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Intra-Firm Inventory Management Model with Transshipments

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Abstract

This paper analyzes the implementation of transshipments among two independent locations which belong to one parent firm and the impact of transshipments, when implemented between these two locations. We examine how the possibility of such transshipments affects the optimal inventory orders at each location and find that, if each location aims to maximize its own profit, their inventory choices will not in general maximize joint profits. However, we find transshipment prices which if implemented by the parent firm as incentive design will make each location behave in order to optimize aggregate profit.

Keywords

Inventory Management, Transshipments, Newsvendor Model

1. Introduction

Inventory management is a critical aspect of enterprise-wide optimization. Inventories are used in production and logistics networks to coordinate supply cycles and to mitigate the risk associated with uncertainty. The importance of inventory in industrial applications derives from the effect of stockouts in the levels of customer satisfaction and the impact of stock out in the economic balance of companies.

Remarkably, the value of U.S inventories was estimated to be over $1707 billion in December 2013 [1], and the opportunity cost ascribed to capital invested in inventories added up to $434 billion in 2012 [2]. Therefore, the potential savings from stockout prevention and inventory related cost offer a huge opportunity for optimization.

Inventory plays a variety of roles. It can be used to buffer tempered mismatches between supply availability, processing rates, and demand in integrated

production networks. It is also useful to satisfy safety requirements, in cases where the availability of hazardous raw materials can be replaced with inventory of its products.

Inventory also plays an important role in the coordination of maintenance turnaround between various facilities across a site. In general, the benefits of inventories derive from their ability to reduce the interdependence of processing units, to mitigate the effects of bottlenecks, and to facilitate capacity utilization.

Many inventory management strategies have been proposed since Harris introduced the Economic Order Quantity (EOQ) model in 1913. Before the 1930s, merchants used the skills of experience and intuition in their shop keeping writing down purchases or looking at how many units were gone at the end of the day and then do their best to forecast for the future needs. However, efficiency and mass production became the main objectives of business after the industrial revolution, along with an improved customer experience at the point of sale.

In the 1960s, the affordable laser technology which allowed for smaller, faster and cheaper readers or scanners as well as the Universal Product Code (UPC) was developed. These systems boosted the efficiency of the computing power which helped in trading and managing inventories better.

In recent years, more flexible inventory systems also allow lateral transshipments within an echelon, thus, between wholesalers or retailers [3]. In this way, members of the same echelon pool their inventories, which can allow them to lower inventory levels and cost whilst still achieving the required service levels.

As a risk-pooling strategy, transshipment provides an efficient and effective approach for firms to reduce inventory carrying costs without negative effects on customer service. For example, Saturn has achieved impressive success with after-sales service through the practice of sharing automobile parts among its dealers [4]. When one dealer is out of a particular Stock Keeping Unit (SKU), this dealer will first contact a nearby Saturn dealer to see if the SKU is available there. If the part is available, transshipments are implemented from the location with excess stock to the location with insufficient supply. Through transshipments, a dealer can quickly receive the part needed and thus satisfy customer requests for either maintenance or repair faster than if the part were backordered from the original supplier or the central distribution center.

The study of multi-location models with transshipment is an important contribution for mathematical inventory theory as well as for inventory practice. The idea of lateral transshipments is not new. The first study dates back to the sixties, the two-location-one-period case with linear cost functions was considered by Aggarwal [5].

Krishnan [6] studies with N-location-one-period model, where the cost parameters are the same for all locations. Johnson [7] incorporated non-negligible replenishment lead times and transshipment lead times among stocking locations to the multi-location model.

The effect of lateral transshipment on the service levels in a two-location-one-period model was studies by Tagaras [8]. In all works only minimization of

the expected total cost is considered.

Our major contribution is to determine the demand variations facing the two locations under which a transshipment policy is likely to increase performance of the parent firm. In particular, we examine how the variance in demand affects inventory levels, revenues and profits of the firm.

This paper is organized as follows. The next section provides a review of literature on inventory management and transshipments. Section 3 sets up the basic framework that allows transshipment between two locations. We look at the classic newsvendor problem as a special case within this framework. Section 4 examines how the possibility of transshipments between these locations affects inventory orders in the case where they are centrally determined by the parent firm. To illustrate the model, we give a numerical analysis in this for normal distribution and compare to the corresponding results of the classical newsvendor model and finally, Section 5 presents the conclusions and future research.

