American Journal of Computational Mathematics, 2016, 6, 267-274
Published Online September 2016 in SciRes. http://www.scirp.org/journal/ajcm
http://dx.doi.org/10.4236/ajcm.2016.63028

Computer Simulation of Transition Regimes of Solitons in Four-Photon Resonant Parametric Processes in Case of Two-Photon Resonance

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Received 30 July 2016; accepted 20 September 2016; published 23 September 2016

Abstract

The transition regimes of solitons in four-photon resonant processes in the case of two-photon absorption of the fundamental radiation are numerically investigated. The standard system of equations for the amplitudes of probability of finding the system in state with certain energy is used to derive the expression for the induced polarization in the nonlinear medium. As for the equations for the amplitudes of the optical pulses, the general case is considered in which both the amplitudes and phases are space-time dependent. We focus on the finite difference methods and the case of simultaneously propagating solitons at all frequencies of the interacting waves (simultons). The obtained results indicate that upon certain threshold conditions all interacting pulses become the solitons of Lorentzian shape. The numerical analysis has also shown that the soliton amplitudes significantly depend on the ratio between the nonlinear polarizability at the fundamental frequency ω_0 and that of combination of ω_0 and the trigger-field frequency $\omega_1\left(2\omega_0+\omega_1\right)$. In the second part of the paper, we apply the method of phase planes to show that at typical values of parameters, the solitons are stable.

Keywords

Solitons, Transition Regime, Stability

1. Introduction

Solitons or self-reinforcing solitary waves can emerge spontaneously in a physical system in which some energy

How to cite this paper: Feshchenko, V. and Feshchenko, G. (2016) Computer Simulation of Transition Regimes of Solitons in Four-Photon Resonant Parametric Processes in Case of Two-Photon Resonance. *American Journal of Computational Mathematics*, **6**, 267-274. http://dx.doi.org/10.4236/ajcm.2016.63028

is fed in, for instance as thermal energy or by an excitation with an electromagnetic wave, even if the excitation does not match exactly the soliton solution. Therefore, if a system possesses the necessary properties to allow the existence of solitons, it is highly likely that any large excitation will indeed lead to their formation [1]-[3]. The field of solitons and related nonlinear phenomena has been substantially advanced and enriched by research and discoveries in nonlinear optics [4]-[7].

In our previous research [8], we established the possibility of the existence of simultons (simultaneously propagating solitons at different frequencies) in the case of nonstationary Raman scattering with excitation of polar optical phonons under the conditions of the interaction of ultrashort pulses of exciting and Stokes radiation in nonlinear crystals. The relevance of this study is connected both with the fact that one can extract additional information on the optical characteristics of matter, and with the possibility of obtaining of ultrashort pulses.

The second topical problem in modern nonlinear optics is the production of coherent and frequency-tunable radiation in the far ultraviolet (UV) and infrared (IR). In these spectral areas, solid materials have broad absorption bands and this narrows down the application of nonlinear crystals for the generation of electromagnetic radiation. Possible ways of overcoming those difficulties are related with the utilization of nonlinear phenomena in gases and metal vapors. The resonant four-photon interaction (RFPI) in the case of two-photon resonance is one of them. Among the advantages of gases are the presence of narrow resonances and possibility of continuous variation of density, width of spectral line, length of the medium, etc. [9]-[11]. Ultrashort pulse propagation in the case of two-photon resonance was first examined in [12] where the two-photon self-induced transparency effect was predicted. This prediction was subsequently confirmed experimentally [13] and by numerical studies [14]. Third-harmonic generation (THG) in media exhibiting resonance behaviour has also attracted considerable attention [15]-[21]. However, RFPIs that are not frequency degenerate are of no less interest; they can be used to transfer the tuning of radiation from one range to another [22] [23].

The present paper is devoted to the computer simulation of transition regimes of RFPI solitons in the case of two-photon resonance. The basic equations describing this process are given in Section 2 [12] [24]. The results of computer simulation are shown in Section 3. The stability of solitons is considered in Section 4.

2. Fundamental Principle

Let us assume that two optical pulses with frequencies $\omega_{0,1}$ propagate in the nonlinear medium at the angles $\theta_{0,1}$ with respect to the z-axis. The value of $2\omega_0$ is close to the frequency of resonant transition between levels 2 and 1 in the medium $(\omega_{21} = 2\omega_0 - \Delta\omega)$. The nonlinear interaction between $\omega_{0,1}$ and the medium results in parametric generation of $\omega_2 = 2\omega_0 + \omega_1$ and $\omega_3 = 2\omega_0 - \omega_1$. The values of $\omega_{0,1,2,3}$ are considered to be in the transparent range of frequencies. We also assume that all electromagnetic waves have the same polarizations.

