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Periodic Solution for Stochastic Predator-Prey Systems with Nonlinear Harvesting and Impulses

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Abstract

In this paper, astochastic predator-prey systems with nonlinear harvesting and impulsive effect are investigated. Firstly, we show the existence and uniqueness of the global positive solution of the system. Secondly, by constructing appropriate Lyapunov function and using comparison theorem with an impulsive differential equation, we study that a positive periodic solution exists. Thirdly, we prove that system is globally attractive. Finally, numerical simulations are presented to show the feasibility of the obtained results.

Keywords

Impulses Perturbations, Periodic Solution, Non-Linear Harvesting, Stochastic Predator-Prey Systems, Globally Attractive

1. Introduction

It is well known that the dynamic relationship between predator and prey has always been one of the main topics in ecology and mathematical ecology. In the past decades, many predator-prey models have been proposed and widely used to describe the food supply relationship between two species [1] [2]. At the same time, it has attracted great attention in many different fields, such as bio-economics. Recently, the interaction of predator-prey with harvesting has been studied. The effect of harvest on population is beneficial to sustainable development and renewable resource management, so many scholars take harvest into account in their models. The capture intensity depends largely on the capture strategy being implemented. Common harvest functions are: constant harvest, proportional harvest and nonlinear harvest. Gupta *et al.* proposed a predator-prey model with nonlinear predator in harvest [3] and discussed the dynamical properties of the following system:

$$\begin{cases} dx = x(t)(r_1 - b_1 x(t)) - ax(t)y(t)dt, \\ dy = y(t)\left(-r_2 + \eta a x(t)y(t) - \dfrac{hy(t)}{1+by(t)}\right)dt, \end{cases} \tag{1.1}$$

On the other hand, the growth of species in nature is often limited by environmental factors. Generally speaking, there are two main types of environmental noise: white noise and colored noise. Wenjie Zuo *et al.* [4] considered the white noise and studied the stationary distribution and periodic solution. However, reading the literature found that studies on the non-linear harvesting of predators and prey are very few literatures [5] [6] [7]. Therefore, the following model is proposed.

$$\begin{cases} dx(t) = x(t)\left[r_1 - a_{11}(t)x(t) - a_{12}(t)y(t) - \dfrac{H(t)}{1+b(t)x(t)}\right]dt \\ \qquad + \sigma_1(t)x(t)dB_1(t), \\ dy(t) = y(t)\left[-r_2 + a_{21}(t)x(t) - a_{22}(t)y(t) - \dfrac{h(t)}{1+b(t)y(t)}\right]dt \\ \qquad - \sigma_2(t)y(t)dB_2(t) - \dfrac{\sigma_3(t)h(t)y(t)}{1+b(t)y(t)}dB_3(t), \end{cases} \tag{1.2}$$

In real life, however, ecosystems are often disturbed by human development or by activities related to natural factors such as drought, floods, earthquakes, and planting. In order to describe this phenomenon more accurately, impulses perturbation is added into the model. To sum up, this paper mainly studies the effects of impulse effect and nonlinear harvesting on predator and prey populations, and proposes the following interesting stochastic system.

$$\begin{cases} \left.\begin{aligned} dx(t) &= x(t)\left[r_1(t) - a_{11}(t)x(t) - a_{12}(t)y(t) - \dfrac{H(t)}{1+b(t)x(t)}\right]dt \\ &\quad + \sigma_1(t)x(t)dB_1(t), \\ dy(t) &= y(t)\left[-r_2(t) + a_{21}(t)x(t) - a_{22}(t)y(t) - \dfrac{h(t)}{1+b(t)y(t)}\right]dt \\ &\quad - \sigma_2(t)y(t)dB_2(t) - \dfrac{\sigma_3(t)h(t)y(t)}{1+b(t)y(t)}dB_3(t), \end{aligned}\right\} t \neq t_k \\ x(t_k^+) - x(t_k) = \alpha_k x(t_k), y(t_k^+) - y(t_k) = \beta_k y(t_k), t = t_k, k = 1,2,3,\cdots \end{cases} \tag{1.3}$$

where $x(t)$ and $y(t)$ represent the density of prey and predator populations respectively. The parameters $r_i(t), a_{ij}(t), (i, j = 1,2)$ are positive, and r_1 is the internal growth rate of prey, and r_2 is the mortality rate of predator. $a_{11}(t)$ and $a_{22}(t)$ represent the intra-specific competition coefficients of prey and predator populations, respectively. The coefficient $a_{12}(t)$ is the predator's capture rate and $a_{21}(t)$ stands for the rate at which nutrients are converted to predators. In addition, $\dfrac{H(t)}{1+b(t)x(t)}, \dfrac{h(t)}{1+b(t)y(t)}$ are the nonlinear harvesting.

Throughout this paper, unless otherwise specified, we suppose $\left(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},\mathbb{P}\right)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions and it is right continuous and increasing, while $\mathcal{F}_0$ contains all $\mathbb{P}$ -null set. All the coefficients are assumed to be T-periodic continuous functions.

The remainder of this paper is organized as follows. In Section 2, we show that the model (1.3) existence of the global positive solution. In Section 3, sufficient conditions are achieved to guarantee the existence of a positive periodic solution of the stochastic system (1.3) by using Itô's formula. In Section 4, we discuss the globally attractive of stochastic model (1.3). In Section 5, we use numerical simulation to illustrate our results.

