Scientific
Research

A New Navigation Force Model for the Earth's Albedo and Its Effects on the Orbital Motion of an Artificial Satellite

Yehia A. Abdel-Aziz[1,2], Afaf M. Abdel-Hameed[1], Khalil I. Khalil[1]
[1]*National Research Institute of Astronomy and Geophysics* (*NRIAG*), *Helwan, Cairo, Egypt*
[2]*Mathematics Department Faculty of Sciences*, *Hail University, Hail, Saudi Arabia*
E-mail: *yehia@nriag.sci.eg*

Abstract

In this paper, we developed a new approach of an analytical model to calculate the radial and transversal components of the acceleration due to the effects of Earth's albedo. Its effects on the orbital motion of an artificial satellite are introduced. It is assumed that the satellite's horizon is illuminated and the sun lies on the equator. The magnitudes of those components are obtained and their effects on orbital evolution have been tested for different satellites elements. The perturbations in orbital elements due to Earth's albedo have been obtained using Lagrange Planetary equation in Gaussian form, in particular the case of LAGEOS satellite, have been found using this new analytical formalism.

1. Introduction

Satellite orbital dynamics is primarily influenced by the earth gravitational field, but there are several other factors which affect orbital motion and must be taken into account in order to prevent escape from the desired orbit and collision with another satellite or space debris in the neighboring orbit. The precise knowledge of the position and velocities of an artificial satellite is essential to the current technologies involving geodetic, communications satellites and GPS system as in Jaggi [1].

Such precisions require accurate models of perturbing gravitational and non-gravitational accelerations which affect the motion of an Earth satellite.

Gravitational perturbations are dominating the force spectrum for most earth orbits. They are caused by non-uniform mass distribution inside the Earth, ocean, atmosphere, Earth tides, and by third body attraction (Sun, Moon, planets). These perturbations can be modeled with a high level of confidence as all of them are conservative causing mainly periodic changes in the orbit energy). A complementary class of orbit perturbation is denoted as non-gravitational. This class comprises aerodynamic forces, direct and indirect radiation pressure effects, thermal radiation, and charged particle drag. Models of these non gravitational forces are affected by

uncertainties in the molecule-surface and photon-surface interaction processes, in molecule and photon flux models, and in the solar and geomagnetic activity levels and their effect on the thermosphere an ionosphere. Some of these perturbations cause a secular, time-proportional decrease of the orbital energy, and hence in the orbital altitude. For low-Earth orbits (LEO), these altitude decays must be compensated by periodic maintenance maneuver.

Earth's albedo effect is one of the most interesting non-gravitational forces, which have significant effects on the orbital motion. Albedo is the fraction of solar energy reflected diffusely from the planet back into space ([2]). It is the measure of the reflectivity of the planet's surface.

Therefore, the Earth albedo can be defined as the fraction of incident solar radiation returned to the space from the Earth's surface as in Marconi [3]

$$Albedo = \frac{radiation\ reflected\ back\ to\ the\ space}{incident\ radiation}$$

A detailed review of Earth's albedo as constant or variable with the changing of the latitude is performed by Green [4] and Sehnal [5]. Sehnal considered the Earth's albedo as the potential function of the Earth.

The Earth's albedo was considered as constant in the

analysis for the reflected radiation pressure in many literatures for example see [6-9]. Moreover [10-14] have taken into account a constant albedo.

Therefore, in this paper we are interested in an accurate analytical model of the acceleration due to the Earth's albedo which will be introduce in section 2. In section 3 we will study the effects of the albedo forces on the orbital motion of Earth's satellite.

2. The Perturbing Acceleration

As in [4], the albedo function can be written as:

$$\zeta = \sum_{n=0}^{\infty}\sum_{m=0}^{n} P_n^m(\sin\varphi)\left(\zeta_{nm}\cos m\lambda + \zeta'_{nm}\sin m\lambda\right) \qquad (1)$$

where $P_n^m(\sin\varphi)$ is the associated Legendre polynomial, φ is the latitude of an arbitrary element of area on the Earth's surface, λ is the geocentric longitude of that element, and ζ_{nm}, ζ'_{nm} are constants to be determined. Using the measures data of the satellites yields to no reliable dependence on the Earth's albedo on the longitude λ, so that the Earth's albedo be written as:

$$\zeta = \sum_{m=0}^{\infty} \zeta_m P_0^m(\sin\varphi) \qquad (2)$$

where ζ_m are constants, to be determined using satellites' observation.

A polynomial fitting of second degree in $\cos\varphi$ is used, $\zeta = \zeta_0 - \zeta_2\cos^2\varphi$, where ζ being the Earth's albedo constants, ζ_0 and ζ_2 to be determined. The data together with the polynomial approximation were provided by [4]; these were done by taking the measurements of the Earth's albedo by Tiros 7 satellite. The results are given for the four seasons of the year.

