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Visualizing Investment Decision on Decision Balls

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Abstract

Decision makers' choices are often influenced by visual background information. This study uses open-ended equity funds in Taiwan to investigate three well-known optimal portfolio models, including the mean-variance, maximin, and minimization of mean absolute deviation. The optimal portfolios are then visualized on Decision Balls to assist investors in making investment decisions. By observing the Decision Balls, investors can see the optimal portfolios, compare the optimal weights provided by the different models, view the cluster of funds, and even find substitute funds if preferred funds are not available.

Keywords: Visualization, Decision Ball, Investment decision, Portfolio

1. Introduction

Decision makers' choices are often influenced by visual background information [1,2]. Visual representations can simplify complex information into meaningful patterns, assist people in comprehending their environment, and allow for simultaneous perceptions of parts as well as the interrelationships between parts [3]. Visual representations are also recognized as being useful to present financial issues. For instance, the efficient frontier [4] is a well known visual representation used to help investors understand relationships between risks and returns.

Several graphic methods have been developed to support the decision-making: for instance, Gower Plots to detect any inconsistencies in a decision maker's preferences and rank alternatives [5,6], and ELECTRE graphs to help decision makers understand investment problems [7]. All these methods, however, use a 2-dimensional plane to illustrate the multidimensional data. A 2-dimensional plane model cannot depict three points that do not obey the triangular inequality (*i.e.* the total length of any two edges must be larger than the length of the third edge) neither can it display four points that are not on the same plane [8].

The method employed here, is the Decision Ball, which has not been used previously for visualizing portfolio. The Decision Ball method [8,9] is based on multidimensional scaling (MDS) [10,11] which has been widely used in marketing and decision-making [12,13]. This study extends the Decision Ball method to visualize optimal portfolios on the surface of a sphere. The distance between two securities is used to represent the correlation between them: the larger the correlation, the shorter the distance. Also, the fund with the higher return is located closer to the North Pole. Mutual funds in Taiwan are taken as an example to demonstrate how to assist investors visualize optimal portfolios on the Decision Ball.

Taiwan's mutual fund industry, which was founded in 1983, has been growing tremendously during the last decade [14,15], with the number of mutual fund corporations increasing from 4 to 38 by 2008. In 1998, there were only 200 funds with a total net asset value of NT$745.97 billion. However, by January 2008, there were 523 funds with a net asset value totaling NT$2,040.91 billion. This shows that the total net asset values of funds have almost tripled during the last decade. In Taiwan, the mutual fund industry is dominated by individual investors who account for over 90% of the market volume. By January 2008, over 1.84 million investors, about 8% of Taiwan's population, had invested in mutual funds.

This study examines 174 open-ended equity mutual funds which were issued and invested in Taiwan's Market from January 2002 to December 2006. Three well-known optimal portfolio models, including the mean- variance [4], the maximin [16], and the minimization of mean absolute deviation [17], are investigated. The optimal portfolios are visualized on the Decision Balls. By studying the Decision Balls, investors can then see the optimal portfolios, compare the optimal weights provided by different models, view the cluster of funds, and even find substitute funds if the preferred funds are not available.

This paper is organized as follows: Section 2 briefly reviews three well-known models for optimal portfolios. Section 3 develops an extended Decision Ball model to allocate funds on the surface of a sphere. Section 4 uses Taiwan's Open-Ended Equity Funds as an example to examine three optimal portfolio models, and Section 5 demonstrates how to visualize optimal portfolios on the Decision Balls.

2. Optimization Models for Portfolio Problem

Three well-known approaches to formulate optimal portfolios are illustrated in this section, including a) a mean-variable model denoted as MinVar, b) a maximin model denoted as MaxiMin, and c) a minimization of mean absolute deviation model denoted as MinMAD.

The mean-variance model, first proposed by Harry Markowitz, is a quadratic programming model to minimize the variance given a required return. Suppose there are n securities, the mean-variance model is formulated as follows:

2.1. Mean-Variance Model (MinVar)

Min $$Var = \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{i,j} w_i w_j$$

subject to: $$\sum_{i=1}^{n} w_i = 1 \quad (1)$$

$$\sum_{i=1}^{n} w_i u_i \geq a \quad (2)$$

$$0 \leq w_i \leq 1 \text{, for all } i \quad (3)$$

where w_i denotes the portfolio allocation of security i; $\sigma_{i,j}$ denotes the covariance between security i and security j; μ_i is the mean return for security i; α is the minimum expected return required by a particular investor.