To find the system of equations that governs the processes of propagation of optical pulses with frequencies $\omega_{0,1,2,3}$ in the medium we take the standard system of equations for the amplitudes of probability a_k of finding the system in state with energy E_k [25]

$$i\frac{\partial a_k}{\partial t} = \frac{1}{\hbar}\sum_l \mathrm{e}^{i\omega_{kl}t} V_{kl} a_l, \tag{1}$$

where $V_{kl} = -\frac{1}{2}\mu_{kl}\sum_m E_m\left(\mathrm{e}^{i\Phi_m} + \mathrm{e}^{-i\Phi_m}\right), \Phi_m = \omega_m t - k_m z + \varphi_m, \left(\Phi_m = -\Phi_{-m}, E_m = E_{-m}\right),$ (2)

μ_{kl} is the dipole moment of the transition $k \to l$; $\omega_m, k_m, E_m, \varphi_m$ are the frequencies, wave vectors, real "slowly-varying amplitudes", and phases of the interacting waves, respectively.

We next use (1) and the theory of perturbations [26] to find a_l $(l \ne 1,2)$ (the perturbation coefficient is of order $\frac{\mu_{lk}E_m}{\hbar\omega_{lk}}$)

$$a_l = \frac{1}{2\hbar}\sum_{m,p=1,2}\frac{\mu_{lp}E_m}{\left(\omega_{lp}+\omega_m\right)}\mathrm{e}^{i\left(\omega_{lp}t+\Phi_m\right)}a_p, \ (l \ne 1,2) \tag{3}$$

To obtain the system of equations for $a_{1,2}$ we introduce the expression (3) into the Equation (1), which becomes

$$i\frac{\partial a_1}{\partial t} = -\left(\sum_{m=0}^{3}\frac{r_{11}^{(m)}}{4\hbar}E_m^2\right)a_1 - \frac{1}{4\hbar}\left(r_{12}^{(1)}E_0^2 + r_{12}^{(2)}E_1E_2\mathrm{e}^{i\Delta_2} + r_{12}^{(3)}E_1E_3\mathrm{e}^{i\Delta_1}\right)\mathrm{e}^{i\Delta}a_2, \tag{4}$$

$$i\frac{\partial a_2}{\partial t} = -\frac{1}{4\hbar}\left(r_{12}^{(1)}E_0^2 + r_{12}^{(2)}E_1E_2e^{-i\Delta_2} + r_{12}^{(3)}E_1E_3\mathrm{e}^{-i\Delta_1}\right)\mathrm{e}^{-i\Delta}a_1 - \left(\sum_{m=0}^{3}\frac{r_{22}^{(m)}}{4\hbar}E_m^2\right)a_2, \tag{5}$$

where $r_{11}^{(m)} = \frac{2}{\hbar}\sum_{l\neq 1,2}\frac{\mu_{1l}\mu_{l1}\omega_{l1}}{\left(\omega_{l1}^2-\omega_m^2\right)}$, $r_{12}^{(1)} = \frac{1}{\hbar}\sum_{l\neq 1,2}\frac{\mu_{1l}\mu_{l2}}{\left(\omega_{l2}+\omega_0\right)}$, $r_{12}^{(2)} = \frac{1}{\hbar}\sum_{l\neq 1,2}\mu_{1l}\mu_{l2}\left(\frac{1}{\omega_{l2}+\omega_2}+\frac{1}{\omega_{l2}-\omega_1}\right)$,

$r_{12}^{(3)} = \frac{1}{\hbar}\sum_{l\neq 1,2}\mu_{1l}\mu_{l2}\left(\frac{1}{\omega_{l2}+\omega_3}+\frac{1}{\omega_{l2}+\omega_1}\right)$, $r_{22}^{(m)} = \frac{2}{\hbar}\sum_{l\neq 1,2}\frac{\mu_{2l}\mu_{l2}\omega_{l2}}{\left(\omega_{l2}^2-\omega_m^2\right)}$, $\Delta = \Delta\omega t - 2k_0^z z + 2\varphi_0$,

$\Delta_1 = \left(2k_0^z - k_1^z - k_3^z\right)z + \varphi_1 + \varphi_3 - 2\varphi_0$, $\Delta_2 = \left(2k_0^z + k_1^z - k_2^z\right)z + \varphi_2 - \varphi_1 - 2\varphi_0$.