In the present work we shall use

$$\zeta = 0.62997 - 0.40893\cos^2\varphi \qquad (3)$$

To illustrate how albedo affects the satellite orbits, see **Figure 1** at which the Earth's center at O, the *OX* axis is directed towards the sun. We denote the *OX* axis by the vector $\boldsymbol{R}_\Theta$. Moreover, the satellite will be at a point S at any moment, $\boldsymbol{r}$ is the position vector of the satellite from O, dE represents an arbitrary small element of area on the Earth's surface, and its position vector is $\boldsymbol{R}$. The angle A_1OA_2 is $-\varphi$ and the angle A_2OA_3 is $+\varphi$, ψ is the angle between the sun and $\boldsymbol{r}$, and *OZ* is perpendicular to the *OXY* plane. In fact, we have three cases:

1) The satellite's horizon lies completely in the illuminated hemisphere, the required condition for the albedo effects on the satellite is $\theta + \varphi \le 90°$.

2) The satellite's horizon lies partially in the illuminated hemisphere and partially in the darkened hemi
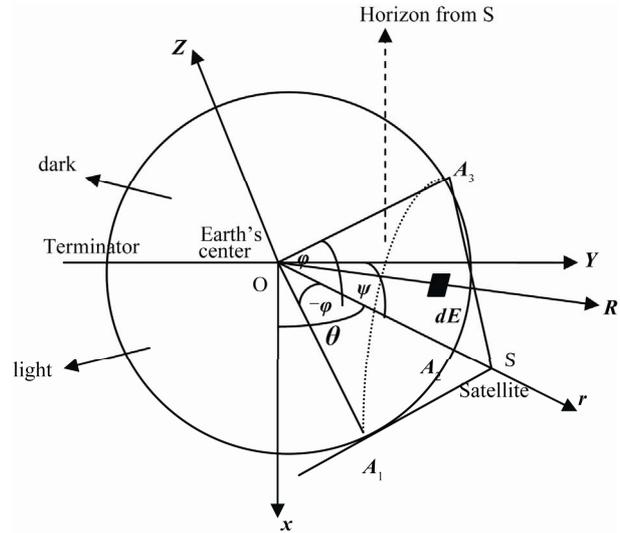
Figure 1. The coordinate system with the Earth's center O, the satellite S, and the sun lies on the *OXY* plane.

sphere and the condition for the albedo effects on the satellite is $\theta - \varphi \le 90° \le \theta + \varphi$.

3) The satellite's horizon lies completely in the darkened hemisphere and in this case $\theta - \varphi \ge 90°$

In the present work, the accuracy of computing the albedo acceleration in the case of constant albedo is increased by using powers of $1/r$ till $(1/r)^5$, while the previous works used powers of $1/r$ till $(1/r)^4$ only.

Therefore, the equations of the radial and transverse components of the reflected radiation pressure are given by

$$a_r^* = N\iint_{G\,D} \zeta\cos^2 D\cos(G+\theta)\left(\cos G\cos D - \frac{1}{r}\right)$$

$$\cdot\left(\frac{r}{\rho}\right)^4\left(1 - \frac{1}{r}\cos G\cos D\right)\mathrm{d}G\mathrm{d}D$$

$$a_T^* = \frac{-N}{r}\iint_{G\,D} \zeta\cos^3 D\cos(G+\theta)\left(\cos G\cos D - \frac{1}{r}\right)\mathrm{d}G\mathrm{d}D$$

where $N = kA/\pi m r^2$, k is the solar constant, and (A/m) is the area to mass ratio of the satellite, and G changes from $-\varphi$ to $+\varphi$ and D changes from $-\omega$ to $+\omega$, and $\omega = \cos^{-1}(\cos\varphi/\cos G)$ (see **Figures 2** and **3**). Also we have;

$$\zeta = \zeta_0 - \zeta_0\cos^2\varphi,$$

where ζ is the variable albedo coefficient and ζ_0 is the constant albedo coefficient, and the normal component of the Earth's albedo is

$$a_N^* =$$

$$\frac{-N}{r}\iint_{G\,D} \zeta\cos^2 D\cos(G+\theta)\left(\frac{r}{\rho}\right)\left(\cos G\cos D - \frac{1}{r}\right)\mathrm{d}G\mathrm{d}D$$