2. Literature Review

2.1. Inventory Management

Inventory can be described as the stock of any item or resource that is used in an organization [9]. There are many reasons for an organization to hold inventory, but a few main reasons are identified by Simchi-Levi [10]. Firstly, to provide for unexpected changes in customer demand; again, to function as a buffer that caters for the presence of uncertainty in some situations; also, to ensure stock availability during supply lead times and finally, to achieve economies of scale offered by transportation companies.

Inventory management under uncertainty originated in the 1950s and the first significant contribution to this field was provided in 1951 by Arrow [11] in which they present a dynamic inventory model with random demand described by a known probability distribution. Their model is commonly referred to as the (s, S) or min max inventory management approach and can be seen as an adaptation of the original Economic Order Quantity (EOQ) model [2] to enable the incorporation of erratic or lumpy demand patterns [12].

The work of Arrow *et al.* [11] was then extended by Dvoretzky [13] [14] in two comprehensive papers. Similar to Arrow *et al.* [11], their first paper focuses on an inventory model where demand is represented by a random variable following a known probability distribution [13]. Their second paper deals with inventory models with uncertain demand and unknown distribution [14].

Since then, many papers on inventory management and modeling under uncertainty have been published as seen in Chu [15]; Handfield [16]; Maiti [17]; Mandal [18]; Ryu [19]; Solyali [20].

A technique often used in the literature to deal with the complexity of inventory management and modeling under uncertainty is to assume that the uncertain parameter, for instance demand, is random and that its probability distribution is known with certainty. In such cases, stochastic programming can be used to incorporate uncertainty into the model. Similar examples may be found in

Maiti *et al.* [17], Ryu *et al.* [19].

2.2. Transshipments

In the field of supply chain management, one of the most important components is meeting customer demand in an effective and efficient manner. Although, there are a number of possible solutions, transshipment is among the most practical approach.

Lateral transshipments within an inventory system are stock movements between locations of the same echelon [3]. There are two main streams of research on lateral transshipment: one is to minimize the total expected cost; lateral transshipments are arranged among all of the stocking locations before customer demands are realized, and two, lateral transshipments are made to meet unsatisfied customers when one of the stocking locations is out of stock and another has extra stock on hand.

In the periodic review system, Krishnan *et al.* [6] may be the first to provide the fundamental framework for a single-period-two-location problem. They analyze transshipment in a setting with a multi-location distribution network that consists of a number of warehouses, characterized by centralized control, independent demand and identical inventory holding and shortage cost for all locations. Their analysis further suggests that, under these conditions, transshipments equalize optimal inventory and service levels throughout stocking locations.

Building on Krishnan *et al.* [6], Tagaras [8] investigates the effect of transshipments on customer service levels, measured by non-stockout probability and fill rate. Tagaras [8] finds that, the complete pooling policy improves the service level in a two-location distribution system and that, the service levels at both locations will be equalized through transshipments under the conditions that the locations have identical market demands and inventory cost structures.

Robinson [21] considers a multi-period, multi-location problem, derives the optimal ordering policy and finds analytical solutions for the two-location case. Hu [22] studies a multiple period setting but focus on the two-location transshipment problem. Zhao [23] studies the dynamic transshipment policies for a multi-location problem. In a recent paper, Heide [24] studies the transshipment of books with online demand to minimize the logistic costs in the library system.

Other recent related transshipment studies can be found in the work of Yu, Tang and Niederloff [25]; Jiang and Anupindi [26], Zhao and Atkins [27], Shen, Hu and Liao [28].

3. The Problem Description and Modeling

Our basic framework follows closely the work of Krishnan *et al.* [6]. We consider one parent firm with two branches serving customers at two distinct locations noted as i and j throughout the paper. Firm i procures inventory Q_i from a central supplier at a fixed unit cost w_i. When placing the orders, the firms do not know the level of demand, D_i, that will be realized at both loca-

tions, however, the joint distribution over demand realizations $(D_1 \times D_2)$ is known. For simple and straightforward analysis, we assume the probability distribution to be twice differentiable and strictly increasing on its support.

Firm i obtains revenue y_i for every unit sold and if it cannot meet the entire demand, it is penalized P_i for every unit of unmet demand. Unsold stock has a salvage value, S_i and we also define the marginal value of additional sales at location i as $M_i = y_i + P_i$.

Table 1 provides the summary of the notations used in this study.