Two important assumptions of the mean-variance model are: the investor prefers a low risk; and the expected return is multivariate normally distributed. The mean-variance model has been widely used in various portfolio problems. However, it may take some time to find optimal solutions with a large number of securities because the objective function is quadratic.

The maximin model [16] is a linear programming model to maximize the minimum portfolio return required by an investor. Denoting P as the minimum required return by an investor for every time period, T as the total number of periods, and $r_{i,t}$ as the return for security i over period t, where $t = 1,\cdots,T$, the maximin linear model is formulated as follows:

2.2. Maximin Linear Model (MaxiMin)

Max $$P$$

subject to: $$\sum_{i=1}^{n} r_{i,t} w_i \geq P \text{,} \quad \forall t \quad (4)$$

$$P \geq 0 \quad (5)$$

(1), (2), (3)

Contrary to the mean-variance model to lower risk by minimizing the variance, the object of this model is to maximize the minimum return over a set of past returns. The major advantage of this model is its capability to deal with portfolio optimization problems involving a large number of securities. Also, according to Young [16], the maximin model is more appropriate than the mean-variance model when data is log-normally distributed or skewed. However, this model may lead to an infeasible solution if the sum of the weighted expected returns is negative for any period of time.

The minimization of mean absolute deviation model [17] is another alternative to simplify the mean-variance model. This model uses the mean absolute deviation as a risk measure. The mean absolute deviation is defined as: $\frac{1}{T}\sum_{t=1}^{T}\left|\sum_{i=1}^{n}\left(r_{i,t}-\mu_i\right)w_i\right|$, Let $Q_t = \left|\sum_{i=1}^{n}\left(r_{i,t}-\mu_i\right)w_i\right|$, the minimization of mean absolute deviation model can be linearized as a linear programming formulation as follows:

2.3. Minimization of Mean Absolute Deviation Model (MinMAD)

Min $$\frac{1}{T}\sum_{t=1}^{T} Q_t$$

subject to: $$Q_t \geq -\sum_{i=1}^{n}\left(r_{i,t}-\mu_i\right)w_i \text{,} \quad \forall t \quad (6)$$

$$Q_t \geq -\sum_{i=1}^{n}\left(r_{i,t}-\mu_i\right)w_i \text{,} \quad \forall t \quad (7)$$

$$Q_t \geq 0 \text{,} \quad \forall t \quad (8)$$

(1), (2), (3)

The complexity of this model is much lower than that of a mean-variance model since the objective function is linear rather than quadratic. This model provides similar results as the mean-variance model if the return is multivariate normally distributed [17].

These three models have been examined by 67 securities over 48 months traded on the Stockholm Stock Exchange [18]. The results show that the maximin model provides the highest return and risk, the mean-variance model yields the lowest risk and return, and the result of the minimization of mean absolute deviation model is close to that of the mean-variance model. This study tries to use mutual funds in Taiwan to examine these three models and then visualize the results on Decision Balls.

3. An Extended Decision Ball Model

In order to visualize the relationships among funds, a Decision Ball model [8] is applied and extended here to display funds on the surface of a hemisphere.

The Decision Ball model is based on the concept of a multidimensional scaling technique. The arc length between two alternatives is used to represent the dissimilarity between them, e.g. the larger the difference, the longer the arc length. However, because the arc length is monotonically related to the Euclidean distance between two points and both approximation methods make little difference to the resulting configuration [19], the Euclidean distance is used for simplification purposes. Also, the alternative with a higher score value is designed to be closer to the North Pole so that alternatives will be located on the concentric circles in scoring order from top view.

In this study, the correlation coefficient is adopted to describe the degree of relationship between two funds because it is one of the most common statistics and it detects linear dependencies between two variables. The linear feature makes it easier to be visualized than covariance. Consider *n* funds denoted as A_i, $i \in \{1,\cdots,n\}$. Denote $\rho_{i,j}$ as the correlation coefficient between securities i and j, where $-1 \le \rho_{i,j} \le 1$ and $\rho_{i,j} = 1$ for all i, j. The closer the coefficient is to either −1 or 1, the stronger the correlation between the variables. If the variables are independent then the correlation is 0.

