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Grey GRM(1, 1) Model Based on Reciprocal Accumulated Generating and Its Application

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ABSTRACT

Aiming the problem of low accuracy during establishing grey model in which monotonically decreasing sequence data and traditional modeling methods are used, this paper applied the reciprocal accumulated generating and the approach optimizing grey derivative which is based on three points to deduce the calculation formulas for model parameters, established grey GRM(1, 1) model based on reciprocal accumulated generating. It provides a new method for the grey modeling. The example validates the practicability and reliability of the proposed model.

Keywords: Reciprocal Accumulated Generating; Grey GRM Model; Data Processing; Grey Modeling

1. Introduction

The main characteristic of grey system theory is the research about small data and uncertainty, and the basic tool is grey generation. Behavioral data of the system may be chaotic and complex, but there is always some kind of law among them. Grey generation is to find the law from these behavioral data, and establish grey model according to the law, further predict the system by solving the model [1]. So the grey generation is the basis establishing grey model. The most commonly method used is accumulated or inverse accumulated generating operation on in the process of modeling. The accumulated generation is able to inverse with the inverse accumulated generation, that is, the one-time accumulated generation sequence can be reverted to the original sequence by the inverse accumulated generation one time. For non-negative discrete sequence $\boldsymbol{X}^{(0)}$, the one-time accumulated generation sequence $\boldsymbol{X}^{(1)}$ is monotonically increasing. When a curve fits $\boldsymbol{X}^{(1)}$, it is reasonable that the curve is monotonically increasing. It is GM(1, 1) to predict. If $\boldsymbol{X}^{(0)}$ itself is monotonically decreasing, $\boldsymbol{X}^{(1)}$ is monotonically increasing and then the model value $\hat{\boldsymbol{X}}^{(1)}$ is also increased. When $\hat{\boldsymbol{X}}^{(1)}$ is inverse accumulated generated to the predicted value of the original sequence $\hat{\boldsymbol{X}}^{(0)}$, there will produce an unreasonable calculation errors. Backward accumulated generation was put forward and GOM(1, 1) based on backward accumulated generation was established [2]. GRM(1, 1) based on reciprocal generation was built after proposing reciprocal generation [3]. GRM(1, 1) was improved to establish the improved grey model CGRM(1, 1) based on reciprocal accumulated generation with better modeling accuracy [4]. Grey models based on reciprocal generation and opposite-direction accumulated generation make the generation sequence $\boldsymbol{X}^{(1)}$ also monotone decreasing, and then fitted $\boldsymbol{X}^{(1)}$ by using the decreasing monotonically curve to obtain the model value $\hat{\boldsymbol{X}}^{(1)}$ of $\boldsymbol{X}^{(1)}$. In this case, the reduction process from $\hat{\boldsymbol{X}}^{(0)}$ to $\boldsymbol{X}^{(0)}$ will not produce the unreasonable error and it improves modeling accuracy. In the paper, the grey derivative was optimized by using three-point grey derivative, and the calculation formulas for model parameters were deduced in the condition that the first component of $\boldsymbol{X}^{(1)}$ was taken as initial condition of grey differential equation in this model on the basis of Ref [3,4]. Grey GRM(1, 1) model based on reciprocal accumulated generating was established. This model with high precision has better practical and theoretical significance. The example validates the practicability and reliability of the proposed model.

2. Grey GRM(1, 1) Model Based on Reciprocal Accumulated Generating

Definition 1. Supposed the original sequence

$$\boldsymbol{X}^{(00)}=\left[x^{(00)}(1),x^{(00)}(2),\cdots,x^{(00)}(n)\right],$$

let $x^{(0)}(k)=\dfrac{1}{x^{(00)}(k)}$, $k=1,2,\cdots,n$, then

$\boldsymbol{X}^{(0)}=\left(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right)$ is named for reciprocal sequence of $\boldsymbol{X}^{(00)}$.