The expression for the polarization induced by the superposition of nonlinear waves is defined by

$$P = \sum_{l\neq 1,2}\left(a_1^*\mu_{1l}a_l\mathrm{e}^{i\omega_{1l}t} + a_2^*\mu_{2l}a_l\mathrm{e}^{i\omega_{2l}t} + c.c.\right). \tag{6}$$

We introduce (3) into (7) and find that the expression for the induced polarization becomes

$$\begin{aligned}
P = &\sum_{m=0}^{3}\left(\Delta r^{(m)}n + x^{(m)}\right)E_m\cos\Phi_m + 2r_{12}^{(1)}E_0P_1\cos\Phi_0 + 2r_{12}^{(1)}E_0P_2\sin\Phi_0\\
&+\left[r_{12}^{(2)}E_2\left(P_1\cos\Delta_2 - P_2\sin\Delta_2\right) + r_{12}^{(3)}E_3\left(P_1\cos\Delta_1 - P_2\sin\Delta_1\right)\right]\cos\Phi_1\\
&+\left[r_{12}^{(2)}E_2\left(-P_2\cos\Delta_2 - P_1\sin\Delta_2\right) + r_{12}^{(3)}E_3\left(P_2\cos\Delta_1 + P_1\sin\Delta_1\right)\right]\sin\Phi_1\\
&+r_{12}^{(2)}E_1\left(P_1\cos\Delta_2 - P_2\sin\Delta_2\right)\cos\Phi_2 + r_{12}^{(2)}E_1\left(P_2\cos\Delta_2 + P_1\sin\Delta_2\right)\sin\Phi_2\\
&+r_{12}^{(2)}E_1\left(P_1\cos\Delta_1 - P_2\sin\Delta_1\right)\cos\Phi_3 + r_{12}^{(2)}E_1\left(P_2\cos\Delta_1 + P_1\sin\Delta_1\right)\sin\Phi_3,
\end{aligned} \tag{7}$$

where $P_1 = \mathrm{Re}\left(a_1^*a_2\mathrm{e}^{i\Delta}\right)$, $P_2 = \mathrm{Im}\left(a_1^*a_2\mathrm{e}^{i\Delta}\right)$, $n = \left|a_2\right|^2 - \left|a_1\right|^2$, $\Delta r^{(m)}, \kappa^{(m)} = 0.5\left(r_{22}^{(m)} \mp r_{11}^{(m)}\right)$.

The system of Equations (4) and (5) can now be rewritten in terms of $P_{1,2}$ and n as follows

$$\frac{\partial P_1}{\partial\tilde t} = -\Delta\Omega\tau_0P_2 - \frac{n}{4}\left(\tilde r_{12}^{(2)}\tilde E_1\tilde E_2\sin\Delta_2 + \tilde r_{12}^{(3)}\tilde E_1\tilde E_3\sin\Delta_1\right), \tag{8}$$

$$\frac{\partial P_2}{\partial\tilde t} = \Delta\Omega\tau_0P_1 - \frac{n}{4}\left(\tilde r_{12}^{(1)}\tilde E_0^2 + \tilde r_{12}^{(2)}\tilde E_1\tilde E_2\cos\Delta_2 + \tilde r_{12}^{(3)}\tilde E_1\tilde E_3\cos\Delta_1\right), \tag{9}$$

$$\frac{\partial n}{\partial\tilde t} = \left(\tilde r_{12}^{(1)}\tilde E_0^2 + \tilde r_{12}^{(2)}\tilde E_1\tilde E_2\cos\Delta_2 + \tilde r_{12}^{(3)}\tilde E_1\tilde E_3\cos\Delta_1\right)P_2 + \left(\tilde r_{12}^{(2)}\tilde E_1\tilde E_2\sin\Delta_2 + \tilde r_{12}^{(3)}\tilde E_1\tilde E_3\sin\Delta_1\right)P_1, \tag{10}$$

where $\Delta\Omega = \sum_{m=0}^{3}\frac{\Delta r^{(m)}}{2\hbar}E_m^2 + \Delta\omega + 2\frac{\partial\varphi_0}{\partial t}$, $\tilde E_m = \frac{E_m}{A_0}$, $\tilde t = \frac{t}{\tau_0}$, $\tilde r_{12}^{(1)} = \frac{r_{12}^{(1)}\tau_0A_0^2}{\hbar}$, $\tilde r_{12}^{(2)} = \frac{r_{12}^{(2)}\tau_0A_0^2}{\hbar}$,

$\tilde r_{12}^{(3)} = \frac{r_{12}^{(3)}\tau_0A_0^2}{\hbar}$; A_0 is the maximum pulse amplitude; τ_0 is the pulse width.