The distance between two funds is used to represent the correlation between them, *i.e.* the larger the correlation, the shorter the distance. The Euclidean distance between A_i and A_j is denoted as $d_{i,j}$, and $\hat{d}_{i,j}$ as the mapped distance of correlation. The relationship between $\hat{d}_{i,j}$ and $\rho_{i,j}$ is defined as below:

$$\hat{d}_{i,j} = s \times \left(1-\rho_{i,j}\right), \tag{9}$$

where *s* is a scaling constant. It is obvious $\hat{d}_{i,j} = \hat{d}_{j,i}$.

The scaling constant can be given as

$$s = \begin{cases} \sqrt{2}/\,\mathrm{Max}\{1\text{-}\rho_{i,j}\}, \text{ if } \mathrm{Max}\{1-\rho_{i,j}\} \ne 0, \forall\, i, j. \\ \sqrt{2} \qquad\qquad, \text{ if } \mathrm{Max}\{1-\rho_{i,j}\} = 0, \forall\, i, j. \end{cases} \tag{10}$$

In Expression (10), $\sqrt{2}$ is used because the distance between the North Pole and the Equator is $\sqrt{2}$ when the radius = 1. From Expressions (9) and (10), if $\rho_{i,j}$ = 1 then $\hat{d}_{i,j}$ = 0; if $\rho_{i,j}$ = 0 then $\hat{d}_{i,j}$ = *s*; if $\rho_{i,j}$ = −1 then $\hat{d}_{i,j} = 2 \times s = \sqrt{2}$. That is, the larger the correlation, the shorter the distance. The range of $\hat{d}_{i,j}$ is $0 \le \hat{d}_{i,j} \le \sqrt{2}$.

The fund with the higher return is designed to be located closer to the North Pole. The coordinates of a fund Ai are denoted on a ball as (x_i, y_i, z_i). Given a radius = 1, the coordinate of the North Pole is expressed as (0, 1, 0). An extended Decision Ball model for portfolio selection is formulated as follows:

Model 1 (An Extended Decision Ball Model for Portfolio Selection)

$$\underset{\{x_i,y_i,z_i\}}{\mathrm{Min}} \quad \sum_{i=1}^{n}\sum_{j>i}^{n}(d_{i,j}-\hat{d}_{i,j})^2$$

Subject to

$$\hat{d}_{i,j} = s \times (1-\rho_{i,j}), \quad \forall i, j > i, \tag{11}$$

$$y_i \ge y_j, \quad \text{if } \mu_i \ge \mu_j, \quad \forall i, j, \tag{12}$$

$$d_{i,j}^2 = (x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2, \quad \forall i, j > i, \tag{13}$$

$$x_i^2 + y_i^2 + z_i^2 = 1, \quad \forall i, \tag{14}$$

$$-1 \le x_i \le 1, \quad 0 \le y_i \le 1, \quad -1 \le z_i \le 1, \quad \forall i \tag{15}$$

The objective of Model 1 is to minimize the sum of the squared differences between $d_{i,j}$ and $\hat{d}_{i,j}$. Constraint (11) is from Expression (9). Constraint (12) is designed for the fund with a higher return to be located closer to the North Pole. Euclidean distance, instead of arc length, is used for simplification purposes (13). All alternatives are graphed on the surface of a sphere (14) and located on the northern hemisphere (15).

The faithfulness of this visual representation can be measured by Stress [20], which is a numerical measure of the closeness between the dissimilarities in the lower dimension and the original spaces formulated as follows:

$$Stress = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j>i}^{n}(d_{i,j}-\hat{d}_{i,j})^2}{\sum_{i=1}^{n}\sum_{j>i}^{n}d_{i,j}^2}} \tag{16}$$

A solution is desirable if its stress value is less than 10%.

Model 1 is a nonlinear model, which can be solved by using some commercial optimization software, such as Global Solver of Lingo 9.0, to obtain an optimum solution. This model has good performance results when the number of funds is small. However, when n becomes large, the computational time will increase greatly since the time complexity of Model 1 is n^2. In practice, in the case of more than 10 funds, we can choose some target funds as anchor points. The coordinates of the anchor points are calculated first, and then the coordinates of the remaining funds can be obtained by calculating the correlations between those funds and the anchor points. Thus, all funds can be displayed on the Decision Ball within a tolerable time frame.