From Table 1, let C_{ij} be the price charged by location i for each unit transshipped to location j. We also assume that, t_{ij} is the cost incurred by location i for every unit it transship to location j. Therefore, location i enjoys a surplus of $C_{ij} - t_{ij}$ for each unit transshipped and would find it profitable to transship to j whenever this surplus is greater than the salvage value, s_i; location j would also find it profitable to accept transshipments whenever it is stocked out and the transshipment price is less than the marginal value of additional sales.

According to the complete pooling assumptions defined by Tagaras [8], transshipments from location i to location j are mutually profitable whenever there is excess demand at location j and surplus stock at location i and $C_{ij} \in \left[s_i + t_{ij}, M_j \right]$. We further assume that, $s_i + t_{ij} < M_j$ for $i, j = 1, 2$. Again, we assume that, $w_i < w_j + t_{ji}, s_i < s_j + t_{ji}$, and $M_i < M_j + t_{ji}$. These restrictions ensure that it is not beneficial to always buy through another location and that transshipment occurs only when there is surplus stock at one location and simultaneously excess demand at the other location.

Table 1. Notation description used in the model.

Notation	Description
y_i, y_j	Unit price of goods sold by Location i and j respectively
w_i, w_j	Procurement cost paid by Location i and j
H_c	Average inventory carrying/holding cost
s_i, s_j	Salvage value for each unit of unsold stock
p_i, p_j	Penalty cost for each unit of unmet demand
t_{ij}, t_{ji}	Cost incurred in transshipping a unit from Location i and j and vice versa
C_{ij}, C_{ji}	Transshipment price charged by i to j and by j to i
Q_i, Q_j	Optimal inventory levels for i and j respectively
X_{ij}, X_{ji}	Quantity of transshipments from i to j and from j to i
R_i, R_j	Actual sales in unit by i and j
U_i, U_j	Amount of unsold stock at i and j respectively
Z_i, Z_j	Amount of unmet demand of i and j
D_i, D_j	Level of demand at locations i and j respectively

Source: Li Zou [29].

Following the constraints above, we define the transshipment quantity from location i to location j as: $X_{ij} = \min\left[\left(D_j - Q_j\right)^+, \left(Q_i - D_i\right)^+\right]$, where $x^+ = \max(x, 0)$. In this expression, the $\left(D_j - Q_j\right)^+$ represents excess demand at location j while surplus stock at location i is represented by $\left(Q_i - D_i\right)^+$.

The transshipment level takes the lower value between the overstock at one location and the under stock at the other location. Transshipment level from location j to location i can similarly be written as:

$$X_{ji} = \min\left[\left(Q_j - D_j\right)^+, \left(D_i - Q_i\right)^+\right].$$

Given the transshipment quantity above, sales for location i can be written as $R_i = \min(D_i, Q_i) + X_{ji}$ units while unsold stock at location i is $U_i = \left(Q_i - D_i - X_{ij}\right)^+$ and unmet demand is $Z_i = \left(D_i - Q_i - X_{ij}\right)^+$.

Similarly, the same expressions can be written for location j as follows:

$$R_j = \min(D_j, Q_j) + X_{ij}; U_j = \left(Q_j - D_j - X_{ji}\right)^+; Z_j = \left(D_j - Q_j - X_{ij}\right)^+$$

The expressions above depend on D_i, D_j, Q_i and Q_j; therefore, expected profit at locationi will be:

$$e_i\left(Q_i, Q_j\right) = E\left[y_i R_i + \left(C_{ij} - t_{ij}\right)X_{ij} - C_{ji}X_{ji} + s_i U_i - p_i Z_i\right] - w_i Q_i \tag{a}$$

Next, we develop a classic newsvendor problem using equation (a) above. This newsvendor has no transshipments but provides very useful benchmark. By setting $X_{ij} = X_{ji} = 0$ and eliminating the subscripts, we get the expected profit for the newsvendor as:

$$\begin{aligned} e^n(Q) &= E\left[yR + sU - pZ\right] - wQ \\ &= E\left[y\min(D,Q) + s(Q-D)^+ - p(D-Q)^+\right] - wQ \end{aligned} \tag{b}$$

Let the probability of stockouts be denoted by $\text{Prob}(D > Q)$, the marginal profitability of increasing the stock level is given by:

$$\frac{\delta e^n}{\delta Q} = y\text{Prob}(D > Q) + s\text{Prob}(D < Q) + p\text{Prob}(D > Q) - w \tag{c}$$

