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Free Convection Flow between Vertical Plates Moving in Opposite Direction and Partially Filled with Porous Medium

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Abstract

The laminar fully developed free-convection flow in a channel bounded by two vertical plates, partially filled with porous matrix and partially with a clear fluid, has been discussed when both the plates are moving in opposite direction. The momentum transfer in porous medium has been described by the Brinkman-extended Darcy model. The affect of Darcy number on flow velocity has been discussed in fluid region, interface region and porous medium with the help of graphs. Analytic method has been adopted to obtain the expressions of velocity and temperature. The skin-friction component has also been determined and presented with the help of tables.

Keywords: Free Convection, Porous Medium, Skin-Friction

1. Introduction

Flow in a region, part of which is occupied by a clear fluid and part by a fluid-saturated porous medium, has recently attracted considerable attention due to its common occurrence in both geophysical and industrial environments, including engineering applications such as thermal-energy storage system, a solar collector with a porous absorber and porous journal-bearings. Flow mechanism at the fluid/porous interface, was first studied by Beavers & Joseph [1] and it was investigated that the velocity gradient on the fluid side of the interface is proportional to the slip velocity at the interface. Taylor [2] & Richardson [3] continued the investigation in which they modeled fluid flow by Darcy number. Further, Vafai & Kim [4] modeled the flow in the porous region utilizing the so-called Brinkman-Forchheimer-extended Darcy equation (Vafai & Kim [5] and Kuznetsov [6]). Alazmi & Vafai [7], Valencia-Lopez and Ochoa-Tapia [8] also investigated fluid flow and heat transfer interfacial conditions of fluid and porous layers.

Furthermore, convection in porous media is applied in utilization of geothermal energy, the control of pollutant spread in groundwater, the design of nuclear reactors, compact heat exchangers, solar power collectors, heat transfer associated with the deep storage of nuclear waste and high performance insulators for buildings. Considerable progress in this area was made by Nield & Bejan [9] and Kaviany [10]. Vafai & Tien [11] also analyzed the effects of a solid boundary and the inertial forces on flow and heat transfer in porous media.

The coupled fluid flow and heat transfer problem in a fully developed composite region of two parallel plates filled with Brinkman-Darcy porous medium was analytically investigated by Kaviany [12]. Rudraiah & Nagraj [13] studied the fully developed free-convection flow of a viscous fluid through a porous medium bounded by two heated vertical plates. Beckerman [14] studied natural convection in vertical enclosures containing simultaneously fluid and porous layers. Recently, Khalili *et al.* [15] studied the instability of superimposed fluid and porous layers with vertical through-flow governed by Darcy-Forchheimer equation. Free convection between vertical walls partially filled with porous medium was investigated by Paul *et al.* [16]. Singh *et al.* [17] analyzed heat and mass transfer phenomena due to natural convection in a composite cavity containing a fluid layer overlying a porous layer saturated with the same fluid, in which the flow in the porous region was modeled using Brinkman-Forchheimer-extended Darcy model that includes both the effect of macroscopic shear (Brinkman effect) and flow inertia (Forchheimer effect).

In most of the flow studies in the channel, the plates are stable. However, to the best of the author's knowledge, the effect of the same idea has not been studied yet, when both the plates are moving. In this paper we extend the problem of [16], when both the plates are moving in the opposite direction.

2. Governing Equations

A channel of two vertical plates partially filled with porous matrix and partially with a clear fluid having interface is shown in **Figure 1**. The laminar fully developed free-convection flow bounded in the channel is discussed, when both the plates are moving in opposite direction and one plate is heated and other is cooled.

The $\bar{x}$-axis is taken along one of the wall and $\bar{y}$-axis normal to it. The plate in the fluid region and the plate in porous region are moving in opposite direction, where $\bar{U}_f$ and $\bar{U}_p$ are the velocities in the direction of $\bar{x}$-axis. The temperature is also considered on the walls $\bar{y}=0$ and $\bar{y}=H$ as $\bar{T}_f=\bar{T}_c+A\left(\bar{T}_h-\bar{T}_c\right)$ and $\bar{T}_p=\bar{T}_c+B\left(\bar{T}_h-\bar{T}_c\right)$ respectively.