To make the system (8) - (10) complete we add Maxwell's equations for the all real "slowly-varying amplitudes" $\tilde E_{0,1,2,3}$ and their phases $\varphi_{0,1,2,3}$. We obtain

$$\frac{\partial\tilde E_0}{\partial\tilde z} + \frac{1}{\tilde z_0^z}\frac{\partial\tilde E_0}{\partial\tilde t} = -2\tilde\alpha_0\tilde E_0P_2, \tag{11}$$

$$\frac{\partial\varphi_0}{\partial\tilde z} + \frac{1}{\tilde v_0^z}\frac{\partial\varphi_0}{\partial\tilde t} = -\tilde\beta_0n - \tilde\gamma_0 - 2\tilde\alpha_0P_1, \tag{12}$$

$$\frac{\partial\tilde E_1}{\partial\tilde z} + \frac{1}{\tilde v_1^z}\frac{\partial\tilde E_1}{\partial\tilde t} = -\tilde\alpha_{12}\tilde E_2\left(-P_2\cos\Delta_2 - P_1\sin\Delta_2\right) - \tilde\alpha_{13}\tilde E_3\left(P_2\cos\Delta_1 + P_1\sin\Delta_1\right), \tag{13}$$

$$\frac{\partial\varphi_1}{\partial\tilde{z}}+\frac{1}{\tilde{v}_1^z}\frac{\partial\varphi_1}{\partial\tilde{t}}=-\tilde{\beta}_1 n-\tilde{\gamma}_1-\tilde{\alpha}_{12}\frac{\tilde{E}_2}{\tilde{E}_1}\left(P_1\cos\Delta_2-P_2\sin\Delta_2\right)-\tilde{\alpha}_{13}\frac{\tilde{E}_3}{\tilde{E}_1}\left(P_1\cos\Delta_1-P_2\sin\Delta_1\right), \tag{14}$$

$$\frac{\partial\tilde{E}_2}{\partial\tilde{z}}+\frac{1}{\tilde{v}_2^z}\frac{\partial\tilde{E}_2}{\partial\tilde{t}}=-\tilde{\alpha}_{21}\tilde{E}_1\left(P_2\cos\Delta_2+P_1\sin\Delta_2\right), \tag{15}$$

$$\frac{\partial\varphi_2}{\partial\tilde{z}}+\frac{1}{\tilde{v}_2^z}\frac{\partial\varphi_2}{\partial\tilde{t}}=-\tilde{\beta}_2 n-\tilde{\gamma}_2-\tilde{\alpha}_{21}\frac{\tilde{E}_1}{\tilde{E}_2}\left(P_1\cos\Delta_2-P_2\sin\Delta_2\right), \tag{16}$$

$$\frac{\partial\tilde{E}_3}{\partial\tilde{z}}+\frac{1}{\tilde{v}_3^z}\frac{\partial\tilde{E}_3}{\partial\tilde{t}}=-\tilde{\alpha}_{31}\tilde{E}_1\left(P_2\cos\Delta_1+P_1\sin\Delta_1\right), \tag{17}$$

$$\frac{\partial\varphi_3}{\partial\tilde{z}}+\frac{1}{\tilde{v}_3^z}\frac{\partial\varphi_3}{\partial\tilde{t}}=-\tilde{\beta}_3 n-\tilde{\gamma}_3-\tilde{\alpha}_{31}\frac{\tilde{E}_1}{\tilde{E}_3}\left(P_1\cos\Delta_1-P_2\sin\Delta_1\right), \tag{18}$$