The optimal inventory choice trades off the expected marginal benefit with its marginal cost, w. There exists a unique order quantity Q^n that maximizes the newsvendor's profit if the distribution over demand is continuous and strictly increasing. Setting Equation (c) to zero and rearranging, we arrive at a similar solution for the newsvendor problem:

$$\text{Prob}(D < Q) = \left(\frac{M - w}{M - s}\right) \tag{d}$$

The expected total profits for the two locations will therefore be:

$$\begin{aligned} e^t\left(Q_1, Q_2\right) = &E\left[y_1 R_1 + y_2 R_2 - t_{21}X_{21} - t_{12}X_{12} + s_1 U_1 + s_2 U_2 - p_1 Z_1 - p_2 Z_2\right] \\ &- w_1 Q_1 - w_2 Q_2 \end{aligned} \tag{e}$$

Next, we calculate the derivative of expression (e) above to obtain the marginal unit of inventory at location i as follows:

$$\frac{\delta e^{t}}{\delta Q_{i}} = \left(y_{i} + p_{i}\right)\text{Prob}\left(D_{i} > Q_{i} + \left(Q_{j} - D_{j}\right)^{+}\right)$$

$$+ \left(y_{j} + p_{j}\right)\text{Prob}\left(D_{j} > Q_{j} + Q_{i} - D_{i}, Q_{i} > D_{i}\right)$$

$$+ t_{ji}\text{Prob}\left(Q_{i} < D_{i} < Q_{i} + Q_{j} - D_{j}\right) - t_{ij}\text{Prob}\left(Q_{i} + Q_{j} - D_{j} < D_{i} < Q_{i}\right)$$

$$+ s_{i}\text{Prob}\left(Q_{i} > D_{i} + \left(D_{j} - Q_{j}\right)^{+}\right) + s_{j}\text{Prob}\left(Q_{j} > D_{j} + D_{i} - Q_{i}, D_{i} > Q_{i}\right) - w_{i}$$

Collecting terms, we have:

$$\frac{\delta e^{t}}{\delta Q_{i}} = M_{i}\left(1 - \text{Prob}\left(D_{i} < Q_{i}\right) - \text{Prob}\left(Q_{i} < D_{i} < Q_{i} + Q_{j} - D_{j}\right)\right)$$

$$+ \left(t_{ji} + s_{j}\right)\text{Prob}\left(Q_{i} < D_{i} < Q_{i} + Q_{j} - D_{j}\right)$$

$$+ \left(M_{j} - t_{ij}\right)\text{Prob}\left(Q_{j} + Q_{i} - D_{j} < D_{i} < Q_{i}\right) \tag{f}$$

$$+ s_{i}\left(\text{Prob}\left(D_{i} < Q_{i}\right) - \text{Prob}\left(Q_{i} + Q_{j} - D_{j} < D_{i} < Q_{i}\right)\right) - w_{i}$$

Expression (f) above has three components:

1) The additional inventory at location i will result in incremental sales at location i which will yield M_{i} per unit except when there is surplus inventory at location i or the effective excess supply at location j could have enabled transshipments from j to i. In this case, the marginal unit of Q_{i} is $t_{ji} + s_{j}$.

2) If there is no stockout at location i but demand exceeds effective supply at location j, the marginal unit can be transshipped to j, which will yield $M_{j} - t_{ij}$.

3) If demand at location j does not exceed effective supply, and there is no stockout at i, the marginal unit is worth only its salvage value s_{i}. We can subtract the marginal procurement cost w_{i} from the above to get the net marginal benefit.