Under usual Boussinesq approximation, the flow in fluid and porous regions is governed by the following equations:

Free Fluid Region:

$$\frac{\mathrm{d}^2\bar{U}_f}{\mathrm{d}\bar{y}^2}+\frac{g\beta}{\upsilon}\left(\bar{T}_f-\bar{T}_c\right)=0 \tag{1}$$

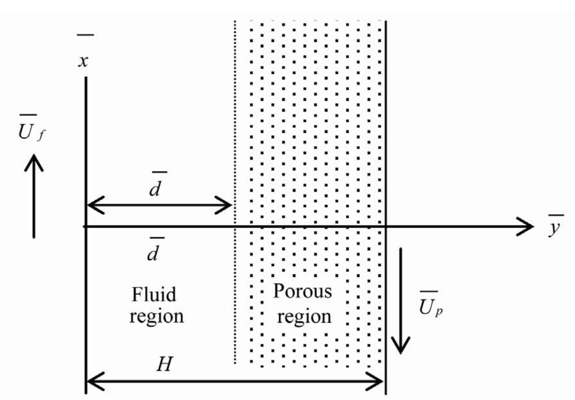

Figure 1. Physical configuration of the system.

$$\left(\frac{\mathrm{d}\bar{U}_f}{\mathrm{d}\bar{y}}\right)^2+\frac{\kappa}{\upsilon}\frac{\mathrm{d}^2\left(\bar{T}_f-\bar{T}_c\right)}{\mathrm{d}\bar{y}^2}=0 \tag{2}$$

Porous Region:

$$\frac{\mathrm{d}^2\bar{U}_p}{\mathrm{d}\bar{y}^2}-\frac{\bar{U}_p}{\bar{k}}+\frac{g\beta}{\upsilon}\left(\bar{T}_p-\bar{T}_c\right)=0 \tag{3}$$

$$\left(\frac{\mathrm{d}\bar{U}_p}{\mathrm{d}\bar{y}}\right)^2+\frac{\bar{U}_p^2}{\bar{k}}+\frac{\kappa}{\upsilon}\frac{\mathrm{d}^2(\bar{T}_p-\bar{T}_c)}{\mathrm{d}\bar{y}^2}=0 \tag{4}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l}
\bar{y}=0:\bar{U}_f=\dfrac{g\beta H^2\left(\bar{T}_h-\bar{T}_c\right)u_0}{\upsilon};\bar{T}_f=\bar{T}_c+A\left(\bar{T}_h-\bar{T}_c\right)\\
\bar{y}=H:\bar{U}_p=-\dfrac{g\beta H^2\left(\bar{T}_h-\bar{T}_c\right)u_0}{\upsilon};\bar{T}_p=\bar{T}_c+B\left(\bar{T}_h-\bar{T}_c\right)\\
\bar{y}=\bar{d}:\bar{U}_f=\bar{U}_p;\dfrac{\mathrm{d}\bar{U}_f}{\mathrm{d}\bar{y}}=\dfrac{\mathrm{d}\bar{U}_p}{\mathrm{d}\bar{y}};\bar{T}_f=\bar{T}_p;\dfrac{\mathrm{d}\bar{T}_f}{\mathrm{d}\bar{y}}=\dfrac{\mathrm{d}\bar{T}_p}{\mathrm{d}\bar{y}}
\end{array}\right\} \tag{5}$$

Introducing following non-dimensional quantities:

$$Da=\frac{\bar{k}}{H^2};\quad y=\frac{\bar{y}}{H};\quad d=\frac{\bar{d}}{H};\quad U_f=\frac{\upsilon\bar{U}_f}{g\beta H^2\left(\bar{T}_h-\bar{T}_c\right)};$$
$$U_p=\frac{\upsilon\bar{U}_p}{g\beta H^2\left(\bar{T}_h-\bar{T}_c\right)};\theta_f=\frac{\left(\bar{T}_f-\bar{T}_c\right)}{\left(\bar{T}_h-\bar{T}_c\right)};$$
$$\theta_p=\frac{\left(\bar{T}_p-\bar{T}_c\right)}{\left(\bar{T}_h-\bar{T}_c\right)};\quad N=\frac{g^2\beta^2H^4\left(\bar{T}_h-\bar{T}_c\right)}{\kappa\upsilon} \tag{6}$$

Equations (1) to (4) become

Fluid Region:

$$\frac{\mathrm{d}^2U_f}{\mathrm{d}y^2}+\theta_f=0 \tag{7}$$

$$\frac{\mathrm{d}^2\theta_f}{\mathrm{d}y^2}+N\left(\frac{\mathrm{d}U_f}{\mathrm{d}y}\right)^2=0 \tag{8}$$