$$(4)$$
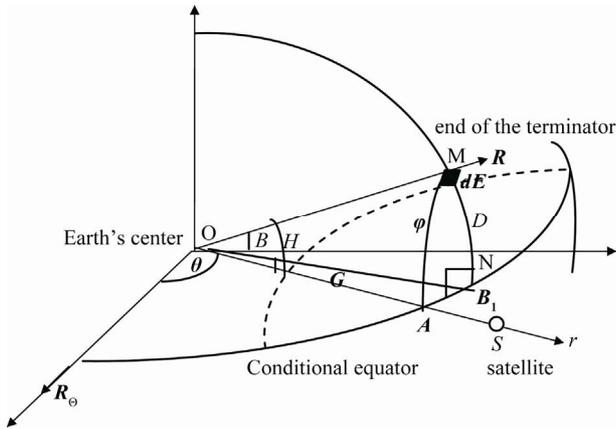
Figure 2. Shows the angles θ, H, G, and D.
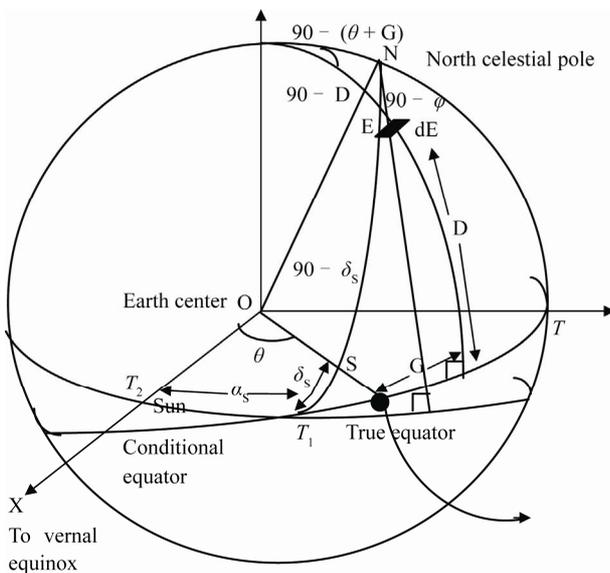
Figure 3. A coordinate system to express φ in terms of *G*, *D*, and θ with the δ_S and a_S for the satellite.

The above equations will be solved when the Earth's albedo is variable and depend on the latitude φ. Then, from the spherical triangle ZNE in **Figure 3**, it is convenient to use the formula:

$$\sin^2 \varphi = \left(1 - A_1^2\right)\sin^2 D + A_1\sqrt{1 - A_1^2}\,\sin 2D \sin\left(\theta + G\right)$$
$$+ A_1^2 \cos^2 D \sin\left(\theta + G\right)$$

where $A_1 = \sin\delta_S / \sin\theta$, put

$$\overline{B}_1 = 1 - A_1^2, \overline{B}_2 = A_1\sqrt{1 - A_1^2}, \overline{B}_3 = A_1^2$$

and

$B_1 = \zeta_2 \overline{B}_1, B_2 = \zeta_2 \overline{B}_2, B_3 = \zeta_2 \overline{B}_3$, also we have
$a_{\tilde{r}}^* = \tilde{a}_r$ (variable albedo) + a_r (constant albedo),
$a_T^* = \tilde{a}_T$ (variable albedo) + a_T (constant albedo),
$a_N^* = \tilde{a}_N$ (variable albedo) + a_N (constant albedo).

So

$$\tilde{a}_r = N\iint_{G\,D} B_1 \sin^2 D + B_2 \sin 2D \sin\left(G + \theta\right)$$

$$+ B_3 \cos^2 D \sin^2\left(G + \theta\right)\cos^2 D \cos\left(G + \theta\right)$$

$$\times \left(\cos G \cos D - \frac{1}{r}\right)$$

$$\times \left(\frac{r}{\rho}\right)^4 \left(1 - \frac{1}{r}\cos G \cos D \left(\frac{r}{\rho}\right)^4\right) dG dD$$

$$+ NB_2 \iint_{G\,D} \sin 2D \cos^2 D \sin(G + \theta)\cos\left(G + \theta\right)$$

$$\times \left(\cos G \cos D - \frac{1}{r}\right)\times\left(1 - \frac{1}{r}\cos G \cos D \left(\frac{r}{\rho}\right)^4\right) dG dD$$

$$+ \frac{\pi}{2r}\sin^2\theta + \frac{1}{r^2}\left(\frac{-22\pi}{105} + \frac{32\pi}{35}\sin^2\theta\right)$$

$$+ \frac{1}{r^3}\left(\frac{-5\pi}{24} + \frac{32}{45}\left(3\sin^2\theta - 1\right)\right)$$

$$+ \frac{1}{r^4}\left(\frac{-338\pi}{315} + \frac{1184\pi}{315}\sin^2\theta\right)$$

$$+ \frac{1}{r^3}\left(\frac{17\pi}{8} - \frac{176}{15}\sin^2\theta + \left(\frac{-283\pi}{240} + \frac{484}{225}\right)\right)$$

$$+ NB_3 \iint_{G\,D} \cos^4 D \cos^2 D \sin^2\left(G + \theta\right)\cos\left(G + \theta\right)$$

$$\times\left(\cos G \cos D - 1\right)$$

$$\times\left(1 - \frac{1}{r}\cos G \cos D\right)\left(\frac{r}{\rho}\right)^4 dG dD$$

$\tilde{a}_r$ may be written as:
$\tilde{a}_r = C_1 + C_2 + C_3$, where

$$C_1 = NB_1 \iint_{G\,D} \sin^2 D \cos^2\left(G + \theta\right)\left(\cos G \cos D - \frac{1}{r}\right)$$

$$\times\left(1 - \frac{1}{r}\cos G \cos D\right)\left(\frac{r}{\rho}\right)^4 dG dD,$$

$$C_2 = NB_2 \iint_{G\,D} \sin^2 D \cos^2 D \sin\left(G + \theta\right)\cos\left(G + \theta\right)$$

$$\times\left(\cos G \cos D - \frac{1}{r}\right)$$

$$\times\left(1 - \frac{1}{r}\cos G \cos D\right)\left(\frac{r}{\rho}\right)^4 dG dD,$$