Figure 1 shows six (6) possible events that may arise with transshipments. Descriptions of the events are as follows:
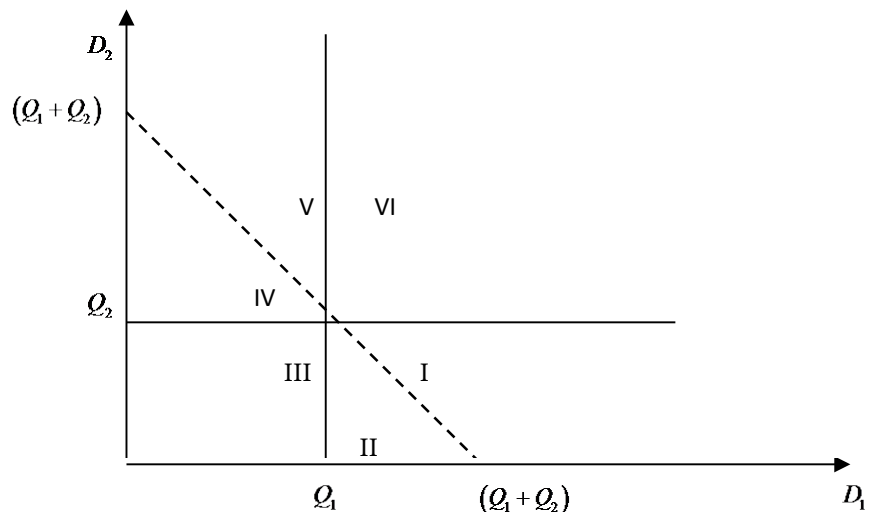

Figure 1. Six (6) possible events with transshipments.

Event I

In this event, location i has demand D_1, which is greater than the stock quantity Q_1; whereas location j has demand D_2 less than its stock quantity Q_2. This event is parallel to event V.

Event II

Location i is out of stock whereas location j is overstocked. This event is parallel to event IV.

Event III

Both location i and j are overstocked in this event and therefore, no transshipments occur in this event.

Event IV

Location j is out of stock, whereas location i is overstocked and the joint stock quantities for both Location i and j is more than the aggregate demand at the two locations.

Event V

In this event, Location j has demand D_2, which is greater than its stock quantity Q_2, whereas Location i has demand D_1 less than its stock quantity Q_1. Hence, Location i has a surplus stock $(Q_1 - D_1)$ and Location j is out of stock by the amount $(D_2 - Q_2)$. Also, in this event, the total demand of Location i and j denoted as $(D_1 + D_2)$, is greater than their aggregate stock $(Q_1 + Q_2)$.

Event VI

Both Location i and j are out of stock in this event and no transshipments occur.

The probability functions of the various events are given in Table 2.

The work of Robinson [21] shows that, the expected profit function is concave in (Q_1, Q_2), so the first-order conditions are sufficient for optimality.

Rearranging expression (f), the conditions characterizing the optimal inventory order are:

$$\alpha_i(Q_i) - \beta_i(Q_i, Q_j)\left(\frac{M_j - s_i - t_{ij}}{M_i - s_i}\right) + \gamma_i(Q_i, Q_j)\left(\frac{M_i - s_j - t_{ji}}{M_i - s_i}\right) = \left(\frac{M_i - w_i}{M_i - s_i}\right) \quad (g)$$

Table 2. Events and associated probability functions.

Event	Description	Probability	Notation	Transshipments
III	Both ations i and j are over stocked	$D_i < Q_i$	$\alpha_i(Q_i)$	$X_{ji} = 0$
V	Location j is out of stock and Location i transships all of its excess stock to j	$Q_i + Q_j - D_j < D_i < Q_i$	$\beta_i(Q_i, Q_j)$	$X_{ji} = 0, X_{ji} = Q_i - L$
II	Location i is out of stock and Location j transships some of its surplus stock to i	$Q_i < D_i < Q_i + Q_j - D_i$	$\gamma_i(Q_i, Q_j)$	$X_{ji} = D_i - Q_i, X_{ij} =$

Source: Li Zou [29].

The right-hand side of Equation (g) above is the adjustment of the newsvendor solution. The left-hand side adjusts Q_i up due to the possibility of transshipment from location i to j; and the third on the left-hand side adjusts Q_i down due to the possibility of transshipments from location j to i.

Next, we continue to calculate the transshipment prices and we ask the following question: does there exist a pair of transshipment prices that maximizes the total profit? If yes, then this can be implemented in many practical situations by the parent firm as incentive design to make each location behave in order to optimize aggregate profit.

There exist a unique set of transshipment prices C_{ij} for $i, j = 1, 2$ that yields the joint optimal solution given by:

$$C_{ij}^* = \left(\frac{M_j \beta_i \beta_j + \left(s_j + t_{ji} - M_i\right)\beta_j \gamma_i - \left(s_i + t_{ij}\right)\gamma_i \gamma_j}{\beta_i \beta_j - \gamma_i \gamma_j} \right) \tag{h}$$

It is sufficient to look for transshipment prices which induce the locations to choose optimal order quantities.