2. Existence and Uniqueness of Global Positive Solution

First, to facilitate the analysis that follows, we make the following tags. When $f(t)$ is a continuous T-periodic function, we define:

$$f^u=\sup_{t\geq 0} f(t),\ f^l=\inf_{t\geq 0} f(t)$$

Moreover, we assume that a product equals unity if the number of factors is zero.

Definition 2.1. [8] Consider an impulsive stochastic differential equation

$$\begin{cases} \mathrm{d}x(t)=f\left(t,x(t)\right)\mathrm{d}t+g\left(t,x(t)\right)\mathrm{d}B(t), t\neq t_k, t>0, \\ x\left(t_k^+\right)-x\left(t_k\right)=\alpha_k x\left(t_k\right), t=t_k, k=1,2,3,\cdots. \end{cases} \tag{2.1}$$

A stochastic process $x(t)=\left(x_1(t),x_2(t),\cdots,x_n(t)\right)^{\mathrm{T}}, t\in[0,+\infty)$ is said to be a solution of ISDE (2.1), if $x(t)$ satisfies

1) $x(t)$ is $\mathcal{F}_t$ adapted and is continuous on $(0,t_1)$ and each interval $(t_k,t_{k+1}), k\in\mathbb{N}$ and $f\left(t,x(t)\right)\in L^1\left(\mathbb{R}^+,\mathbb{R}^n\right), g\left(t,x(t)\right)\in L^2\left(\mathbb{R}^+,\mathbb{R}^n\right)$;

2) $x(t)$ obeys the equivalent integral equation of (2.1) for almost every $t\in\mathbb{R}_+\setminus t_k$ and satisfies the impulsive conditions at each $t\in\mathbb{R}_+, k\in\mathbb{N}$ *a.s.*;

3) For each $t_k, k\in\mathbb{N}$, $x\left(t_k^+\right)=\lim_{t\to t_k^+}x(t)$ and $x\left(t_k^-\right))=\lim_{t\to t_k^-}x((t))$ exist and $x\left(t_k^-\right)=x\left(t_k\right)$ with probability one.

We give the main results of system (1.3) as follows.

Theorem 2.1. For any initial value $\left(x_0,y_0\right)\in R_+^2$ the system (1.3) has a unique global positive solution $\left(x(t),y(t)\right)$ for $t\geq 0$ and the solution remains in $\mathbb{R}_+$ with probability one.

Proof. First, we construct the following SDE without impulses:

$$\begin{cases} \mathrm{d}x_1(t)=x_1(t)\left[r_1(t)+\dfrac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)-a_{11}(t)A_1(t)x_1(t)-a_{12}(t)A_2(t)x_2(t)\right. \\ \qquad\left.-\dfrac{H(t)}{1+b(t)A_1(t)x_1(t)}\right]\mathrm{d}t+\sigma_1(t)x_1(t)\mathrm{d}B_1(t), \\ \mathrm{d}x_2(t)=x_2(t)\left[-r_2(t)+\dfrac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)+a_{21}(t)A_1(t)x_1(t)-a_{22}(t)A_2(t)x_2(t)\right. \\ \qquad\left.-\dfrac{h(t)}{1+b(t)A_2(t)x_2(t)}\right]\mathrm{d}t-\sigma_2(t)x_2(t)\mathrm{d}B_2(t)-\dfrac{\sigma_3(t)h(t)x_2(t)}{1+b(t)x_2(t)}\mathrm{d}B_3(t), \end{cases} \tag{2.2}$$

with the initial value $(x_1, x_2) = (x_0, y_0)$, where

$$A_1(t) = \left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-\frac{t}{T}} \prod_{0 \le t_k < t}(1+\alpha_k), \quad A_2(t) = \left(\prod_{j=1}^{p}(1+\beta_j)\right)^{-\frac{t}{T}} \prod_{0 \le t_k < t}(1+\beta_k)$$

Then it is obvious that $A_1(t)$, $A_2(t)$ are positive T-periodic functions. In fact,

$$\begin{aligned}\frac{A_1(t+T)}{A_2(t)} &= \frac{\left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-\frac{t+T}{T}} \prod_{0 \le t_k < t+T}(1+\alpha_k)}{\left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-\frac{t}{T}} \prod_{0 \le t_k < t}(1+\alpha_k)} \\ &= \left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-1} \prod_{t \le t_k < t+T}(1+\alpha_k).\end{aligned} \tag{2.3}$$

For any $t \ge 0$, there is an integer n, such that

$$nT \le t \le (n+1)T.$$

The limited mathematical induction procedures, together with $t_{k+p} = t_k + T$, $\alpha_{k+p} = \alpha_k$ induce that

$$t_{k+np} = t_{k+(n-1)p} + T = \cdots = t_k + nT, \alpha_{k+np} = \alpha_{k+(n-1)p} = \cdots = \alpha_k \tag{2.4}$$

According to $[0,T) \cap \{t_k, k \in \mathbb{Z}\} = \{t_1, t_2, \cdots, t_p\}$, there exists $l = \{1, 2, \cdots, p\}$ such that

$$\begin{aligned}&t_{l+np}, t_{l+1+np}, \cdots, t_{p+np} \in [t, (n+1)T), \\ &t_{1+(n+1)p}, t_{2+(n+1)p}, \cdots, t_{l-1+(n+1)p} \in [(n+1)T, t+T).\end{aligned} \tag{2.5}$$

Thus, combining (2.2)-(2.4), we obtain

$$\begin{aligned}A_1(t+T) &= A_1(t)\left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-1} \prod_{k=l}^{p}(1+\alpha_{k+np}) \prod_{k=1}^{l-1}(1+\alpha_{k+(n+1)p}) \\ &= A_1(t)\left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-1} \prod_{k=l}^{p}(1+\alpha_{k+np}) \prod_{k=1}^{l-1}(1+\alpha_k) \\ &= A_1(t)\left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-1} \prod_{k=1}^{p}(1+\alpha_k) \\ &= A_1(t),\end{aligned}$$

Similarly, $A_2(t+T) = A_2(t)$.