To evaluate C_1, we have

$$\left(r/\rho\right)^4 = a_0 + a_1 \cos H + a_2 \cos^2 H + a_3 \cos^3 H$$
$$+ a_4 \cos^4 H + a_5 \cos^5 H + a_6 \cos^6 H$$

where $\cos H = \cos G \cos D$, and

AM

$$a_0 = -\frac{1}{r} + \frac{2}{r^3} - \frac{3}{r^5}, \quad a_1 = 1 - \frac{5}{r^2} + \frac{13}{r^4},$$

$$a_2 = \frac{3}{r} - \frac{18}{r^3} + \frac{57}{r^5}, \quad a_3 = \frac{8}{r^2} - \frac{56}{r^4},$$

$$a_4 = \frac{20}{r^3} - \frac{160}{r^5}, \quad a_5 = \frac{80}{r^4}; a_6 = \frac{112}{r^5},$$

$$C_1 = NB_1 \int_{-\varphi}^{+\varphi} \cos(G+\theta)\, dG \int_{D=-\omega}^{+\omega} \sin^2 D \cos^2 D$$

$$\left\{ a_0 + a_1 \cos G \cos D + a_2 \cos^2 G \cos^2 D \right.$$

$$+ a_3 \cos^3 G \cos^3 D + a_4 \cos^4 G \cos^4 D$$

$$\left. + a_5 \cos^5 G \cos^5 D + a_6 \cos^6 G \cos^6 D \right\}\, dD$$

where $\omega = \cos^{-1}\dfrac{1}{r\cos G}$.

Two more integrals are required, which are

$$\int \cos^9 D\, dD = \frac{\sin D}{315}\left(128 + 64\cos^2 D + 48\cos^4 D \right.$$

$$\left. + 40\cos^6 D + 35\cos^8 D\right),$$

and

$$\int \cos^{10} D\, dD = \frac{63 D}{256} + \frac{\sin D}{1280}\left(315\cos D + 210\cos^3 D\right.$$

$$\left. + 168\cos^5 D + 144\cos^7 D + 128\cos^8 D\right)$$

Finally arranging the terms w. r. to. $(1/r)$, this yields:

$$C_1 = NB_1 \cos\theta\left\{ \frac{2\pi}{15} + \frac{1}{r^2}\left(\frac{-22\pi}{105}\right) + \frac{1}{r^3}\left(\frac{-\pi}{24} + \frac{7}{45}\right)\right.$$

$$\left. + \frac{1}{r^4}\left(\frac{338\pi}{315}\right) + \frac{1}{r^5}\left(\frac{-43\pi}{240} + \frac{301}{450}\right)\right\}$$

Evaluating C_2,

$$C_2 = NB_2 \int_{G=-\varphi}^{+\varphi} \sin(\theta+G)\cos(\theta+G)\, dG \int_{D=-\omega}^{+\omega} \sin 2D$$

$$\cos^2 D\left(\cos G \cos D - \frac{1}{r}\right)$$

$$\times \left(1 - \frac{1}{r}\cos G \cos D\right)\left(\frac{r}{\rho}\right)^4 dD$$

Finally, we can see that $C_2 = 0$

Evaluating C_3

$$C_3 = NB_3 \int_{G=-\varphi}^{+\varphi} \sin^2(\theta+G)\cos(\theta+G)\, dG \int_{D=-\omega}^{+\omega} \cos^4 D$$

$$\times \left(\cos G \cos D - \frac{1}{r}\right)\left(1 - \frac{1}{r}\cos G \cos D\right)\left(\frac{r}{\rho}\right)^4 dD$$

this yields:

$$C_3 = NB_3 \cos\theta\left\{ \frac{2\pi}{15} + \frac{\pi}{2r}\sin^2\theta \right.$$

$$+ \frac{1}{r^2}\left(\frac{-22\pi}{105} + \frac{32\pi}{35}\sin^2\theta\right)$$

$$+ \frac{1}{r^3}\left(\frac{-5\pi}{24} + \frac{32}{45}\right)\left(3\sin^2\theta - 1\right)$$

$$+ \frac{1}{r^4}\left(\frac{-338\pi}{315} + \frac{1184\pi\sin^2\theta}{315}\right)$$

$$\left. + \frac{1}{r^5}\left(\frac{17\pi}{8} - \frac{176}{15}\right)\sin^2\theta + \left(\frac{-283\pi}{240} + \frac{848}{225}\right)\right\}$$