4. Empirical Study of Taiwan Open-Ended Equity Funds

This study takes 174 open-ended equity funds, which were issued and invested in the Taiwan Market from January 2002 to December 2006 for example to investigate the three optimization models. From these 174 funds, 39 funds are excluded for the following two reasons: 1) 9 funds were not listed at the starting period 2) 30 funds left for various reasons over the examined time period. The monthly returns and rankings of the 135 open-ended equity funds are listed in **Table 1**. Fund number 129 has the highest monthly return of 0.0170; whereas, fund number 46 yields the lowest return, –0.0014. The average monthly return of the 135 funds is 0.0078.

To simplify, suppose our investors are only interested in the top 30 performance funds (*i.e.*, rank 1 ~ 30) and request monthly returns of at least 1%, and short selling is not allowed. The descriptive statistics for the top 30 funds are listed in **Table 2**. The third, fourth, fifth, and sixth columns of **Table 2** show the mean of the monthly return, the standard deviation, and both the minimum and maximum values of the funds. The last column describes the ranking of funds.

The MinVar, MaxiMin, and MinMAD models are all examined using the top 30 funds. However, the MaxiMin model yields infeasible solutions. The reason is that all top 30 funds exhibited negative monthly returns in some months. For instance, in September 2002, affected by that year's stock market downturn across the United States, Europe, and Asia, the average monthly return of the top 30 funds was –0.0892 ranging from –0.1242 to –0.0577.

The results of the MinVar and MinMAD models are listed in **Table 3**. Only those funds which appear in portfolios at least once, *i.e.* the funds numbering 12, 86, 129, and 133, are shown. The weights of the funds in an optimal portfolio for four different α are exhibited. Because α ranging from 1% to 1.3% yields the same portfolio weights, $1.0 \le \alpha \le 1.3\%$ is presented for short. Also, since the maximum mean of monthly returns for the top 30 funds is 0.0170 (exactly, 0.01695), $\alpha \ge 1.7\%$ is neglected because none of top 30 funds yields the mean of monthly returns greater than or equal to 1.7%. The bottom two rows of **Table 3** indicate the portfolio return and variance.

As shown in **Table 3**, the portfolio weights in both the MinVar and MinMAD models remain unchanged for low α values ranging from 1.0% to 1.3%. In this range, the MinVar model yields an optimal portfolio return of 0.0132, a variance of 0.0023, and weights w_{86} = 0.053, w_{129} = 0.243, and w_{133} = 0.704. Whereas, the MinMAD model yields an optimal portfolio return of 0.0130, a portfolio variance of 0.0023, and portfolio weights w_{129} = 0.219 and w_{133} = 0.781. As we can see, the MinVar model provides a higher expected return than required, and higher also than that of the MinMAD model. Given α = 1.4%, the MinVar and MinMAD models yield exactly the same solutions with a portfolio return of 0.014, a variance of 0.0024, and with weights w_{129} = 0.413 and w_{133} = 0.587. Given α = 1.5% and 1.6%, an optimal portfolio of the MinVar and MinMAD models consists of the same funds with different weights. The expected portfolio return and variance are the same.

In this study, the outcome of the MinMAD model is quite close to that of the MinVar model. This result is the same as the conclusions of Papahristodoulou and Dotzauer [18], in which 67 shares traded on the Stockholm Stock Exchange between January 1997 and December 2000 were examined. Both models provide optimal portfolio suggestions. However, the investors cannot tell directly, the correlations among funds through table- listing. The next section will demonstrate how to visualize the optimal portfolio on Decision Balls.

5. Visualizing Optimal Portfolios on Decision Balls

An extended Decision Ball model uses the distance between two funds to represent the correlation between them, *i.e.* the larger the correlation, the shorter the distance. Also, the fund with the higher return is located closer to the North Pole. At first, a correlation matrix of funds is calculated. From Expression (9), $\hat{d}_{i,j}$ for all *i*, *j* can be calculated. Since $\text{Max}\{1-\rho_{i,j}\}$ = 0.3064 for all *i*, *j*, from Expression (10), the scaling constant s is given as 4 in this example. If the number of funds being considered is small, then Model 1 can be applied directly to yield the coordinates of all funds. However, when the number of funds is large, the computational time for Model 1 will increase greatly.