where $\tilde{z}=\frac{z}{z_0}$, $\tilde{v}_{0,1,2,3}^z=\frac{v_{0,1,2,3}^z}{c}$, $\tilde{\alpha}_{0,12,21,31}=\alpha_{0,12,21,31}c\tau_0$, $\alpha_0=\frac{2\pi\omega_0 N}{cn_0}r_{12}^{(1)}$, $\alpha_{12}=\frac{2\pi\omega_1 N}{cn_1}r_{12}^{(2)}$, $z_0=c\tau_0$, $\alpha_{21}=\frac{2\pi\omega_2 N}{cn_2}r_{12}^{(2)}$, $\alpha_{31}=\frac{2\pi\omega_3 N}{cn_3}r_{12}^{(2)}$, $\beta_0=\frac{2\pi\omega_0 N}{cn_0}\Delta r^{(0)}$, $\beta_1=\frac{2\pi\omega_1 N}{cn_1}\Delta r^{(1)}$, $\beta_2=\frac{2\pi\omega_2 N}{cn_2}\Delta r^{(2)}$, $\beta_3=\frac{2\pi\omega_3 N}{cn_3}\Delta r^{(3)}$, $\gamma_0=\frac{2\pi\omega_0 N}{cn_0}\kappa^{(0)}$, $\gamma_1=\frac{2\pi\omega_1 N}{cn_1}\kappa^{(1)}$, $\gamma_2=\frac{2\pi\omega_2 N}{cn_2}\kappa^{(2)}$, $\gamma_3=\frac{2\pi\omega_3 N}{cn_3}\kappa^{(3)}$, $\tilde{\beta}_{0,1,2,3}=\beta_{0,1,2,3}c\tau_0$, $\tilde{\gamma}_{0,1,2,3}=\gamma_{0,1,2,3}c\tau_0$, N is the number of molecules in cm^3, $c=3\times10^{10}$ cm/s.

3. Transition Regime

We carried out the computer simulation of the system (8) - (10) and (11) - (18) for the following parameters of electromagnetic radiation and medium (gases): $\omega=10^{15}\ \text{s}^{-1}$, $N=10^{19}\ \text{cm}^{-3}$, $A_0=10^4$ esu $\left(I=10^{11}\ \text{W/cm}^2\right)$, $\mu=10^{-18}$ esu, $r=10^{-24}\ \text{cm}^3$, and $\tau_0=10^{-11}$ s [27]-[29]. The optical pulses on the pump and trigger frequencies $\omega_{0,1}$ were chosen to be of Gaussian shape. The accuracy of numerical results was based upon monitoring the conservation of energy of the system at every cross-sectional area in the medium. We considered the following trends: the phase locking and confinement; the length of medium needed for soliton formation; the distribution of energy during the transition regime; the energy limitations on solitons formation; the effect of slowing down of solitons in resonant systems; the relationship between the soliton speed and characteristics of incoming electromagnetic waves and nonlinear medium; the conditions leading to formation solitary wave at one frequency (instead of generation of sequence of them); the connection between the amplitude of soliton and parameters upon consideration. The space-time evolution of the normalized intensities ($A=\tilde{E}_0^2\left(\tilde{z},\tilde{t}\right)/\tilde{E}_{0,\max}^2$, $B=\tilde{E}_1^2\left(\tilde{z},\tilde{t}\right)/\tilde{E}_{1,\max}^2$, $C=\tilde{E}_2^2\left(\tilde{z},\tilde{t}\right)/\tilde{E}_{2,\max}^2$, and $D=\tilde{E}_3^2\left(\tilde{z},\tilde{t}\right)/\tilde{E}_{3,\max}^2$) is shown in **Figures 1-4**.

4. Stability

To investigate the stability of solitons we perform the summation of the Equations (12) (14) (16) and (18) for phases $\varphi_{0,1,2,3}$ and transform them to the equations for $\Delta_{1,2}$. We assume that both processes $\omega_{2,3}=2\omega_0\pm\omega_1$ occur at the conditions of synchronism, so that $k_3^7\approx 2k_0^7-k_1^7$ and $k_2^7\approx 2k_0^7+k_1^7$. This condition is usually satisfied in gases [29]. Moreover, we also suggest that the phase differences $\Delta_{1,2}$ are locked to $2\pi n$ or $\left(2n+1\right)\pi$ due to the nonlinear effects [30]. In this case $P_1=0$ ($\Delta\Omega\tau_0\ll 1$). Let $\Delta_{1,2}$ be $\Delta_{1,2}=2\pi n+\tilde{\Delta}_{1,2}$ or $\Delta_{1,2}=\left(2n+1\right)\pi+\tilde{\Delta}_{1,2}$, where $\tilde{\Delta}_{1,2}$ are some small phase fluctuations. Finally, the modified system of (12) (14) (16) and (18) can be written in terms of $\tilde{\Delta}_{1,2}$ as follows