Then, the radial component $\tilde{a}_r$ can be written as:

$$\tilde{a}_r = C_1 + 0 + C_3$$

$$= NB_1 \cos\theta\left[\frac{2\pi}{15} + \frac{1}{r^2}\left(\frac{-22\pi}{105}\right) + \frac{1}{r^3}\left(\frac{-\pi}{24} + \frac{7}{45}\right)\right.$$

$$\left. + \frac{1}{r^4}\left(\frac{338\pi}{315}\right) + \frac{1}{r^5}\left(\frac{-43\pi}{240} + \frac{301}{450}\right)\right]$$

$$+ \left[NB_3 \cos\theta\frac{2\pi}{15} + \frac{\pi}{2r}\sin^2\theta \right.$$

$$+ \frac{1}{r^2}\left(\frac{-22\pi}{105} + \frac{32\pi}{35}\sin^2\theta\right)$$

$$+ \frac{1}{r^3}\left(\frac{-5\pi}{24} - \frac{32}{45}\right)\left(3\sin^2\theta - 1\right)$$

$$+ \frac{1}{r^4}\left(\frac{-338\pi}{315} + \frac{1184\pi\sin^2\theta}{315}\right)$$

$$\left. + \frac{1}{r^5}\left(\frac{17\pi}{8} + \frac{176}{15}\right)\sin^2\theta + \left(\frac{-283\pi}{240} + \frac{848}{225}\right)\right]$$

Finally $\tilde{a}_r$ can be taken the form:

$$\tilde{a}_r = NB_1 \cos\theta\left[\frac{2\pi}{15} + \frac{1}{r^2}\left(\frac{-22\pi}{105}\right) + \frac{1}{r^3}\left(\frac{-\pi}{24} + \frac{7}{45}\right)\right.$$

$$+ \frac{1}{r^4}\left(\frac{338\pi}{315}\right) + \frac{1}{r^5}\left(\frac{-43\pi}{240} + \frac{301}{450}\right)\bigg]$$

$$+ NB_3 \cos\theta\left[\frac{2\pi}{15} + \frac{\pi}{2r}\sin^2\theta + \frac{1}{r^2}\left(\frac{-22\pi}{105} + \frac{32\pi}{35}\sin^2\theta\right)\right.$$

$$+ \frac{1}{r^3}\left(\frac{-5\pi}{24} + \frac{32}{45}\right)\left(3\sin^2\theta - 1\right)$$

$$+ \frac{1}{r^4}\left(\frac{338\pi}{315} + \frac{1184\pi\sin^2\theta}{315}\right)$$

$$\left. + \frac{1}{r^5}\left(\frac{17\pi}{8} - \frac{176}{15}\right)\sin^2\theta + \left(\frac{-283\pi}{240} + \frac{848}{225}\right)\right]$$

and

$$\zeta_3 = \zeta_0 - \zeta_2, \quad \zeta_2 = 0.40893, \quad \zeta_0 = 0.62997.$$

AM

So, the variable Earth's albedo
$$\zeta = 0.62997 - 0.40893\cos^2\varphi .$$

Similarly, the final form of the parameter $\tilde{a}_T$ is given by

$$\tilde{a}_{\tilde{T}} = -NB_1\cos\theta\left[\frac{\pi^2}{32r} + \frac{1}{r^2}\left(\frac{88}{315}\right) + \frac{1}{r^3}\left(\frac{-3\pi^2}{256}\right)\right.$$
$$+ \frac{1}{r^4}\left(\frac{4013}{9450} - \frac{\pi}{24}\right) + \frac{1}{r^5}\left(\frac{-5\pi^2}{256}\right)\bigg]$$
$$-NB_3\cos\theta\left[\frac{5\pi^2}{128r} + \frac{1}{r^2}\left(\frac{416}{1575} + \frac{2656}{1575}\sin^2\theta\right)\right.$$
$$+ \frac{1}{r^3}\left(\frac{-15\pi^2}{512} + \frac{105\pi^2}{256}\sin^2\theta\right)$$
$$+ \frac{1}{r^4}\left(\frac{5\pi}{8} + \frac{208}{147}\right)\sin^2\theta + \left(\frac{-5\pi}{24} + \frac{19856}{33075}\right)$$
$$+ \frac{1}{r^5}\left(\frac{1095\pi^2}{2048} + \frac{175\pi^2}{1024}\sin^2\theta\right)\bigg]$$

3. Perturbation in Orbital Elements Due to Earth's Albedo

Using Lagrange Planetary equations in Gaussian form:

$$\dot{a} = 2\sqrt{\frac{a^3}{\mu(1-e^2)}}\left[e\sin f\tilde{a}_r + \frac{a(1-e^2)}{r}\tilde{a}_T\right],$$

$$\dot{e} = \sqrt{\frac{a(1-e^2)}{\mu}}\left[\sin f\tilde{a}_r + \left(e + \cos f\left(\frac{r}{a} + 1\right)\right)\tilde{a}_T\right],$$

$$i = \frac{r\cos(\omega+f)}{\sqrt{\mu a(1-e^2)}}\tilde{a}_N ,$$

$$\dot{\Omega} = \frac{r\sin(\omega+f)}{\sin i\sqrt{\mu a(1-e^2)}}\tilde{a}_N ,$$

$$\dot{\omega} = -\dot{\Omega}\cos i - \frac{1}{e}\sqrt{\frac{a(1-e^2)}{\mu}}$$
$$\times\left[\cos f\tilde{a}_r - \left(1 + \frac{r}{a(1-e^2)}\right)\sin f\tilde{a}_T\right],$$

$$\dot{M} = n - \frac{2r}{\sqrt{\mu a}}\tilde{a}_r - \sqrt{1-e^2}\left(\dot{\omega} + \dot{\Omega}\cos i\right),$$