By the same method as [9] and standard proof [10], Equation (2.2) has a unique global positive Solution $(x_1(t), x_2(t))$.

Next we will show that $(x(t), y(t))$ is the solution of system (2.2), which is continuous on each interval $(t_k, t_{k+1}) \in R^+$. For any $t \ne t_k$.

Let

$$x(t) = A_1(t)x_1(t), \quad y(t) = A_2(t)x_2(t).$$

$$\begin{aligned}\mathrm{d}x(t) &= A_1'(t)x_1(t)\mathrm{d}t + A_1(t)\mathrm{d}x_1(t) \\ &= x(t)\left[r_1(t) - a_{11}(t)x(t) - a_{12}(t)y(t) - \frac{H(t)}{1+b(t)x(t)}\right] \\ &\quad + \sigma_1(t)x_1(t)\mathrm{d}w_1(t),\end{aligned}$$

$$\mathrm{d}y(t) = y(t)\left[-r_2(t) + a_{21}(t)x(t) - a_{22}(t)y(t) - \frac{h(t)}{1+b(t)y(t)}\right]\mathrm{d}t$$
$$-\sigma_2(t)y(t)\mathrm{d}w_2(t) - \frac{\sigma_3(t)h(t)y(t)}{1+b(t)y(t)}\mathrm{d}w_3(t).$$

And, for every $k \in N$,

$$x(t_k^+) = \lim_{t\to t_k^+} A_1(t)x_1(t) = \left(\prod_{j=1}^{p}(1+\alpha_j)\right)^{-\frac{t_k}{\gamma}} \prod_{0\le t_k < t_k}(1+\alpha_k)x_1(t_k) = (1+\alpha_j)x(t_k),$$
$$x(t_k^-) = \lim_{t\to t_k^-} A_1(t)x_1(t) = A_1(t_k)x_1(t_k^-) = A_1(t_k)x_1(t_k) = x(t_k).$$

Similarly, we can show that,

$$y(t_k^+) = (1+\beta_j)y(t_k), y(t_k^-) = y(t_k).$$

Therefore, $(x(t), y(t))$ is a solution that satisfies system (1.3) Finally, we prove the nonnegative uniqueness of the solution of system (1.3) (more details see [11]).

Then the proof is completed.

3. Existence of Periodic Solutions of the System

In this section, we give the existence of the positive periodic solution of the stochastic system (1.3) with impulses. For convenience of readers, we first give the definition of the periodic solution of the impulsive stochastic differential equation in the sense of distribution and the results of the existence of periodic solutions (see [10] [11]).

Definition 3.1. [12] A stochastic process $\xi(t) = \xi(t,w)$ is said to be periodic with period *T*, if for every finite sequence of numbers $t_1, t_2, \cdots, t_n$ the joint distribution of random variables $\xi(t_1+h), \xi(t_2+h), \cdots, \xi(t_n+h)$ is independent of *h*, where $h = kT\,(k = \pm1, \pm2, \cdots)$.

Consider the following periodic stochastic differential equation without impulse:

$$\mathrm{d}x(t) = f(t, x(t))\mathrm{d}t + g(t, x(t))\mathrm{d}B(t), t \ge 0 \tag{3.1}$$

where $g(t,x(t))_{n\times l}$ is a $n\times l$ matrix function, $f(t,x(t))$ and the matrix $g(t,x(t))_{n\times l}$ are *T*-periodic in *t*.

Lemma 3.1. [12] [13] Assume that the system (3.1) has a global solution, and there exists a *T*-periodic function $V(t,x)$ such that the following conditions hold:

1) $LV(t,x) \le -1$ on the outside of some compact set;

2) $\inf_{|x|>R} \to \infty$, as $R \to \infty$.

Then Equation (3.1) has a *T*-periodic solution.

According to Lemma 3.2, we can obtain the main result in this section.

Theorem 3.1. Assume

$$\lambda_1 = \frac{1}{T}\int_0^T \left(a_{21}^l A_1^u \left(r_1(t) + \frac{1}{T}\sum_{j=1}^p \ln(1+\alpha_j) - H(t) - \frac{\sigma_1^2(t)}{2} \right) \right.$$

$(H1)$: $$\left. -a_{11}^u A_1^l \left(-r_2(t) + \frac{1}{T}\sum_{j=1}^p \ln(1+\beta_j) - h(t) - \frac{\sigma_2^2(t)}{2} - \frac{\sigma_3^2(t)h^2(t)}{2} \right) \right) dt$$

$$> 0,$$

$(H2)$: $$\lambda_2 = \frac{1}{T}\int_0^T \left(r_2(t) - \frac{1}{T}\sum_{j=1}^p \ln(1+\beta_j) - \frac{\sigma_2^2(t)}{2} - \frac{\sigma_3^2(t)h^2(t)}{2} \right) dt > 0,$$

$(H3)$: $$\frac{a_{21}^l A_1^u}{A_1^l} a_{11}^u A_1^u \le a_{11}^u A_1^l a_{21}^l A_1^l,$$

$(H4)$: $$\frac{CA_2^u \left(a_{21}^l a_{12}^u A_1^u + a_{11}^u a_{22}^u \right)}{r_2^l A_1^l T} \sum_{j=1}^p \ln(1+\beta_j) \le \frac{CA_2^u \left(a_{21}^l a_{12}^u A_1^u + a_{11}^u a_{22}^u \right)}{A_1^l}.$$

Then system (1.4) has a positive T-periodic solution.