Porous Region:

$$\frac{\mathrm{d}^2U_p}{\mathrm{d}y^2}-\frac{U_p}{Da}+\theta_p=0 \tag{9}$$

$$\frac{\mathrm{d}^2\theta_p}{\mathrm{d}y^2}+N\left(\frac{\mathrm{d}U_p}{\mathrm{d}y}\right)^2+\frac{N}{Da}U_p^2=0 \tag{10}$$

In Equation (9), The momentum transfer in porous domain is described based on Brinkman-extended Darcy model [18].

Where Da is the Darcy number, d the distance of interface from the plate $y=0$, g the acceleration due to gravity, H the distance between vertical plates, $\bar{k}$ the permeability of the porous matrix, κ the thermal conductivity, N the Buoyancy parameter, β the coefficient of thermal expansion, μ the dynamic viscosity, ν the kinematic viscosity, ρ the density, and θ is the temperature. The subscripts *f*, represent fluid layer, *p* porous layer, *h* hot plate and *c*, the cold plate.

The boundary and matching conditions (5) in dimensionless form are:

$$\begin{aligned} &y=0: U_f=u_0; \theta_f=A \\ &y=1: U_p=-u_0; \theta_f=B \\ &y=d: U_f=U_p; \frac{\mathrm{d}U_f}{\mathrm{d}y}=\frac{\mathrm{d}U_p}{\mathrm{d}y} \\ &y=d: \theta_f=\theta_p; \frac{\mathrm{d}\theta_f}{\mathrm{d}y}=\frac{\mathrm{d}\theta_p}{\mathrm{d}y} \end{aligned} \tag{11}$$

where, the matching conditions for velocity are due to continuity of velocity and shear stress at the interface. The continuity of temperature and heat flux at the interface has been considered as matching conditions for temperature.

3. Solution

It can be observed that problem is non-linear due to viscous and Darcy dissipation terms. This problem can be tackled by using a perturbation method as N is small in most of the practical problems. Accordingly we assume, for small N, the expansions:

$$\begin{aligned} &U_f=U_{0f}+NU_{1f}+O\left(N^2\right) \\ &U_p=U_{0p}+NU_{1p}+O\left(N^2\right) \\ &\theta_f=\theta_{0f}+N\theta_{1f}+O\left(N^2\right) \\ &\theta_p=\theta_{0p}+N\theta_{1p}+O\left(N^2\right) \end{aligned} \tag{12}$$

Substituting (12) in the Equations (7) to (10) gives

$$\frac{\mathrm{d}^2U_{0f}}{\mathrm{d}y^2}+\theta_{0f}=0 \tag{13}$$

$$\frac{\mathrm{d}^2U_{1f}}{\mathrm{d}y^2}+\theta_{1f}=0 \tag{14}$$

$$\frac{\mathrm{d}^2\theta_{0f}}{\mathrm{d}y^2}=0 \tag{15}$$

$$\frac{\mathrm{d}^2\theta_{1f}}{\mathrm{d}y^2}+\left(\frac{\mathrm{d}U_{0f}}{\mathrm{d}y}\right)^2=0 \tag{16}$$

$$\frac{\mathrm{d}^2U_{0p}}{\mathrm{d}y^2}-\frac{1}{Da}U_{0p}+\theta_{0p}=0 \tag{17}$$

$$\frac{\mathrm{d}^2U_{1p}}{\mathrm{d}y^2}-\frac{1}{Da}U_{1p}+\theta_{1p}=0 \tag{18}$$

$$\frac{\mathrm{d}^2\theta_{0p}}{\mathrm{d}y^2}=0 \tag{19}$$

$$\frac{\mathrm{d}^2\theta_{1p}}{\mathrm{d}y^2}+\left(\frac{\mathrm{d}U_{0p}}{\mathrm{d}y}\right)^2+\frac{1}{Da}U_{0p}^2=0 \tag{20}$$