Definition 2. Supposed the original sequence

$$X^{(00)}=\left[x^{(00)}(1),x^{(00)}(2),\cdots,x^{(00)}(n)\right],$$

let $x^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)$, where $x^{(0)}(i)=\frac{1}{x^{(00)}(i)}$, $i=1,2,\cdots,n$, then $X^{(1)}=\left(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\right)$ is called as one-time reciprocal accumulated generation of $X^{(00)}$.

Definition 3. Supposed the original sequence

$$X^{(00)}=\left[x^{(00)}(1),x^{(00)}(2),\cdots,x^{(00)}(n)\right],$$

let $x^{(-1)}(k)=x^{(0)}(k)-x^{(0)}(k-1)$, where $x^{(0)}(k)=\frac{1}{x^{(00)}(k)}$, $k=2,\cdots,n$, then $X^{(-1)}=\left(x^{(-1)}(2),x^{(-1)}(3),\cdots,x^{(-1)}(n)\right)$ is called as one-time reciprocal regressive generation of $x^{(00)}$. The inverse accumulated generation is the inverse of accumulated generation, and they meet that

$$x^{(1)}(k)-x^{(1)}(k-1)=x^{(0)}(k)=\frac{1}{x^{(00)}(k)},\ k=2,\cdots,n.$$

It is known that the solution of equation

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}+ax^{(1)}(t)=b \tag{1}$$

is $x^{(1)}(t)=ce^{-at}+f$. When this curve is used to fit $X^{(1)}$, the key is how to deal with the derivative signal of discrete points. We take three points $A_{k-1}\left(k-1,x^{(1)}(k-1)\right)$, $A_k\left(k,x^{(1)}(k)\right)$ and $A_{k+1}\left(k+1,x^{(1)}(k+1)\right)$ in the exponential curve with monotone decreasing and up-concave $x^{(1)}(t)=ce^{-at}+f$. It is known easily that the slope of the curve at the point $A_k\left(k,x^{(1)}(k)\right)$ is between the ones of $A_{k-1}A_k$ and A_kA_{k+1}, namely,

$$\begin{aligned}\left.\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}\right|_{t=k}&=\alpha\left[x^{(1)}(k)-x^{(1)}(k-1)\right]\\&\quad+(1-\alpha)\left[x^{(1)}(k+1)-x^{(1)}(k)\right]\\&=\alpha x^{(0)}(k-1)+(1-\alpha)x^{(0)}(k)(k=2,3,\cdots,n)\end{aligned} \tag{2}$$

The albino equation of grey differential equation $\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}+az^{(1)}(t)=b$ is $\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t}+ax^{(1)}(t)=b$, so it can be discretized into:

$$\begin{gathered}\alpha x^{(0)}(k-1)+(1-\alpha)x^{(0)}(k)+ax^{(1)}(k)=b,\\k=2,3,\cdots,n,\end{gathered} \tag{3}$$

where, α is the related coefficient with a, a is development coefficient and b is the control coefficient.

Supposed

$$B=\begin{bmatrix}-x^{(1)}(2) & 1\\-x^{(1)}(3) & 1\\\cdots & \cdots\\-x^{(1)}(n) & 1\end{bmatrix},$$

$$Y=\begin{bmatrix}\alpha x^{(0)}(1)+(1-\alpha)x^{(0)}(2)\\\alpha x^{(0)}(2)+(1-\alpha)x^{(0)}(3)\\\cdots\\\alpha x^{(0)}(n-1)+(1-\alpha)x^{(0)}(n)\end{bmatrix}$$

and $a=\begin{bmatrix}a\\b\end{bmatrix}$. Equation (3) can be expressed as $Y=Ba$. The following equation can be obtained by using the least squares method:

$$\hat{a}=\left(B^TB\right)^{-1}B^TY \tag{4}$$

When the first component of $X^{(1)}$ is taken as initial condition of grey differential equation, the continuous solution of albino differential equation in the initial conditions is:

$$\hat{x}^{(1)}(t)=\left[x^{(1)}(1)-\frac{b}{a}\right]e^{-a(t-t_1)}+\frac{b}{a} \tag{5}$$