In order to increase computational efficiency, the four funds listed in **Table 3**, *i.e.* the funds numbered 12, 86, 129, and 133, can be chosen as the target funds. These four funds, in which investors may be the most interested, are suggested in an optimal portfolio for both the MinVar and MinMAD models. The correlation matrix of the target funds is calculated first. Applying **Model 1** to these four funds yields the coordinates of them, the so called anchor points. The coordinates of funds numbered 12, 86, 129, and 133 are {0.820, 0.565, 0.083}, {0.292, 0.365, 0.883},

Table 1. The expected monthly returns from 135 mutual funds.

No	Quote	Return	Rank	No	Quote	Return	Rank	No	Quote	Return	Rank
1	0001	0.000 95	133	46	IC25	–0.001 43	135	91	ML09	0.010 17	38
2	0002	0.008 07	63	47	II01	0.005 57	93	92	NC13	0.006 13	87
3	0003	0.005 23	100	48	II05	0.008 82	53	93	YC07	0.002 76	124
4	0004	0.007 75	67	49	II13	0.007 15	74	94	ML15	0.010 92	30
5	0005	0.008 83	52	50	JF51	0.003 85	114	95	UI05	0.002 61	126
6	0012	0.002 58	127	51	JS01	0.005 33	96	96	AI05	0.005 28	99
7	0013	0.003 27	120	52	JS03	0.014 24	7	97	BR05	0.008 70	55
8	0014	0.012 43	18	53	KG01	0.008 93	50	98	CA09	0.012 22	22
9	0017	0.011 71	27	54	KY01	0.015 80	2	99	CI07	0.004 22	109
10	0018	0.003 53	117	55	ML01	0.014 14	8	100	CP03	0.009 62	41
11	0021	0.010 13	39	56	ML02	0.005 71	91	101	CP14	0.010 71	33
12	0025	0.015 51	3	57	ML06	0.010 53	35	102	CS07	0.004 65	107
13	0026	0.009 87	40	58	ML12	0.006 88	77	103	DD04	0.006 39	82
14	AI01	0.005 08	103	59	NC03	0.006 84	78	104	DF07	0.007 52	69
15	AP01	0.003 67	115	60	NC07	0.006 26	84	105	DS03	0.002 39	128
16	AP04	0.006 84	79	61	NC09	0.008 58	57	106	FH06	0.012 15	24
17	BR01	0.007 30	73	62	PS01	0.006 52	81	107	FP11	0.005 87	90
18	CA03	0.009 17	46	63	PS02	0.007 06	76	108	FP16	0.008 04	64
19	CA04	0.003 96	112	64	PS03	0.009 59	43	109	GC05	0.012 35	20
20	CF01	0.008 87	51	65	PS09	0.006 26	85	110	GC17	0.006 26	86
21	CI01	0.005 14	102	66	PS14	0.014 08	9	111	IC06	0.007 08	75
22	CI04	0.005 23	101	67	TC01	0.008 96	49	112	IC30	0.002 83	123
23	CI05	0.008 23	61	68	TI02	0.007 61	68	113	II11	0.009 61	42
24	CI12	0.004 10	111	69	TI07	0.012 81	16	114	JF76	0.003 57	116
25	CP04	0.004 23	108	70	TR01	0.002 08	130	115	JS07	0.005 89	89
26	CP07	0.002 64	125	71	TS02	0.002 86	122	116	KY06	0.009 20	45
27	CS02	0.007 83	65	72	TS08	0.008 39	59	117	ML07	0.002 94	121
28	CS09	0.005 31	97	73	TS13	0.006 80	80	118	NC16	0.013 22	13
29	CT01	0.007 35	72	74	YT02	0.013 29	12	119	PS10	0.010 76	31
30	CY01	0.008 73	54	75	YT03	0.009 04	48	120	TC03	0.005 67	92
31	CY14	0.002 39	129	76	YT04	0.012 44	17	121	TC19	0.013 17	14
32	DD01	0.005 30	98	77	YT11	0.015 38	4	122	TR04	0.005 90	88
33	DS04	0.001 15	132	78	YT12	0.013 70	10	123	TS09	0.007 41	71
34	FD01	0.005 34	95	79	0023	0.011 58	28	124	UI03	–0.000 48	134
35	FE01	0.006 38	83	80	0029	0.004 67	106	125	YC02	0.007 82	66
36	FH01	0.003 51	118	81	BR03	0.008 53	58	126	YT09	0.009 11	47
37	FH03	0.008 68	56	82	CI11	0.005 00	105	127	0015	0.005 07	104
38	FP03	0.012 23	21	83	CP10	0.011 43	29	128	FP05	0.012 13	25
39	FP04	0.010 59	34	84	CY03	0.014 79	5	129	JS06	0.016 95	1
40	FP06	0.01076	32	85	DF05	0.004 16	110	130	TI09	0.003 86	113
41	FP10	0.014 50	6	86	FH08	0.012 18	23	131	FD02	0.009 52	44
42	GC01	0.008 29	60	87	IC08	0.010 43	36	132	FP15	0.013 34	11
43	IC01	0.005 38	94	88	II10	0.003 29	119	133	JF85	0.011 92	26
44	IC04	0.007 43	70	89	JF83	0.008 21	62	134	NC17	0.012 85	15
45	IC22	0.001 85	131	90	JS04	0.012 43	19	135	YT16	0.010 31	37