The corresponding boundary conditions are

$$\begin{aligned} &y=0:\quad U_{0f}=u_0; U_{1f}=0; \theta_{0f}=A; \theta_{1f}=0; \\ &y=1:\quad U_{0p}=u_0; U_{1p}=0; \theta_{0p}=B; \theta_{1p}=0; \\ &y=d: \\ &\quad U_{0f}=U_{0p}; U_{1f}=U_{1p}; \frac{\mathrm{d}U_{0f}}{\mathrm{d}y}=\frac{\mathrm{d}U_{0p}}{\mathrm{d}y}; \frac{\mathrm{d}U_{1f}}{\mathrm{d}y}=\frac{\mathrm{d}U_{1p}}{\mathrm{d}y} \\ &\quad \theta_{0f}=\theta_{0p}; \theta_{1f}=\theta_{1p}; \frac{\mathrm{d}\theta_{0f}}{\mathrm{d}y}=\frac{\mathrm{d}\theta_{0p}}{\mathrm{d}y}; \frac{\mathrm{d}\theta_{1f}}{\mathrm{d}y}=\frac{\mathrm{d}\theta_{1p}}{\mathrm{d}y} \end{aligned} \tag{21}$$

Solving Equations (13) to (20) using boundary conditions (21) gives the following velocity and temperature components

$$U_{0f}=-\frac{Ay^2}{2}+\left(A-B\right)\frac{y^3}{6}+u_0+K_5y$$

$$U_{0p}=\frac{A+\left(B-A\right)y}{m^2}+K_4\mathrm{e}^{my}+K_3\mathrm{e}^{-my}$$

$$\theta_{0f}=\theta_{0p}=A+\left(B-A\right)y$$

$$\begin{aligned}\theta_{1f}=&-\frac{\left(A-B\right)^2y^6}{120}+\frac{A\left(A-B\right)y^5}{20}-\frac{K_5\left(A-B\right)y^4}{12} \\ &-\frac{A^2y^4}{12}+\frac{AK_5y^3}{3}-\frac{K_5^2y^2}{2}+K_{13}y\end{aligned}$$

$$\begin{aligned}\theta_{1p}=&-\frac{K_4^2\mathrm{e}^{2my}}{2}-\frac{K_3^2\mathrm{e}^{-2my}}{2}-\frac{A^2y^2}{2m^2}-\frac{\left(B-A\right)^2y^2}{2m^4} \\ &-\frac{\left(B-A\right)^2y^4}{12m^2}-\frac{A\left(B-A\right)y^3}{3m^2}-\frac{2K_4\mathrm{e}^{my}}{m^2}\left(A-\frac{B-A}{m}\right) \\ &-\frac{2K_3\mathrm{e}^{-my}}{m^2}\left(A+\frac{B-A}{m}\right)-\frac{2K_4\left(B-A\right)y\mathrm{e}^{my}}{m^2} \\ &-\frac{2K_3\left(B-A\right)y\mathrm{e}^{-my}}{m^2}+K_{12}y+K_{11}\end{aligned}$$

$$U_{1f} = \frac{(A-B)^2 y^8}{6720} - \frac{A(A-B)y^7}{840} + \frac{K_5(A-B)y^6}{360} + \frac{A^2y^6}{360} - \frac{AK_5y^5}{60} + \frac{K_5^2y^4}{24} - \frac{K_{13}y^3}{6} + K_{22}y$$

$$U_{1p} = K_{21}\mathrm{e}^{my} + K_{20}\mathrm{e}^{-my} + \frac{K_4^2\mathrm{e}^{2my}}{6m^2} + \frac{K_3^2\mathrm{e}^{-2my}}{6m^2} - \frac{1}{2m^6}(m^2y^2+2)\left(A^2 + \frac{(A-B)^2}{m^2}\right) - \frac{(B-A)^2}{12m^8} \cdot (m^4y^4 + 12m^2y^2 + 24) + \frac{K_4}{2m^4}\left(A - \frac{(B-A)}{m}\right) \cdot \mathrm{e}^{my}(2my-1) - \frac{A(B-A)y}{3m^6}(m^2y^2+6) - \frac{K_3}{2m^4}\left(A + \frac{(B-A)}{m}\right)\mathrm{e}^{-my}(2my+1) + \frac{K_4(B-A)e^{my}}{4m^5} \cdot (2m^2y^2 - 2my + 1) - \frac{K_3(B-A)\mathrm{e}^{-my}}{4m^5} \cdot (2m^2y^2 + 2my + 1) + \frac{K_{12}y + K_{11}}{m^2}$$