4. Numerical Analysis

In this section, we compute explicit numerical example to illustrate the analytical results. Assuming demand realizations at the two locations are independent and distributed normally, with mean 100 and standard deviation 50. Thus, $D_i \sim N(100, 50)$. We truncate the distribution at 0 and redistribute this proportionally to the positive part of the distribution in order to rule out negative realizations of the demand variable. It should be noted that, this yields a slightly higher mean and slightly lower variance.

As provided in **Table 3**, the procurement cost, w_i is assumed to be 20 per unit, the selling price y_i is assumed to be twice as high as the procurement cost. Also, salvage value for each part is 25% of the product value, penalty for lost sales is zero and transshipment cost which is assumed to be equal for the two locations is 2. It should be noted that, all the parameter values used in this analysis is assumed to be symmetric.

The optimal order quantities and average profits are illustrated in **Figure 2** and **Table 4**.

Table 3. Assumed values for parameters used in numerical analysis.

Cost parameter	Notation	Assumed values
Procurement cost	w_i, w_j	20.00
Selling price	y_i, y_j	40.00
Salvage value	s_i, s_j	10.00
Penalty cost	p_i, p_j	0
Transshipment cost	t_{ij}, t_{ji}	2

Source: Li Zou [29].

Table 4. Inventory decisions with $D_i \sim N(100,50)$.

		Optimal Inventory Q	Average Profit at each location, e
Without transshipment	Newsvendor	122.5	1530
With transshipments, thus $C_{ij} = C_{ji}$	12	107.0	1660
	18	112.3	1672
	20	114.1	1674
	22	115.9	1675
$C_{ij}^{*} = C_{ji}^{*}$	23.3	117.1	1676
	26	119.4	1675
	35	127.0	1661
	40	130.9	1648

Source: Rudi, Kapur, and Pyke [30].
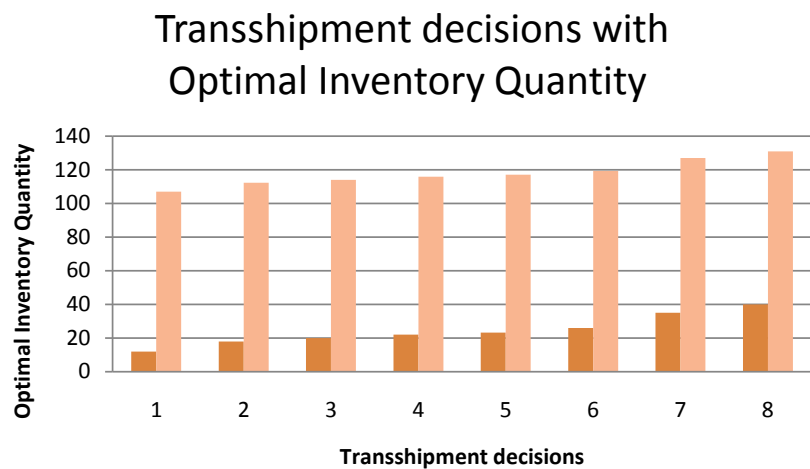
Figure 2. Transshipment decisions with Optimal Inventory Quantity.

As the numbers in Figure 2 and Table 4, transshipments improve profitability considerably over the case where transshipments are not permitted and we arrive at the highest increment in profitability when inventory orders are centrally coordinated as per this study. It could further be seen that, the equilibrium order quantity is increasing in the transshipment price, and there exist a coordinating transshipment price that decentralizes the joint-profit maximizing outcome. If the parent firm could however, impose a transshipment price of 23.3, it would result in inventory orders that maximizes aggregate profits.

5. Conclusion and Future Research

In conclusion, this study analyses the role that transshipments play in modern inventory management and its benefits between two independent locations belonging to one parent firm. It is found in this study that, the use of an appropriate transshipment price is an effective tool for these locations to maximize their

profits and optimize inventory level decisions. There exists a unique transshipment price that is optimal for both locations.

We show in the study how this also affects the optimal inventory orders at each location and found that, if each location aims to maximize its own profit, their inventory choices will not in general, maximize joint profits. In the case where inventory choice varies with transshipment prices, we find transshipment prices that induce the locations to choose inventory levels consistent with joint-profit maximization.

For future research, it is worthwhile to consider larger system consisting of more than two firms since our study only considered a two-location newsvendor problem with and without transshipments for a single replenishment period.

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