Table 2. Descriptive statistics for the top 30 mutual funds.

No	Quote	Return	STD	Min	Max	Rank	No	Quote	Return	STD	Min	Max	Rank
8	0014	0.0124	0.0662	–0.1157	0.1625	18	83	CP10	0.0114	0.0626	–0.1214	0.1508	29
9	0017	0.0117	0.0614	–0.1118	0.1472	27	84	CY03	0.0148	0.0587	–0.1157	0.1379	5
12	0025	0.0155	0.0573	–0.1118	0.1426	3	86	FH08	0.0122	0.0591	–0.1366	0.1951	23
38	FP03	0.0122	0.0632	–0.1279	0.1529	21	90	JS04	0.0124	0.0731	–0.1405	0.241	19
41	FP10	0.0145	0.0672	–0.123	0.1985	6	94	ML15	0.0109	0.0721	–0.1263	0.2299	30
52	JS03	0.0142	0.072	–0.1224	0.1905	7	98	CA09	0.0122	0.0695	–0.1418	0.1755	22
54	KY01	0.0158	0.071	–0.1139	0.1478	2	106	FH06	0.0121	0.0573	–0.096	0.1554	24
55	ML01	0.0141	0.0674	–0.1097	0.1909	8	109	GC05	0.0123	0.0664	–0.1297	0.1663	20
66	PS14	0.0141	0.0679	–0.1141	0.2176	9	118	NC16	0.0132	0.0694	–0.1325	0.1995	13
69	TI07	0.0128	0.0688	–0.1091	0.1728	16	121	TC19	0.0132	0.0673	–0.1252	0.1621	14
74	YT02	0.0133	0.0654	–0.1131	0.1948	12	128	FP05	0.0121	0.0687	–0.1219	0.2403	25
76	YT04	0.0124	0.0658	–0.1486	0.1709	17	129	JS06	0.017	0.0545	–0.0898	0.1418	1
77	YT11	0.0154	0.0608	–0.1042	0.1757	4	132	FP15	0.0133	0.0712	–0.1271	0.1989	11
78	YT12	0.0137	0.0723	–0.1359	0.2337	10	133	JF85	0.0119	0.0494	–0.1108	0.1099	26
79	0023	0.0116	0.0618	–0.1202	0.1698	28	134	NC17	0.0128	0.0616	–0.1109	0.1739	15

Table 3. Portfolios of the top 30 mutual funds by the MinVar and MinMAD models.

		$1.0\% \le \alpha \le 1.3\%$		$\alpha = 1.4\%$		$\alpha = 1.5\%$		$\alpha = 1.6\%$	
No	Quote	MinVar	MinMAD	MinVar	MinMAD	MinVar	MinMAD	MinVar	MinMAD
12	0025	0	0	0	0	0.084	0.165	0.216	0.298
86	FH08	0.053	0	0	0	0	0	0	0
129	JS06	0.243	0.219	0.413	0.413	0.552	0.494	0.657	0.598
133	JF85	0.704	0.781	0.587	0.587	0.364	0.341	0.127	0.104
P. Return		0.0132	0.013	0.014	0.014	0.015	0.015	0.016	0.016
P. Variance		0.0023	0.0023	0.0024	0.0024	0.0025	0.0025	0.0026	0.0026

Table 4. Coordinates of the top 30 mutual funds.