where

$$m = (Da)^{-1/2};$$

$$K_1 = \frac{Ad^2}{2} - \frac{(A-B)d^3}{3} + u_0;$$

$$K_2 = (1+md)\mathrm{e}^{m(1-d)} - (1-md)\mathrm{e}^{-m(1-d)};$$

$$K_3 = \frac{1}{K_2}\left[\mathrm{e}^m\left(K_1 - \frac{A}{m^2}\right) + (1-md)\mathrm{e}^{md}\left(\frac{B}{m^2} + u_0\right)\right];$$

$$K_4 = \left(-u_0 - \frac{B}{m^2}\right)\mathrm{e}^{-m} - K_3\mathrm{e}^{-2m};$$

$$K_5 = \frac{1}{d}\left[K_4\mathrm{e}^{md} + K_3\mathrm{e}^{-md} + \frac{A+(B-A)d}{m^2} + \frac{Ad^2}{2} - \frac{(A-B)d^3}{6} - u_0\right];$$

$$K_6 = -\frac{(A-B)^2 d^5}{20} + \frac{A(A-B)d^4}{4} - \frac{A^2d^3}{3} - \frac{K_5(A-B)d^3}{3} + AK_5d^2 - K_5^2 d;$$

$$K_7 = -\frac{(A-B)^2 d^6}{120} + \frac{A(A-B)d^5}{20} - \frac{A^2d^4}{12} - \frac{K_5(A-B)d^4}{12} + \frac{AK_5d^3}{3} - \frac{K_5^2d^2}{2};$$

$$K_8 = -\frac{K_4^2\mathrm{e}^{2m}}{2} - \frac{K_3^2\mathrm{e}^{-2m}}{2} - \frac{A^2}{2m^2} - \frac{(B-A)^2}{2m^4} - \frac{(B-A)^2}{12m^2} - \frac{A(B-A)}{3m^2} - \frac{2K_4\mathrm{e}^m}{m^2}\left(A - \frac{B-A}{m}\right) - \frac{2K_3\mathrm{e}^{-m}}{m^2}\left(A + \frac{B-A}{m}\right) - \frac{2K_4(B-A)e^m}{m^2} - \frac{2K_3(B-A)\mathrm{e}^{-m}}{m^2};$$

$$K_9 = -\frac{K_4^2\mathrm{e}^{2md}}{2} - \frac{K_3^2\mathrm{e}^{-2md}}{2} - \frac{A^2d^2}{2m^2} - \frac{(B-A)^2 d^2}{2m^4} - \frac{(B-A)^2 d^4}{12m^2} - \frac{A(B-A)d^3}{3m^2} - \frac{2K_4\mathrm{e}^{md}}{m^2}\left(A - \frac{B-A}{m}\right) - \frac{2K_3\mathrm{e}^{-md}}{m^2}\left(A + \frac{B-A}{m}\right) - \frac{2K_4(B-A)de^{md}}{m^2} - \frac{2K_3(B-A)d\mathrm{e}^{-md}}{m^2};$$

$$K_{10} = -mK_4^2\mathrm{e}^{2md} + mK_3^2\mathrm{e}^{-2md} - \frac{A^2d}{m^2} - \frac{(B-A)^2 d}{m^4} - \frac{(B-A)^2 d^3}{3m^2} - \frac{A(B-A)d^2}{m^2} - \frac{2AK_4\mathrm{e}^{md}}{m} + \frac{2AK_3\mathrm{e}^{-md}}{m} - \frac{2K_4(B-A)d\mathrm{e}^{md}}{m} + \frac{2K_3(B-A)d\mathrm{e}^{-md}}{m};$$

$$K_{11} = (K_{10} - K_6)d - K_9 + K_7;$$

$$K_{12} = -K_8 - K_{11};$$

$$K_{13} = K_{10} - K_6 + K_{12};$$

$$K_{14} = \frac{K_4^2\mathrm{e}^{2m}}{6m^2} + \frac{K_3^2\mathrm{e}^{-2m}}{6m^2} - \frac{1}{2m^6}(m^2+2)\left(A^2 + \frac{(A-B)^2}{m^2}\right) - \frac{(A-B)^2}{12m^8}(m^4 + 12m^2 + 24) + \frac{K_4}{2m^4}\left(A - \frac{(B-A)}{m}\right) \cdot \mathrm{e}^m(2m-1) - \frac{A(B-A)}{3m^6}(m^2+6) - \frac{K_3}{2m^4} \cdot \left(A + \frac{(B-A)}{m}\right)\mathrm{e}^{-my}(2m+1) + \frac{K_4(B-A)}{4m^5} \cdot (2m^2 - 2m + 1) - \frac{K_3(B-A)}{4m^5}(2m^2 + 2m + 1) + \frac{K_{12} + K_{11}}{m^2};$$