And according to the components of the acceleration of the albedo force a_{albedo} we find out the perturbation in the orbital elements due to the effects of Earth's albedo as the following:

$$\langle\dot{a}\rangle_f = \frac{1}{2\pi}\int_0^{2\pi}\dot{a}\,df$$

$$\langle\dot{e}\rangle_f = \frac{1}{2\pi}\int_0^{2\pi}\dot{e}\,df$$

$$\langle\dot{\omega}\rangle_f = \frac{1}{2\pi}\int_0^{2\pi}\dot{\omega}\,df$$

4. Numerical Realization

Depending on latitude φ and for constant albedo as in [14], we shall consider $\alpha_s = 0$ from the right ascension, of the satellite and $a_N = 0$ for constant albedo. Also we study the case of $\delta_s = 0$, after that $a_N^* = 0$, so we have the declination of the satellite and putting $\bar{M} = k(A/m)$ = 5.23×10^{-5} cm^2/sec, and the Earth's radius $R = 1$.

Using these conditions and value of the parameters, we obtained the following results

1) The acceleration a_{albedo} is considered as two components only (radial and transverse), which $a_{albedo} = \sqrt{\tilde{a}_{\tilde{r}}^2 + \tilde{a}_{\tilde{T}}^2}$.

2) **Figure 4** illustrates the change in the acceleration a_{albedo} with different values of r (r change from 1.04 R to 7 R) and θ which is the angle between the Sun's position and the radius vector of the satellite (θ change from 0 to π), with $\bar{M} = k(A/m)$ = 5.23×10^{-5} cm^2/sec, where R is the equatorial radius of the Earth

3) **Figures 5-7** represent the variation of the acceleration a_{albedo} versus θ for the following satellites: a) GFO with semi major axis = 7162 km, b) LAGEOS1 with semi major axis = 12160 km, and c) ETALON1 with semi major axis = 255000 km, the figures shows the magnitude of the acceleration is increased in LOE and decreased in MEO, but in the case of LAGEOS1 satellite,
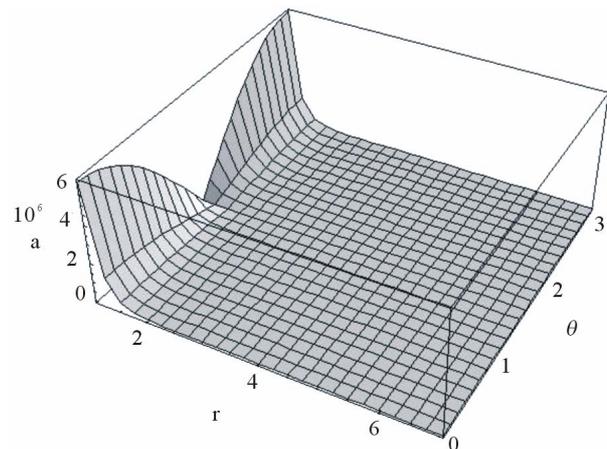
Figure 4. The variation of the acceleration a_{albedo} with respect to r and θ (rad).
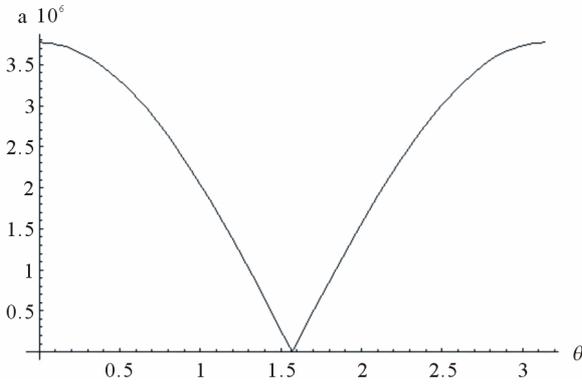
Figure 5. The variation of the acceleration a_{albedo} with respect to θ (rad) for GFO satellite.
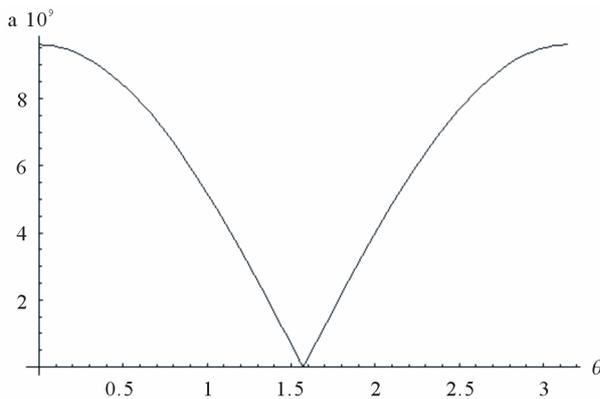
Figure 6. The variation of the acceleration a_{albedo} with respect to θ (rad) for Lageos1 satellite.
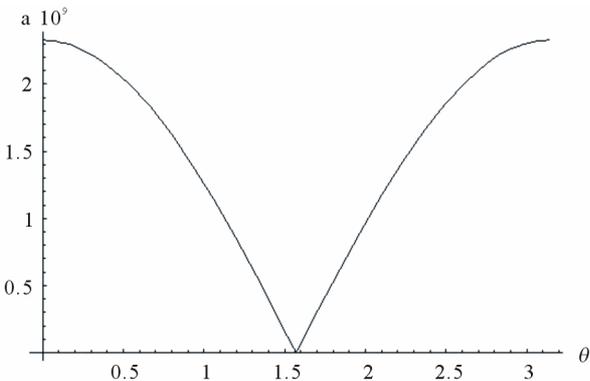
Figure 7. The variation of the acceleration a_{albedo} with respect to θ (rad) for Etalon1 satellite.