Its discrete solution is:

$$\hat{x}^{(1)}(k)=\left[x^{(1)}(1)-\frac{b}{a}\right]e^{-a(k-1)}+\frac{b}{a} \tag{6}$$

The model value $\hat{x}^{(0)}(k)(k=1,2,\cdots,n)$ of the originnal sequence can be obtained by regressive generation.

$$\hat{x}^{(0)}(k)=\begin{cases}\left(1-e^{-a}\right)\left[x^{(1)}(1)-\frac{b}{a}\right]e^{-a(k-1)} & (k=2,\cdots,n)\\x^{(1)}(1) & (k=1)\end{cases} \tag{7}$$

Then the model value $\hat{x}^{(00)}(k)(k=1,2,\cdots,n)$ of the original sequence by using Definition 1 is obtained.

Presumed that $X^{(1)}$ is in the exponential curve $x^{(1)}(t)=ce^{-at}+f$, the accurate conditions during modeling is that two equations between Equation (3) and Equation (7) are satisfied at the same time. Equation (3) substituted by Equation (7) is simplificated, and then a relationship between α and a can be established as:

$$\alpha=\frac{e^a(a-1)+1}{\left(e^a-1\right)^2} \tag{8}$$

Since that Y is the function of α in Equation (4) and α is the function of a in Equation (8), as long as giving an initial value a_0 of a, α can be obtained in Equation (8). Substituting again into Equation (8) will obtain

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α and into Equation (4) calculate a. After iterating several times the exact value $\hat{\boldsymbol{a}}$ will be found. After defining the absolute error $q(k)=x^{(00)}(k)-\hat{x}^{(00)}(k)$, the relative error $e(k)=\frac{q(k)}{x^{(0)}(k)}\times 100\%$ and the mean relative error $\frac{\sum_{k=1}^{n}|e(k)|}{n}$, we wrote the Matlab program named as GRM for grey GRM(1, 1) model based on reciprocal accumulated generating, where as long as inputting the known data, the corresponding error and accuracy of the model can be obtained.

3. Example

There are the fatigue experimental data (Mpa) in [5]: $\boldsymbol{X}^{(00)}$ = [560, 540, 523, 500, 475], corresponds temperature (˚C): $\boldsymbol{T}$ = [100, 150, 200, 250, 300], the number corresponding to the temperature: k = 1, 2, 3, 4, 5. The model was obtained by using this method proposed in this paper:

$$\hat{x}^{(1)}(k)=0.044858e^{0.040394(k-1)}-0.04307 .$$

The fitting value of the data is

$\hat{\boldsymbol{X}}^{(00)}$ = [560, 540.8374, 19.4263, 498.8629, 479.1135].

The relative error (%) is

$\boldsymbol{e}(k)$ = [0, −0.15508, 0.6833, 0.22743, −0.86599].

The mean of the relative error is 0.38636%.

This model has high precision.

The mean relative error in the non-homogeneous model based on traditional accumulated generating in reference [5] is 0.33666%. After the original data were pre-processed by using $t=\frac{T-50}{50}$ and $X^{(0)}=\frac{\sigma_{-1}-400}{50}$ in reference [6], the maximum relative error is 4.86% and the mean relative error is 3.19%. The model was established by using the function transformation method in reference [7] and the mean relative error is 0.6587%. Homogeneous exponent function fitting one-time accumulated generating sequence was used in reference [8] and it is 0.9765%. Thus, the examples validate the adaptability and the scientific of the proposed model.

4. Conclusion

This paper applied the reciprocal accumulated generating and the approach optimizing grey derivative which is based on three points to deduce the calculation formulas for model parameters in the condition that the first component of $\boldsymbol{X}^{(1)}$ was taken as initial condition of grey differential equation, established homogeneous GRM(1, 1) model based on reciprocal accumulated generating. This model with high precision has better theoretical and practical significance. Example validates the practicability and reliability of the proposed model.

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