$$K_{15} = \frac{(A-B)^2 d^8}{6720} - \frac{A(A-B)d^7}{840} + \frac{K_5(A-B)d^6}{360} + \frac{A^2 d^6}{360} - \frac{AK_5 d^5}{60} + \frac{K_5^2 d^4}{24} - \frac{K_{13} d^3}{6};$$

$$K_{16} = \frac{K_4^2 e^{2md}}{6m^2} + \frac{K_3^2 e^{-2md}}{6m^2} - \frac{1}{2m^6}\left(m^2 d^2 + 2\right) \cdot \left(A^2 + \frac{(A-B)^2}{m^2}\right) - \frac{(B-A)^2}{12m^8}\left(m^4 d^4 + 12m^2 d^2 + 24\right) + \frac{K_4}{2m^4}\left(A - \frac{(B-A)}{m}\right)e^{md}(2md-1) - \frac{A(B-A)d}{3m^6} \cdot \left(m^2 d^2 + 6\right) - \frac{K_3}{2m^4}\left(A + \frac{(B-A)}{m}\right)e^{-md}(2md+1) + \frac{K_4(B-A)e^{md}}{4m^5}\left(2m^2 d^2 - 2md + 1\right) - \frac{K_3(B-A)e^{-md}}{4m^5} \cdot \left(2m^2 d^2 + 2md + 1\right) + \frac{K_{12} d + K_{11}}{m^2}$$

$$K_{17} = \frac{(A-B)^2 d^7}{840} - \frac{A(A-B)d^6}{120} + \frac{K_5(A-B)d^5}{60} + \frac{A^2 d^5}{60} - \frac{AK_5 d^4}{12} + \frac{K_5^2 d^3}{6} - \frac{K_{13} d^2}{2};$$

$$K_{18} = \frac{K_4^2 e^{2md}}{3m} - \frac{K_3^2 e^{-2md}}{3m} - \frac{d}{m^4}\left(A^2 + \frac{(A-B)^2}{m^2}\right) - \frac{(B-A)^2}{3m^6} d\left(m^2 d^2 + 6\right) - \frac{A(B-A)}{m^6}\left(m^2 d^2 + 2\right) + \frac{K_4}{2m^3}\left(A - \frac{(B-A)}{m}\right)e^{md}(2md+1) - \frac{K_3}{2m^3} \cdot \left(A + \frac{(B-A)}{m}\right)e^{-md}(1-2md) + \frac{K_4(B-A)}{2m^4} \cdot e^{md}(2md-1) + \frac{K_4(B-A)e^{md}}{4m^4}\left(2m^2 d^2 - 2md + 1\right) + \frac{K_3(B-A)e^{-md}}{4m^4}\left(2m^2 d^2 + 2md + 1\right) - \frac{K_3(B-A)e^{-md}}{2m^4}(2md+1) + \frac{K_{12}}{m^2};$$

$$K_{19} = (1-md)K_{14}e^{md} - \left[\left(K_{16} - K_{15}\right) - d\left(K_{18} - K_{17}\right)\right]e^{m};$$

$$K_{20} = K_{19}/K_2;$$

$$K_{21} = -K_{14}e^{-m} - K_{20}e^{-2m};$$

$$K_{22} = mK_{21}e^{md} - mK_{20}e^{-md} + K_{18} - K_{17}.$$

Skin-Friction Components

At plate $y = 0$:

$$\tau_1 = K_5 + NK_{22}$$

At plate $y = 1$:

$$\tau_2 = -\left[\frac{(B-A)}{m^2} + mK_4 e^{m} - mK_3 e^{-m}\right] - N\left[\frac{K_4^2 e^{2m}}{3m} - \frac{K_3^2 e^{-2m}}{3m} - \frac{1}{m^4}\left(A^2 + \frac{(A-B)^2}{m^2}\right) - \frac{(B-A)^2}{3m^6}\left(m^2+6\right) - \frac{A(B-A)}{m^6}\left(m^2+2\right) + \frac{K_4}{2m^3} \cdot \left(A - \frac{(B-A)}{m}\right)e^{m}(2m+1) - \frac{K_3}{2m^3}\left(A + \frac{(B-A)}{m}\right) \cdot e^{-m}(1-2m) + \frac{K_4(B-A)}{2m^4}e^{m}(2m-1) + \frac{K_4(B-A)e^{m}}{4m^4} \cdot \left(2m^2 - 2m + 1\right) + \frac{K_3(B-A)e^{-m}}{4m^4}\left(2m^2 + 2m + 1\right) - \frac{K_3(B-A)e^{-m}}{2m^4}(2m+1) + \frac{K_{12}}{m^2} + mK_{21}e^{m} - mK_{20}e^{-m}\right]$$