Proof. We only need to prove the existence of a periodic solution of the equivalent system (2.2) without impulses. The global existence of the solution has been ensured by Theorem 1. Then, we only have to verify the conditions of by Lemma 3.1.

Define a C^2-function $V(t,x,y): \mathbb{R}_+^2 \to \mathbb{R}_+$:

$$V(t,x,y) = C\left(-\frac{a_{21}^l A_1^u}{A_1^l} \ln x - a_{11}^u A_1^u \ln y + w_1(t) + \frac{A_2^u \left(a_{21}^l a_{12}^u A_1^u + a_{11}^u a_{22}^u \right)}{r_2^l A_1^l} y \right)$$

$$+ \mathrm{e}^{w_2(t)} \frac{\left(\frac{a_{21}^u}{a_{12}^l} A_1^u x + A_2^l \right)^2 y}{2\left(A_2^l\right)^2} \tag{3.2}$$

$$\triangleq V_1(t,x,y) + V_2(t,x,y).$$

where $C > 0$ will be determined later. Here, $w_i(t)(i=1,2)$ satisfies

$$w_1'(t) = a_{21}^l A_1^u \left(r_1(t) - H(t) - \frac{\sigma_1^2(t)}{2} \right) - a_{11}^u A_1^l \left(-r_2(t) - \frac{\sigma_2^2(t) + \sigma_3^2(t)h^2(t)}{2} \right)$$
$$- \left\langle a_{21}^l A_1^u \left(r_1(t) - H(t) - \frac{\sigma_1^2(t)}{2} \right) - a_{11}^u A_1^l \left(-r_2(t) - \frac{\sigma_2^2(t) + \sigma_3^2(t)h^2(t)}{2} \right) \right\rangle_T \tag{3.3}$$

$$w_2'(t) = 2r_2(t) - \sigma_2^2(t) - \sigma_3^2(t)h^2(t) - \left\langle 2r_2(t) - \sigma_2^2(t) - \sigma_3^2(t)h^2(t) \right\rangle_T \tag{3.4}$$

Which λ_1 and λ_2 are defined by (H1), (H2). Obviously, $w_i(t)$ are T-periodic functions. And $w_2'(t)$ is a bounded function. Thus there is $K > 0$ such that;

$$\left| w_2'(t) \right| \le K, \quad \forall t \ge 0 \tag{3.5}$$

In order to confirm the condition (2) of Lemma 3.1, we only need to prove that

$$\inf_{(t.x.y)\in[0,\infty)\times\left(R_+^2 \setminus U_k\right)} V(t,x,y) \to \infty, \text{ as } k \to \infty.$$

where $U_k=\left(\frac{1}{k},k\right)\times\left(\frac{1}{k},k\right)$, here the coefficients of the quadratic term x^2,y^2 of $V(t,x,y)$ are all positive.

Next, we verify the condition (1) of Lemma 3.2. By Itô's formula, we have:

$$\begin{aligned}
LV_1 \le{}& \frac{Ca_{21}^l A_1^u}{A_1^l}\left(-r_1(t)-\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+\frac{\sigma_1^2(t)}{2}+a_{11}(t)A_1(t)x\right.\\
&\left.+a_{12}(t)A_2(t)y+H(t)\right)+Cw_1'(t)+Ca_{11}^u A_1^l\left(r_2(t)-\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)\right.\\
&\left.-a_{21}A_1(t)x+a_{22}(t)A_2(t)y+h(t)\frac{\sigma_2^2(t)+\sigma_3^2(t)}{2}\left(\frac{h(t)}{1+b(t)A_2(t)y}\right)^2\right)\\
&+\frac{CA_2^u\left(a_{21}^l a_{12}^u A_1^u+a_{11}^u a_{22}^u\right)}{r_2^l A_1^l}y\left(-r_2(t)+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)+a_{21}(t)A_1(t)x\right.\\
&\left.-a_{22}(t)A_2(t)y-\frac{h(t)}{1+b(t)A_2(t)y}\right)\\
\le{}& \frac{Ca_{21}^l A_1^u}{A_1^l}\left(-r_1(t)+Cw_1'(t)+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+H(t)+\frac{\sigma_1^2(t)}{2}\right)\\
&+Ca_{11}^u A_1^l\left(r_2(t)-\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)+h(t)+\frac{\sigma_2^2(t)+\sigma_3^2(t)h(t)^2}{2}\right)\\
&+Cw_1'(t)+\frac{Ca_{21}^u A_1^u A_2^u\left(a_{21}^l a_{12}^u A_1^u+a_{11}^u a_{22}^u\right)}{r_2^l A_1^l}xy\\
&+\left(\frac{Ca_{21}^l A_1^u}{A_1^l}a_{11}(t)A_1(t)-Ca_{11}^u A_1^l a_{21}(t)A_1(t)\right)x\\
&+\left(\frac{Ca_{21}^l A_1^u}{A_1^l}a_{12}(t)A_2(t)+\frac{CA_2^u\left(a_{21}^l a_{12}^u A_1^u+a_{11}^u a_{22}^u\right)}{r_2^l A_1^l T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)\right.\\
&\left.-\frac{CA_2^u\left(a_{21}^l a_{12}^u A_1^u+a_{11}^u a_{22}^u\right)}{A_1^l}\right)y\\
\le{}& C\left(-\lambda_1+\frac{a_{21}^u A_1^u A_2^u\left(a_{21}^l a_{12}^u A_1^u+a_{11}^u a_{22}^u\right)}{r_2^l A_1^l}xy+\frac{a_{21}^l A_1^u}{A_1^l}a_{12}(t)A_2(t)y\right).
\end{aligned}$$