the acceleration have a significant variation with respect to the angle θ.

4) **Figures 8** and **13** represent the variation in the orbital elements (semi major axis, eccentricity and the argument of perigee) versus the angle θ for LAGEOS1 and STARLETTE (semi major axis = 7334.092) satellites. **Figures 8** and **9** show that the variation in the semi

major axises due to albedo force for LAGEOS and STARLETTE satellites is in order (10^{-9}) which is a significant effects. Also, **Figures 10** and **11** show the variation in the eccentricity due to albedo force for LAGEOS and STARLETTE satellites is in order (10^{-14}, 10^{-13}) which means the albedo force can affect the eccentricity. Moreover, **Figures 12** and **13** show the variation in the argument of perigee due to albedo force for LAGEOS and STARLETTE satellites is in order (10^{-10}, 10^{-11}) which means the albedo force can have a significant effects on the argument of perigee.
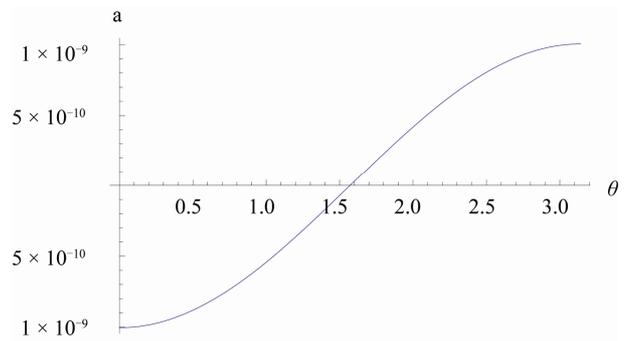
Figure 8. The Albedo perturbation in semi-major axis for Lageos1 satellite.
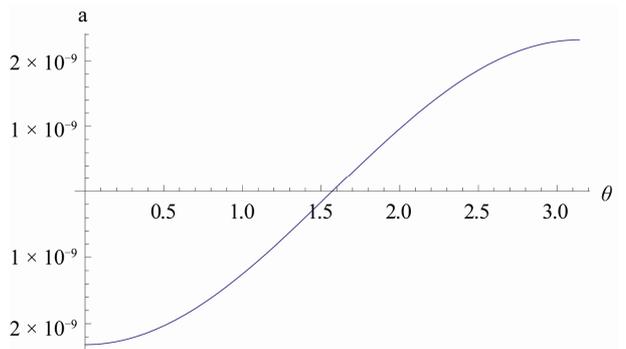
Figure 9. The Albedo perturbation in semi-major axis for Starlet satellite.
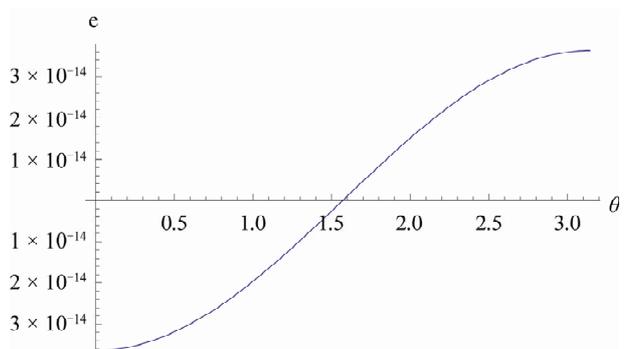
Figure 10. The Albedo perturbation in eccentricity for Lageos1 satellite.
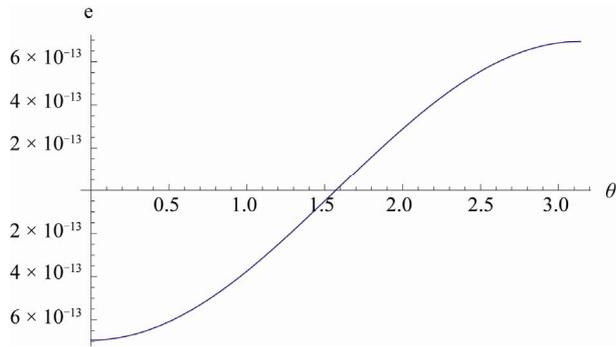
Figure 11. The Albedo perturbation in eccentricity for Starlet satellite.
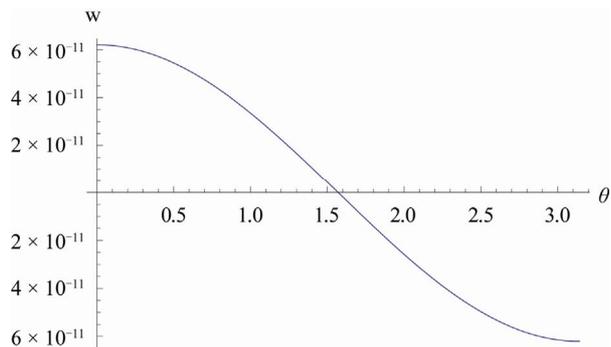
Figure 12. The Albedo perturbation in argument of perigee for Lageos1 satellite.
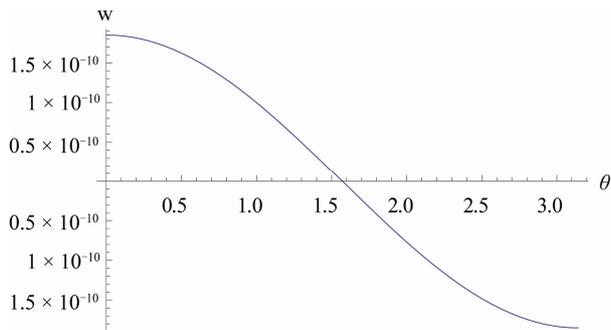
Figure 13. The Albedo perturbation in argument of perigee for Starlet satellite.