4. Discussions and Conclusion

Natural convection in a vertical channel partially filled with porous medium has been discussed in the preceding sections when it is assumed that plates are moving in opposite direction. The governing equations having non-linear nature have been solved by analytical method. Different types of interfacial conditions between a porous medium and fluid layer are analyzed in detail. Three primary regions were found likewise, fluid region (near wall $y = 0$), interface region and porous region (near the wall $y = 1$). The affect of Darcy number on the flow has been discussed.

When viscous and Darcy dissipation are zero, velocity profiles between vertical plates have been illustrated in **Figure 2**, for the cases when $A = 1$, $B = 1$ (plate $y = 0$ is heated and plate $y = 1$ is cooled) and $A = 0$, $B = 1$ (plate $y = 0$ is cooled and $y = 1$ is heated). It is observed that in first region, the fluid velocity increases with increasing Darcy number (Da), whereas the reversal phenomenon occurs in third region. Near the interface region in porous medium, the velocity is constant for Da = 10^{-3} and effect of Brinkman term is almost negligible, and flow is only characterized by classical Darcy

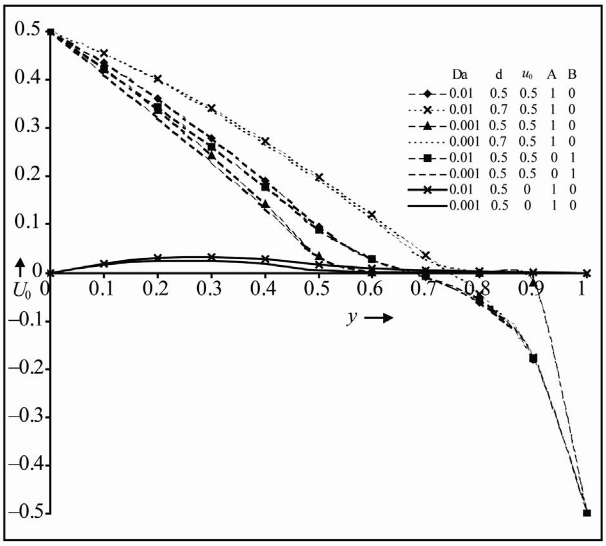

Figure 2. Graph of U_0 against *y*.

law [19], but for Da = 10^{-2}, the velocity does not show the constant nature. It is also observed that the fluid velocity near the heated wall $y = 0$ (*i.e.*, $A = 1$, $B = 0$) is more than the cooled wall $y = 0$ (*i.e.*, $A = 0$, $B = 1$), *i.e.*, heat in the porous medium exerts the restraining force on the fluid.

Figure 3 and **4** show the effect of viscous and Darcy dissipation terms on the velocity and temperature respectively. The governing equations become non-linear because of existence of these terms whereas very small influence of these terms on both temperature and velocity fields, is observed.

The numerical values of skin-friction on both the plates are also obtained and shown in **Tables 1** and **2** for $u_0 = 0$ and $u_0 > 0$ respectively. τ_1 is the skin-friction when $A = 1$, $B = 0$ and τ_2 is the skin friction when $A = 0$, $B = 1$. Similar effects of Darcy number on skin-friction are observed as mentioned in [16] for $u_0 = 0$, as

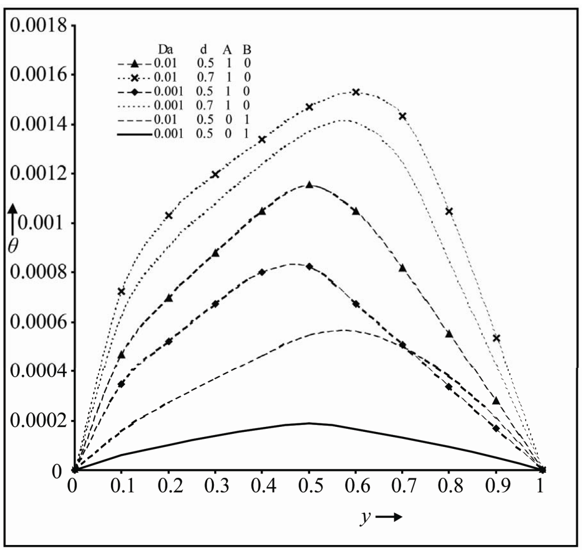

Figure 3. Graph of θ against *y*.