As $V_2(t,x,y)=\mathrm{e}^{w_2(t)}\dfrac{\left(\dfrac{a_{21}^u}{a_{12}^l}A_1^u x(t)+A_2^l\right)^2}{2\left(A_2^l\right)^2}y$ so that:

$$V_2(t,x,y)=\frac{1}{2}\mathrm{e}^{w_2(t)}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}x\right)^2+\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\mathrm{e}^{w_2(t)}xy+\frac{1}{2}\mathrm{e}^{w_2(t)}y^2$$

Let $V_3(t,x,y)=\frac{1}{2}\mathrm{e}^{w_2(t)}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}x\right)^2$, $V_4(t,x,y)=\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\mathrm{e}^{w_2(t)}xy$, $V_5(t,x,y)=\frac{1}{2}\mathrm{e}^{w_2(t)}y^2$, we have

$$
\begin{aligned}
&LV_3(t,x,y)\\
&=\mathrm{e}^{w_2(t)}\left(\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\right)^2\left[x^2\left(\frac{w_2'(t)}{2}+r_1(t)+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+\frac{\sigma_1^2(t)}{2}\right)\right.\\
&\quad\left.+\left(-a_{11}(t)A_1(t)x^3-a_{12}(t)A_2(t)x^2y-\frac{H(t)x^2}{1+b(t)A_1(t)x}\right)\right]\\
&\le\mathrm{e}^{w_2(t)}\left(\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\right)^2\left[x^2\left(\frac{K}{2}+r_1^u+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+\frac{\left(\sigma_1^2\right)^u}{2}\right)-a_{11}^lA_1^lx^3-a_{12}^lA_2^lx^2y\right],
\end{aligned}
$$

$$
\begin{aligned}
&LV_4(t,x,y)\\
&=\mathrm{e}^{w_2(t)}\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\left[w_2'(t)+r_1(t)+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)-r_2(t)+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)\right.\\
&\quad-\left(a_{11}(t)A_1(t)x^2y+a_{12}(t)A_2(t)y^2x-a_{21}(t)A_1(t)x^2y+a_{22}(t)A_2(t)y^2x\right)\\
&\quad\left.-\left(\frac{H(t)}{1+b(t)A_1(t)x}+\frac{h(t)}{1+b(t)A_2(t)y}\right)xy\right]\\
&\le\mathrm{e}^{w_2(t)}\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\left[K+r_1^u+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+r_2^u\right.\\
&\quad\left.+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)+a_{21}^uA_1^ux^2y-a_{12}^lA_2^ly^2x\right],
\end{aligned}
$$

$$
\begin{aligned}
&LV_5(t,x,y)\\
&=\mathrm{e}^{w_2(t)}\left(\frac{w_2'(t)}{2}-r_2(t)+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)+\frac{\sigma_2^2(t)}{2}+\frac{\sigma_3^2(t)h^2(t)}{2}\right)y^2\\
&\quad+\mathrm{e}^{w_2(t)}\left(-a_{22}(t)A_2(t)y^3+a_{21}(t)A_1(t)y^2x-\frac{h(t)y^2}{1+b(t)A_2(t)y}\right)\\
&\le-\mathrm{e}^{w_2(t)}\lambda_2y^2+\mathrm{e}^{w_2(t)}\left(-a_{22}(t)A_2(t)y^3+a_{21}^uA_1^uy^2x\right).
\end{aligned}
$$

Then

$$
\begin{aligned}
&LV_2(t,x,y)\\
&\le\mathrm{e}^{\left|w_2^u\right|}\left(\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\right)^2x^2\left[\frac{K}{2}+r_1^u+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+\frac{\left(\sigma_1^2\right)^u}{2}\right]\\
&\quad+\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\mathrm{e}^{\left|w_2^u\right|}xy\left[K+r_1^u+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right)+r_2^u+\frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)\right]\\
&\quad-\mathrm{e}^{w_2^l}\left(\left(\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\right)^2a_{11}^lA_1^lx^3+a_{22}^lA_2^ly^3+\lambda_2y^2\right).
\end{aligned}
\tag{3.7}
$$

Then

$$
\begin{aligned}
&LV(t,x,y)\\
&=LV_1(t,x,y)+LV_2(t,x,y)\\
&=-C\lambda_1+m_1xy+m_2x^2-\mathrm{e}^{w_2^l}\left(\left(\frac{a_{21}^uA_1^u}{a_{12}^lA_2^l}\right)^2a_{11}^lA_1^lx^3+a_{22}^lA_2^ly^3+\lambda_2y^2\right).
\end{aligned}
$$

where:

$$m_1 = \frac{Ca_{21}^u A_1^u A_2^u \left(a_{21}^l a_{12}^u A_1^u + a_{11}^u a_{22}^u\right)}{r_2^l A_1^l} + \frac{a_{21}^u A_1^u e^{|w_2^u|}}{a_{12}^l A_2^l}\left[K + r_1^u + \frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right) + r_2^u + \frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\beta_j\right)\right],$$

$$m_2 = e^{|w_2^u|}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2\left[\frac{K}{2} + r_1^u + \frac{1}{T}\sum_{j=1}^{p}\ln\left(1+\alpha_j\right) + \frac{\left(\sigma_1^2\right)^u}{2}\right]$$