No	Quote	x	y	z	No	Quote	x	y	z
8	0014	0.854	0.415	–0.312	83	CP10	0.525	0.189	0.83
9	0017	0.677	0.325	0.659	84	CY03	0.476	0.545	0.689
12	0025	0.82	0.565	0.083	86	FH08	0.292	0.365	0.883
38	FP03	0.621	0.385	0.682	90	JS04	0.085	0.405	0.911
41	FP10	0.503	0.535	0.678	94	ML15	0.598	0.179	0.782
52	JS03	0.154	0.525	0.836	98	CA09	0.56	0.375	0.738
54	KY01	0.59	0.575	0.566	106	FH06	0.596	0.355	0.719
55	ML01	0.667	0.515	0.538	109	GC05	0.545	0.395	0.739
66	PS14	0.597	0.505	0.622	118	NC16	0.159	0.465	0.87
69	TI07	-0.043	0.435	0.899	121	TC19	0.449	0.455	0.768
74	YT02	0.486	0.475	0.733	128	FP05	0.456	0.345	0.82
76	YT04	0.841	0.425	-0.334	129	JS06	0.183	0.856	0.483
77	YT11	0.616	0.555	0.558	132	FP15	0.641	0.485	0.594
78	YT12	0.463	0.495	0.734	133	JF85	0.914	0.335	0.227
79	0023	0.507	0.199	0.838	134	NC17	0.674	0.445	0.589

{0.183, 0.856, 0.483}, and {0.914, 0.335, 0.227} respectively. The coordinates for the remaining funds can be obtained by calculating the correlations between those funds and the target funds. The locations of all top 30 funds are listed in **Table 4** and depicted in **Figure 1**.

In **Figure 1**, the four target funds are shown as bold font. Since the expected return is $\mu_{129} > \mu_{12} > \mu_{86} > \mu_{133}$, fund 129 is closest to the northern point followed by funds 12, 86, 133. Also, the distance between the two funds is used to represent the correlation between them: the larger the correlation, the shorter the distance. Take fund 12 for example, because the correlation between funds 12 and 133 ($\rho_{12,133} = 0.9087$) is higher than that between funds 12 and 129 ($\rho_{12,129} = 0.7501$), then the distance between the former is shorter than that of the later.

The optimal portfolios obtained by the MinVar and MinMAD models are graphed on the Decision Ball and represented respectively with "dotted" and "dash" circles. The scale of the circle represents the weight of the fund: the higher the weight, the larger the circle. Given a required return $\alpha \le 1.3\%$, the optimal portfolio weights of the MinVar and MinMAD models are {w_{86} = 0.053, w_{129} = 0.243, w_{133} = 0.704} and {w_{129} = 0.219, w_{133} = 0.781} respectively, as listed in **Table 3**. This result is depicted in **Figure 2** where funds 86, 129, and 133 are marked by dotted circles, and funds 129 and 133 are marked by dash circles. Since the weight of fund 133 by the MinMAD model is higher than the one by the MinVar model, the dash circle of fund 133 is bigger than the dotted circle. The optimal portfolios suggested by both models are quite diversified because the locations of the selected funds (funds 86, 129, and 133) are far apart from each other.

Given α = 1.4% and 1.6%, the Decision Balls for optimal portfolios are shown in **Figures 2** and **3**. In **Figure 2**, the optimal portfolio weights by the MinVar and MinMAD models are exactly the same, {w_{129} = 0.413, w_{133} = 0.587}. In **Figure 3**, the optimal portfolio weights by MinVar and MinMAD are {w_{12} = 0.216, w_{129} = 0.657, w_{133} = 0.127} and {w_{12} = 0.298, w_{129} = 0.598, w_{133} = 0.104} respectively.

When comparing **Figures 1, 2**, and **3**, the circle of fund 129 becomes bigger and fund 133's circle becomes smaller when the given expected return α is increased. That is, the optimal portfolio weights shift from the lower to the upper part of the Ball because the funds located in the upper part imply a higher return. Also, there is an obvious cluster, including most of the top 30 funds, except for funds 129, 12, 133, 76, and 8. The correlation between the funds can be examined, both visually and directly, through the Decision Balls. Take fund 86 for instance, if the selected fund 86 is not available, funds 121, 109, 118, or 90 may be good substitutes because they have a high correlation with fund 86 plus a higher return.