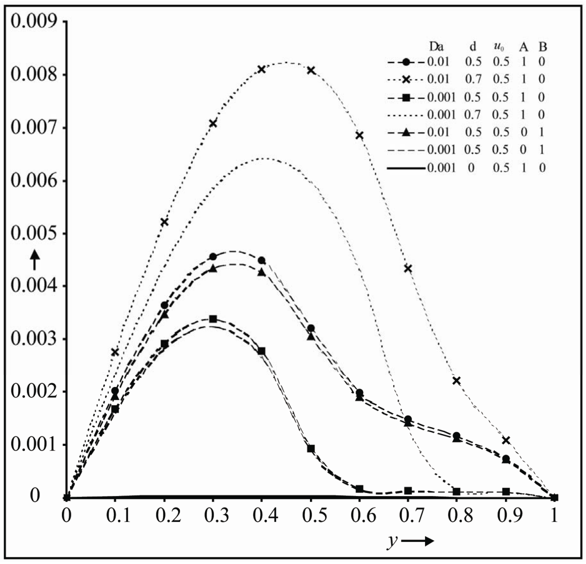

Figure 4. Graph of U_1 against *y*.

Table 1. Values of skin-friction for U_0 = 0.

N	Da	*d*	*A* = 1.0, *B* = 0.0		*A* = 0.0, *B* = 1.0	
			1		1	
0	0.1	0.3	0.2601317	0.0772881	0.0947285	0.2196207
		0.5	0.2960825	0.0916772	0.122357	0.2310641
		0.7	0.3221576	0.1221194	0.1504263	0.2647968
	0.01	0.3	0.18	0.0101185	0.0324544	0.090041
		0.5	0.2427759	0.011647	0.0653299	0.0909297
		0.7	0.2940734	0.0249912	0.1107526	0.1038333
0.1	0.1	0.3	0.2601631	0.077311	0.0947545	0.2196449
		0.5	0.2961261	0.0917077	0.1223924	0.231094
		0.7	0.3222115	0.1221609	0.1504735	0.2648389
	0.01	0.3	0.1800062	0.0101198	0.032457	0.0900422
		0.5	0.2427957	0.0116499	0.065338	0.0909317
		0.7	0.2941141	0.025	0.1107776	0.1038401

Table 2. Values of skin-friction for U_0 = 0.5.

N	Da	*d*	*A* = 1.0, *B* = 0.0		*A* = 0.0, *B* = 1.0	
			1		1	
0	0.1	0.3	–0.7386463	1.836859	–0.9040495	1.9791916
		0.5	–0.5860769	1.8926462	–0.7598024	2.0320331
		0.7	–0.5739595	1.8944114	–0.7456908	2.0370888
	0.01	0.3	–1.0722803	5.0123941	–1.2198258	5.0923165
		0.5	–0.6017996	5.0225739	–0.7792456	5.1018567
		0.7	–0.3934316	5.0685534	–0.5767524	5.1473955
0.1	0.1	0.3	–0.7358637	1.839478	–0.9014259	1.9816794
		0.5	–0.5827951	1.8955703	–0.7567184	2.0348034
		0.7	–0.5700895	1.8980549	–0.7420603	2.0405343
	0.01	0.3	–1.0708265	5.0132408	–1.2184213	5.0931465
		0.5	–0.5997182	5.023457	–0.7772541	5.1027177
		0.7	–0.3906199	5.0697341	–0.5740707	5.148541

clearly shown in **Table 1**. Increasing values of skin-friction are observed with increasing width of fluid layer in **Table 2**. It is also found in **Table 2**, that increasing Darcy number and dissipation results a very small increment in skin-friciton. The effect of temperature on skin-friction on both the plates is also studied and found that the skin friction on both the plates increases when those are heated.

5. References

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