Let, we take

$$C = \frac{2}{\lambda_1}\max\left\{2, m_2 x^2 - \frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 - \frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 - \lambda_2 y^2\right\} \quad (3.8)$$

To confirm the condition (1) of Lemma 3.2, we choose a sufficiently small constant ε such that:

$$0 < \varepsilon \le \left\{\frac{C\lambda_1}{4m_1}, \frac{e^{w_2^l}\left(a_{21}^u A_1^u\right)^2 a_{11}^l A_1^l}{2m_1\left(a_{12}^l A_2^l\right)^2}, \frac{e^{w_2^l} a_{22}^l A_2^l}{2m_1}\right\} \quad (3.9)$$

$$-C\lambda_1 + C_1 + 1 \le e^{w_2^l}\min\left\{\frac{\left(a_{21}^u A_1^u\right)^2 a_{11}^l A_1^l}{2\left(a_{12}^l A_2^l\right)^2 \varepsilon^3}, \frac{a_{22}^l A_2^l}{2\varepsilon^3}\right\} \quad (3.10)$$

where

$$C_1 = \max_{(x,y)\in\mathbb{R}_+^2}\left\{\frac{2}{5}m_1 x^{\frac{5}{2}} + \frac{3}{5}m_1 y^{\frac{5}{3}} + m_2 x^2 - \frac{e^{w_2^l}}{2}\left(\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 + a_{22}^l A_2^l y^3\right) + e^{w_2^l}\lambda_2 y^2\right\} \quad (3.11)$$

Define a bounded open set

$$D_\varepsilon = \left\{(x,y) \mid \varepsilon < x < \frac{1}{\varepsilon}, \varepsilon < y < \frac{1}{\varepsilon}\right\}.$$

and denote

$$D_\varepsilon^1 = \left\{(x,y) \mid 0 < x \le \varepsilon\right\},\ D_\varepsilon^2 = \left\{(x,y) \mid 0 < y \le \varepsilon\right\},$$

$$D_\varepsilon^3 = \left\{(x,y) \mid x \ge \frac{1}{\varepsilon}\right\},\ D_\varepsilon^4 = \left\{(x,y) \mid y \ge \frac{1}{\varepsilon}\right\}.$$

It is obvious that $D_\varepsilon^c = D_\varepsilon^1 \cup D_\varepsilon^2 \cup D_\varepsilon^3 \cup D_\varepsilon^4$. Next, $LV(t,x,y) \le -1$ on $[0,\infty)\times D_\varepsilon^c$ must be shown.

Case 1: If $(t,x,y)\in D_\varepsilon^1$, then $xy \le \varepsilon y \le \varepsilon\left(1+y^3\right)$, we have:

$$LV(t,x,y) \le -\frac{C\lambda_1}{4} + \left(-\frac{C\lambda_1}{4} + m_1\varepsilon\right) - \frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 - \left(\frac{e^{w_2^l}}{2}a_{22}^l A_2^l - \varepsilon\right)y^3 + \left\{-\frac{C\lambda_1}{2} + m_2 x^2 - \frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 - \frac{e^{w_2^l}}{2}a_{22}^l A_2^l y^3 - \lambda_2 y^2\right\},$$

Using (3.8) and (3.9), we obtain

$$LV(t,x,y) \le -\frac{C\lambda_1}{4} - \frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 \le -\frac{C\lambda_1}{4} \le -1.$$

Case 2: If $(t,x,y) \in D_\varepsilon^2$, then $xy \le \varepsilon x \le \varepsilon(1+x^3)$, we have:

$$\begin{aligned} &LV(t,x,y) \\ &\le -\frac{C\lambda_1}{4} + \left(-\frac{C\lambda_1}{4} + m_1\varepsilon\right) - \left(\frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l - \varepsilon\right)x^3 - \frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 \\ &\quad + \left\{-\frac{C\lambda_1}{2} + m_2 x^2 - \frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 - \frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 - \lambda_2 y^2\right\}, \end{aligned}$$

By the definition (3.8) of C and the inequalities (3.9), we have:

$$LV(t,x,y) \le -\frac{C\lambda_1}{4} - \frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 \le -\frac{C\lambda_1}{4} \le -1.$$

By Young inequality, we have $xy \le \frac{2}{5} x^{\frac{5}{2}} + \frac{3}{5} y^{\frac{5}{3}}$. Then by equality (3.11), the following inequality is obvious:

$$\begin{aligned} &LV(t,x,y) \\ &\le -C\lambda_1 + m_2 x^2 - e^{w_2^l}\left(\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 + a_{22}^l A_2^l y^3 + \lambda_2 y^2\right) + \frac{2}{5} m_1 x^{\frac{5}{2}} + \frac{3}{5} m_2 y^{\frac{5}{3}} \\ &\le -C\lambda_1 - \frac{e^{w_2^l}}{2}\left(\frac{a_{21}^u A_1^u}{a_{12}^l A_2^l}\right)^2 a_{11}^l A_1^l x^3 - \frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 + C_1. \end{aligned}$$

Case 3: If $(t,x,y) \in D_\varepsilon^3$, from (3.9) and (3.10), we obtain

$$LV(t,x,y) \le -C\lambda_1 - \frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 + C_1 \le -C\lambda_1 e^{w_2^l} \frac{\left(a_{21}^u A_1^u\right)^2 a_{11}^l A_1^l}{2\left(a_{12}^l A_2^l\right)^2 \varepsilon^3} + C_1 \le -1.$$

Case 4: If $(t,x,y) \in D_\varepsilon^4$, from (3.9) and (3.10), we obtain

$$LV(t,x,y) \le -C\lambda_1 - -\frac{e^{w_2^l}}{2} a_{22}^l A_2^l y^3 + C_1 \le -C\lambda_1 e^{w_2^l} \frac{a_{11}^l A_1^l}{2\varepsilon^3} + C_1 \le -1.$$

Thus, we obtain $LV(t,x,y) \le -1$ on $[0,\infty) \times D_\varepsilon^c$, and the condition (1) of Lemma 3.2 is satisfied. Therefore, by Lemma 3.2, system (1.3) has a positive T-periodic solution.

