ'', γ''' and η_y exist, and b, c, C_1 and C_2 are arbitrary constants.

Case 2. When $\lambda^2 - 4\mu < 0$, by the general solutions of Equation (5) we can derive

$$\frac{G'}{G} = -\frac{\lambda}{2} + \Delta 2\frac{-C_1\sin\Delta 2(\gamma+\eta) + C_2\cos\Delta 2(\gamma+\eta)}{C_1\cos\Delta 2(\gamma+\eta) + C_2\sin\Delta 2(\gamma+\eta)}. \quad (24)$$

So the trigonometric solutions of the system (1)-(2) are expressed as follows:

$$u_2 = -\frac{2b\gamma'^3\Delta_2^2 + b\gamma''' + c\gamma'}{4b\gamma'} + \frac{3}{2}\gamma''\Delta_2\frac{-C_1\sin\Delta_2(\gamma+\eta) + C_2\cos\Delta_2(\gamma+\eta)}{C_1\cos\Delta_2(\gamma+\eta) + C_2\sin\Delta_2(\gamma+\eta)}$$
$$-\frac{3}{2}\gamma'^2\Delta_2^2\left[\frac{-C_1\sin\Delta_2(\gamma+\eta) + C_2\cos\Delta_2(\gamma+\eta)}{C_1\cos\Delta_2(\gamma+\eta) + C_2\sin\Delta_2(\gamma+\eta)}\right]^2, \quad (25)$$

$$v_2 = -\frac{3}{2}\gamma'\eta_y\Delta_2^2\left\{1 + \left[\frac{-C_1\sin\Delta_2(\gamma+\eta) + C_2\cos\Delta_2(\gamma+\eta)}{C_1\cos\Delta_2(\gamma+\eta) + C_2\sin\Delta_2(\gamma+\eta)}\right]^2\right\}, \quad (26)$$

where $\Delta_2 = \frac{\sqrt{4\mu - \lambda^2}}{2}$, $\gamma' \neq 0$, γ', γ'' and η_y exist, and b, c, C_1 and C_2 are arbitrary constants.

Case 3. When $\lambda^2 - 4\mu = 0$, by the general solutions of Equation (5) we can derive

$$\frac{G'}{G} = -\frac{\lambda}{2} + \frac{C2}{C_1 + C_2(\gamma+\eta)}. \quad (27)$$

In this case, the rational solutions of the system (1)-(2) are showed as:

$$u_3 = -\frac{\gamma'''}{4\gamma'} - \frac{c}{4b} + \frac{3}{2}\gamma''\frac{C_2}{C_1 + C_2(\gamma+\eta)}$$
$$-\frac{3}{2}\gamma'^2\left[\frac{C_2}{C_1 + C_2(\gamma+\eta)}\right]^2, \quad (28)$$

$$v_3 = -\frac{3}{2}\gamma'\eta_y\left[\frac{C_2}{C_1 + C_2(\gamma+\eta)}\right] \quad (29)$$

where $\gamma' \neq 0$, γ', γ'' and η_y exist, and b, c, C_1 and C_2 are arbitrary constants.

3. Soliton Structure Excitation of the System (1)-(2)

Due to the arbitrary functions $\gamma(x)$ and $\eta(y+ct)$ in the solutions (22)-(29), it is convenient to excite abundant soliton structures. We take the solution (23) as an example to study the soliton excitations for the (2+1)-dimensional breaking soliton system (1)-(2). For instance, if we choose γ and η as

$$\gamma = k_1 + k_2 x \operatorname{sech}(k_3 x),$$
$$\eta = r_1 + r_2 (y + ct) \operatorname{sech}[r_3 (y + ct)], \tag{30}$$

where $k_1, k_2, k_3, r_1, r_2, r_3$ are arbitrary constants, and all are non-zero.

Substituting (30) into (23) leads to a soliton structure for the system (1)-(2). **Figures 1(a)-(d)** are the evolution plots of the solution (23) with time under the parameters as

$$C_1 = 1, C_2 = 2, k_1 = 0.15, k_2 = 0.1, k_3 = 0.3, r_1 = 0.15,$$
$$r_2 = 0.3, r_3 = 0.3, c = 1, \lambda = 1, \mu = 0.1. \tag{31}$$

Figures 2(a)-(b) are the plots with $t = 0$, and we choose the following special values of the parameters $C_1, C_2, k_1, k_2, k_3, r_1, r_2, r_3, c, \lambda, \mu$. There are

$$C_1 = 1, C_2 = 2, k_1 = 0.15, k_2 = 0.1, k_3 = 0.3,$$
$$r_1 = 0.15, r_2 = 0.3, r_3 = 0.3, c = 1, \lambda = 1, \mu = 0.1. \tag{32}$$

$$C_1 = 1, C_2 = 2, k_1 = 0.15, k_2 = 3, k_3 = 0.3, r_1 = 0.15,$$
$$r_2 = 3, r_3 = 0.3, c = 1, \lambda = 1, \mu = 0.1. \tag{33}$$

Above we show the excitation process of a special dromion soliton structure of the solution (23) for the (2+1)-dimensional breaking soliton system (1)-(2). It is clear