5. Conclusions

The net acceleration, a_{albedo}, arising from the effect of the diffusion of reflected radiation on the satellites, decreases with the distance between the Earth and the satellites. This acceleration is in the order of 10^{-9} for the satellites in low earth orbit which means that this force decreases when r increases. We could conclude that this force is in the same order of the air drag force, radiation pressure and the effect of Sun and Moon on the satellite. This is the reason for including the albedo force on the orbital elements of the satellite. However, in our

cases we could conclude that the best representation of the Earth's albedo function is to consider the Earth's albedo as a function of latitude φ, which inters the equation of motion of the satellite. We found out that the Albedo force have a significant effects on the orbital elements of the satellites.

6. References

[1] A. Jaggi, R. Dach, *et al.*, "Phase Center Modeling for LAGEOS GPS Receiver Antennas and Its Impact on Precise Orbit Determination," *Journal of Geodesy*, Vol. 83, 2009, pp. 1145-1162.

[2] M. Harris and R. Lyle, "Spacecraft Radiation Torque," NASA Space Vechile Design Criteria (Guidance and Control) NASA SP-8027, October 1969, pp. 1-37.

[3] E. M. Rocco, "Evaluation of the Terrestial Albedo Applied to Some Scientific Missions," *Space Science Review*, Vol. 151, No. 1-3, 2009, pp. 135-147. doi:10.1007/s11214-009-9622-6

[4] J. S. A. Green, "Reflections on the Earth's Albedo: A Collection of Scattered Thoughts," *Weather*, Vol. 57, No. 12, 2002, pp. 431-439. doi:10.1256/wea.223.01

[5] L. Sehnal, "The Earth Albedo Model in Spherical Harmonics," *Bulletin of the Astronomical Institution of Czechoslovakia*, Vol. 30, No. 4, 1979, pp. 199-204.

[6] R. Bowman, "Orbit Perturbation Analysis of West Ford Needles Clusters," American Institute of Aeronautics and Astronautics, Worldwide, 2000.

[7] I. Pater and J. Lissauer, "Planetary Science," Cambridge University Press, Cambridge, 2001.

[8] K. H. Nisancioglu, "Modeling the Impact of Atmospheric Moisture Transport on Global Ice Volume," Ph D. Thesis, Massachusetts Institute of Technology, Cambridge, 2004.

[9] L. Anselmo, P. Farinella, A. Milani and A. M. Nobili, "Effects of the Earth-Reflected Sunlight on the Orbit of the LAGEOS Satellite," *Astronomy and Astrophysics*, Vol. 117, No. 1, 1983, pp. 1-3.

[10] E. Levin, "Reflected Radiation Received by an Earth Satellite," *AES Journal*, Vol. 32, No. 9, 1962, pp. 1328-1331.

[11] S. Wyatt and P. Springer, "The Effect of Terrestrial Radiation Pressure on Satellite Orbits," In: M. Roy, Ed., *Dynamics of Satellites*, Paris, May 1962, pp. 186-196.

[12] L. Sehnal, "The Perturbations of the Orbital Elements Caused by the Pressure of the Radiation Reflected from the Earth," *Proceeding of Symposium*, Paris, 1965.

[13] D. E. Smith, "Earth Reflected Radiation Pressure," Dynamics of Satellites, Prague, 1969.

[14] R. M. L. Baker, "Radiation on a Satellite in the Presence of Diffuse and Partially Specular Reflecting Body," *Proceeding of Trajectories of Artificial Celestial bode as Determined from Observations*, Paris, April 1965, pp. 85-150.

AM