The proof is confirmed.

4. Globally Attractive

Theorem 4.1. [14] [15] If system (1.3) satisfies $a_{11}^l - a_{21}^u - \frac{H^u}{b_l^2} > 0$, $a_{22}^l - a_{12}^u - \frac{h^u}{b_l^2} > 0$, then the system (1.3) is globally attractive.

Proof. Let $x(t)=(x_1(t),x_2(t))$, $y(t)=(y_1(t),y_2(t))$ be two arbitrary solutions of model (1.3) with initial values $x(t)>0, y(t)>0$.

We defined the following Lyapunov function

$$V(t)=\left|\ln x_1(t)-\ln y_1(t)\right|+\left|\ln x_2(t)-\ln y_2(t)\right|$$

Then by calculating the right differential $D^{+}V(t)$ and employing Ito's formula.

When $t\neq t_k$, we have:

$$\begin{aligned}
D^{+}V(t)&=\operatorname{sign}(x_1(t)-y_1(t))\mathrm{d}(\ln x_1(t)-\ln y_1(t))\\
&\quad+\operatorname{sign}(x_2(t)-y_2(t))\mathrm{d}(\ln x_2(t)-\ln y_2(t))\\
&=\operatorname{sign}(x_1(t)-y_1(t))\Bigg(-a_{11}(t)(x_1(t)-y_1(t))-a_{12}(t)(x_2(t)-y_2(t))\\
&\quad+\frac{H(t)(x_1(t)-y_1(t))}{[1+b(t)x_1(t)][1+b(t)y_1(t)]}\Bigg)\mathrm{d}t\\
&\quad+\operatorname{sign}(x_2(t)-y_2(t))\Bigg(a_{21}(t)(x_1(t)-y_1(t))-a_{22}(t)(x_2(t)-y_2(t))\\
&\quad+\frac{h(t)(x_2(t)-y_2(t))}{[1+b(t)x_2(t)][1+b(t)y_2(t)]}\Bigg)\mathrm{d}t\\
&=\Bigg(-a_{11}(t)\left|x_1(t)-y_1(t)\right|+a_{12}(t)\left|x_2(t)-y_2(t)\right|\\
&\quad+\frac{H(t)\left|x_1(t)-y_1(t)\right|}{[1+b(t)x_1(t)][1+b(t)y_1(t)]}\Bigg)\mathrm{d}t\\
&\quad+\Bigg(a_{21}(t)\left|x_1(t)-y_1(t)\right|-a_{22}(t)\left|x_2(t)-y_2(t)\right|\\
&\quad+\frac{h(t)\left|x_2(t)-y_2(t)\right|}{[1+b(t)x_2(t)][1+b(t)y_2(t)]}\Bigg)\mathrm{d}t\\
&\leq\Bigg[-\left(a_{11}(t)-a_{21}(t)-\frac{H(t)}{b^2(t)}\right)\left|x_1(t)-y_1(t)\right|\\
&\quad-\left(a_{22}(t)-a_{12}(t)-\frac{h(t)}{b^2(t)}\right)\left|x_2(t)-y_2(t)\right|\Bigg]\mathrm{d}t\\
&\leq-\left[\left(a_{11}^{l}-a_{21}^{u}-\frac{H^{u}}{b_l^2}\right)\left|x_1(t)-y_1(t)\right|+\left(a_{22}^{l}-a_{12}^{u}-\frac{h^{u}}{b_l^2}\right)\left|x_2(t)-y_2(t)\right|\right]\mathrm{d}t,
\end{aligned}$$

when $t=t_k, k\in N$, we get

$$\begin{aligned}
V(t_k^{+})&=\left|\ln x_1(t_k^{+})-\ln y_1(t_k^{+})\right|+\left|\ln x_2(t_k^{+})-\ln y_2(t_k^{+})\right|\\
&=\left|\ln(1+\alpha_k)x_1(t_k)-\ln(1+\alpha_k)y_1(t_k)\right|\\
&\quad+\left|\ln(1+\beta_k)x_2(t_k)-\ln(1+\beta_k)y_2(t_k)\right|\\
&=\left|\ln x_1(t_k)-\ln y_1(t_k)\right|+\left|\ln x_2(t_k)-\ln y_2(t_k)\right|\\
&=V(t_k).
\end{aligned}$$

Integrating both sides and then taking the expectation yields that

$$V(t) \le V(0) - \left(a_{11}^l - a_{21}^u - \frac{H^u}{b_l^2}\right)\int_0^t \left|x_1(s) - y_1(s)\right| ds$$
$$-\left(a_{22}^l - a_{12}^u - \frac{h^u}{b_l^2}\right)\int_0^t \left|x_2(s) - y_2(s)\right| ds$$