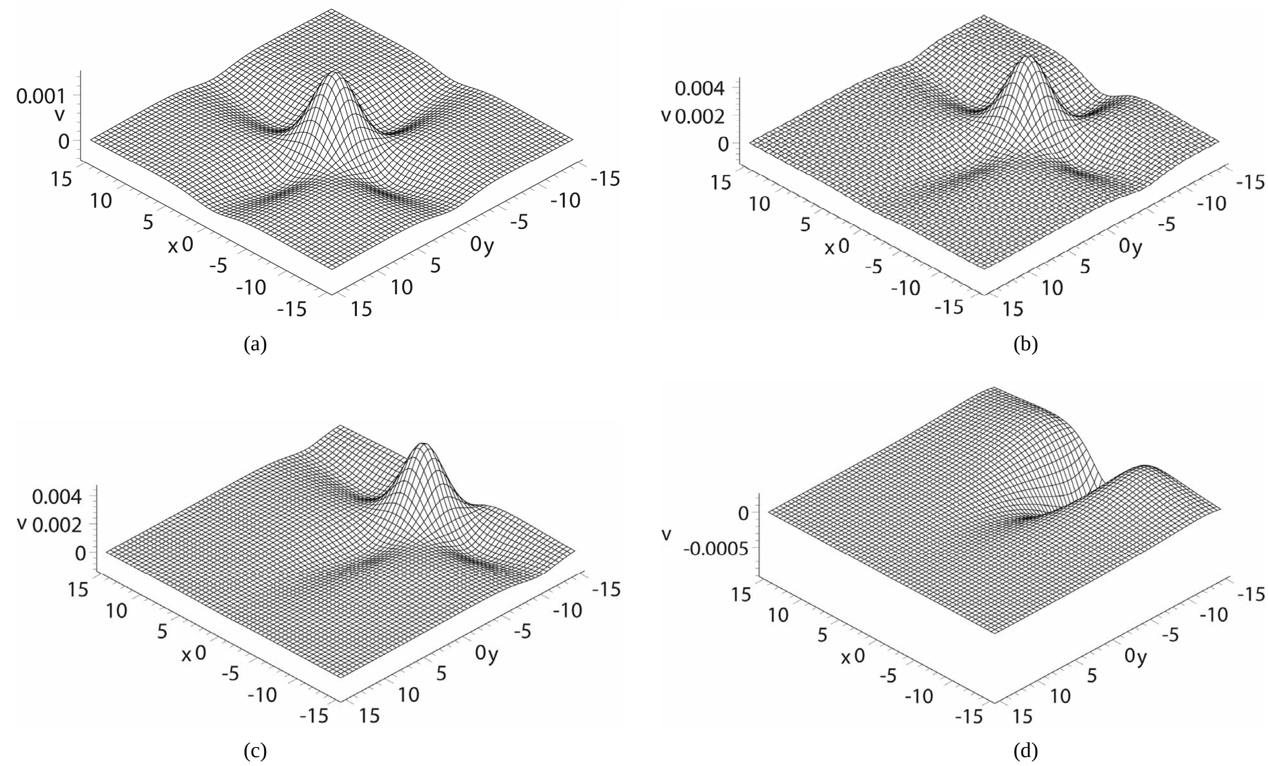

Figure 1. The evolution plots of the solution (23) under the parameters (31) with time: (a) t = 0; (b) t = 5; (c) t = 10; and (d) t = 20.

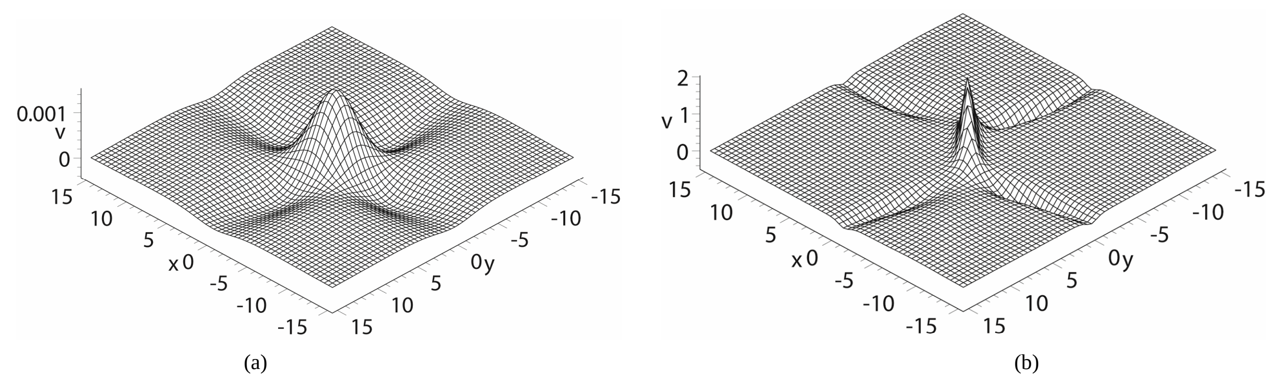

Figure 2. (a) A plot of the soliton for the Equation (23) with condition (32) at t = 0; (b) A plot of the soliton for the Equation (23) with condition (33) at t = 0.

that other selections of the arbitrary $\gamma(x)$ and $\eta(y+ct)$ in (23) may generate rich localized soliton structures. On the other hand, the solutions (22), (25)-(26), (28)-(29) may also be used to excite abundant soliton structures.

4. Summary and Discussion

In summary, via extending the (G'/G)-expansion method, more rich types explicit and exact non-traveling wave solutions of the (2+1)-dimensional breaking soliton system (1)-(2) are found out, and the traveling wave solutions are included by these non-traveling wave solutions. So the non-traveling wave solutions are more general. Furthermore, by choosing appropriately the arbitrary function $\xi(x,y,t)$ included in its solutions, one can study various interesting localized soliton excitations. Since the wide applications of the soliton theory, to learn more about the localized excitations and their applications in reality is worthy of study further.

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