6. Conclusions

This study uses open-ended equity funds in Taiwan to in-

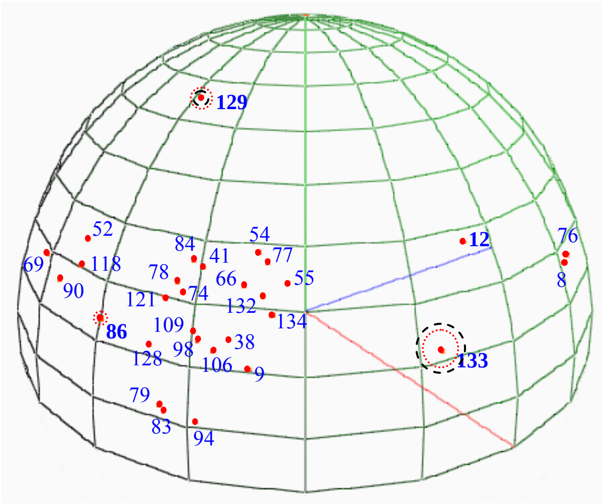

Figure 1. Given $a \le 1.3\%$ yields MinVar (dotted circle) and MinMAD (dash circle).

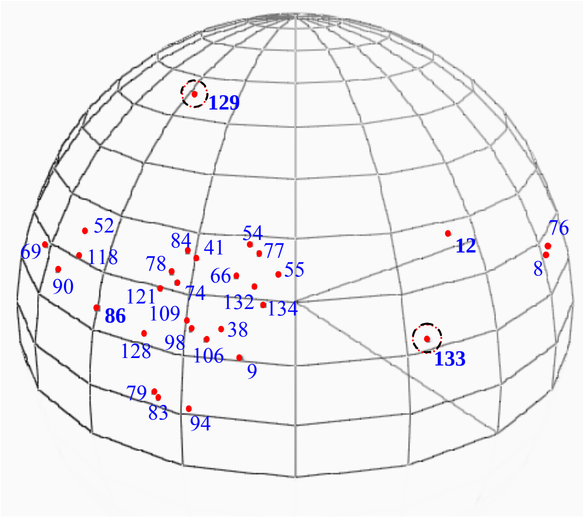

Figure 2. Given $a \le 1.4\%$ yields MinVar (dotted circle) and MinMAD (dash circle).

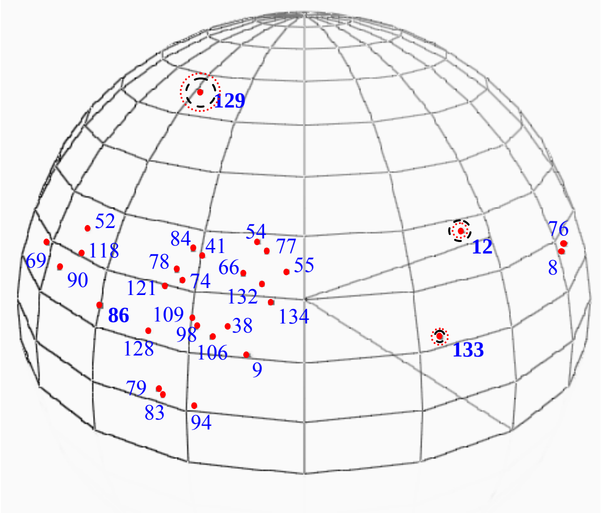

Figure 3. Given $a \le 1.6\%$ yields MinVar (dotted circle) and MinMAD (dash circle).

vestigate three well-known optimal portfolio models, including the mean-variance, maximin, and minimization of mean absolute deviation. The maximin model yields infeasible solutions because all top 30 funds exhibit negative monthly returns in some months during the examined time period. The outcome of the minimization of mean absolute deviation model is quite close to that of the mean-variance model. This result is the same as the conclusions for the study by Papahristodoulou and Dotzauer [18], in which securities traded on the Stockholm Stock Exchange were examined. An extended Decision Ball model is proposed to visualize optimal portfolios on the surface of a sphere, where the distance between two funds indicates the correlation between them, and the fund with a higher return is located closer to the North Pole. The scale of optimal portfolio weights is represented by the size of the circle of the selected fund. By observing the Decision Balls, investors can see the optimal portfolio, compare the optimal weights provided by the different models, view the cluster of funds, and even find substitute funds if the preferred funds are not available. In future studies, the question of how to linearize this non-linear model in order to general a global optimal solution can be addressed.

7. References

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