That is

$$V(t) + \left(a_{11}^l - a_{21}^u - \frac{H^u}{b_l^2}\right)\int_0^t \left|x_1(s) - y_1(s)\right| ds$$
$$+\left(a_{22}^l - a_{12}^u - \frac{h^u}{b_l^2}\right)\int_0^t \left|x_2(s) - y_2(s)\right| ds < V(0) < \infty$$

Then, in the view of $V(t) > 0$ and $a_{11}^l - a_{21}^u - \frac{H^u}{b_l^2} > 0, a_{22}^l - a_{12}^u - \frac{h^u}{b_l^2} > 0$ that $\lim_{t\to\infty} V(t) = 0$. Thus, it is easy to see from Lemmas 6.1 [15]

$$\lim_{t\to\infty}\left|x_i(t) - y_i(t)\right| = 0, i = 1, 2 \quad a.s.$$

The proof is complete.

5. Computer Simulations

In this section, we will prove our theoretical results by some examples with the help of the Matlab software [16] and reveal the influence of impulses and the white noise.

Example 1.

Let

$$a_{11} = 1.1 + \cos t, a_{12} = 1.2 + \sin t, r_1 = 1 + \sin t, H = 0.3 + 0.1\sin t,$$
$$a_{21} = 0.05 + 0.1\sin t, a_{22} = 1.1 + \sin t, T = 2\pi, r_2 = 0.15 + \sin t,$$
$$h = 1.5 + \sin t, b = 2 + \sin t, \sigma_1 = 0.03 + 0.1\sin t, \sigma_2 = \sigma_3 = 0.02 + 0.1\sin t,$$

then

$$a_{21}^l A_1^u \left(r_1(t) + \frac{1}{T}\sum_{j=1}^p \ln\left(1+\alpha_j\right) - H(t) - \frac{\sigma_1^2(t)}{2}\right)$$
$$+ a_{11}^u A_1^l \left(r_2(t) - \frac{1}{T}\sum_{j=1}^p \ln\left(1+\beta_j\right) + h(t) + \frac{\sigma_2^2(t)}{2} + \frac{\sigma_3^2(t)h^2(t)}{2}\right) = 3.13 > 0,$$

$$r_2(t) - \frac{1}{T}\sum_{j=1}^p \ln\left(1+\beta_j\right) - \frac{\sigma_2^2(t)}{2} - \frac{\sigma_3^2(t)h^2(t)}{2} \approx 1.65 > 0.$$

Thus, the conditions of **Theorem 3.1.** hold. Then the model (1.3) has a positive 2π-periodic solution. **Figure 1** confirms the results.

Example 2. Set $a_{11}^l = 1.1, a_{21}^u = 0.05, a_{22}^l = 1.2, a_{12}^u = 1$. Making condition of the Theorem 4.1 is satisfied. We get that system (1.3) is globally attractive (see **Figure 2**).

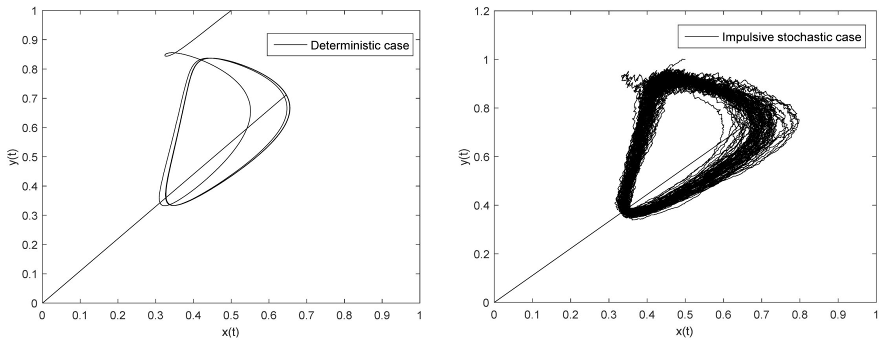

Figure 1. A solution of system (1.4) with the initial value $(x_0, y_0) = (0.6, 0.3)$. The left is the phase diagram of the stochastic system, and the right is the phase diagram of the deterministic system.

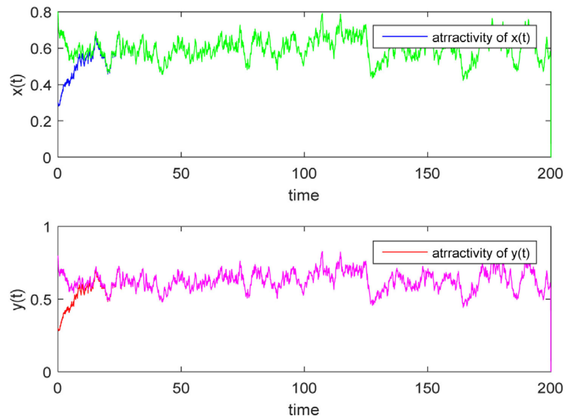

Figure 2. The figure shows the attractiveness of system (1.4), the blue and red lines represent the solution of prey and predator species.

6. Conclusion

In this paper, we propose a stochastic predator-prey system with nonlinear harvesting and impulsive perturbations. Firstly, we show that there is a unique positive solution in system (1.3). Secondly, the system has a positive periodic solution under a certain condition. Result shows that when the impulses are sufficiently large such that $\lambda_1 > 0, \lambda_2 > 0$ then the predator and prey will tend to exhibit periodicity. It is verified by constructing the appropriate Lyapunov functions and using Itô's formula. Moreover, these methods used in this study can be extended to more complex and realistic models.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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