$$\frac{\mathrm{d}\tilde{\Delta}_1}{\mathrm{d}\tilde{\xi}}=\left(\pm1\right)\left(f_{11}\sin\tilde{\Delta}_1+f_{12}\sin\tilde{\Delta}_2\right)\sin\Phi, \tag{19}$$

$$\frac{\mathrm{d}\tilde{\Delta}_2}{\mathrm{d}\tilde{\xi}}=\left(\pm1\right)\left(f_{21}\sin\tilde{\Delta}_1+f_{22}\sin\tilde{\Delta}_2\right)\sin\Phi, \tag{20}$$

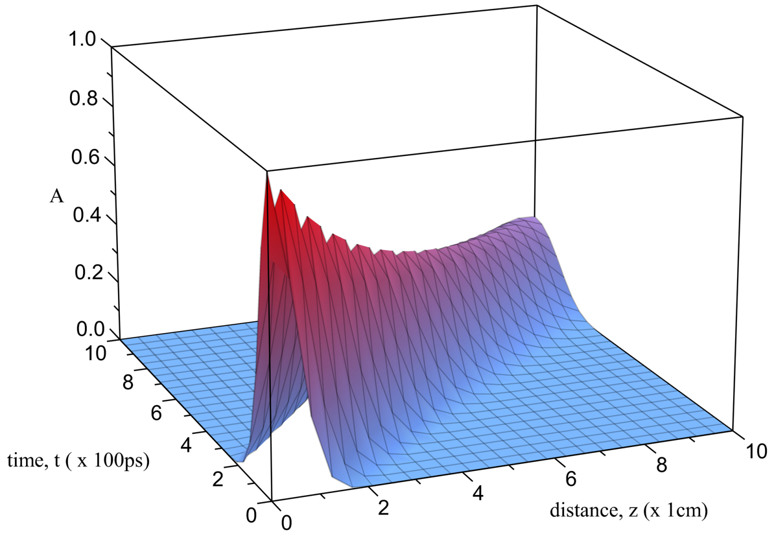

Figure 1. The formation of the soliton at frequency ω_0.

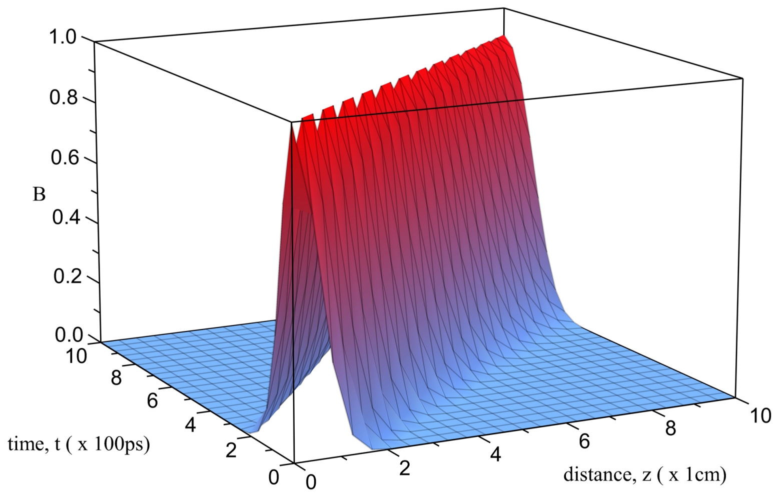

Figure 2. The formation of the soliton at frequency ω_1.

where $f_{11} = \dfrac{\kappa_3\tilde{\alpha}_{31}p + \kappa_1\tilde{\alpha}_{13}a_{31}}{2\sqrt{pa_{31}}}$, $f_{12} = \dfrac{\kappa_1\tilde{\alpha}_{12}a_{21}}{2\sqrt{pa_{31}}}$, $f_{21} = -\dfrac{\kappa_1\tilde{\alpha}_{13}a_{31}}{2\sqrt{pa_{31}}}$, $s\left(\cos\Delta_{1,2}\right) = \pm 1$,
$f_{22} = \dfrac{\kappa_2\tilde{\alpha}_{21}pa_{31} - \kappa_1\tilde{\alpha}_{12}a_{21}^2}{2a_{21}\sqrt{pa_{31}}}$, $\tilde{\xi} = \tilde{t} - \dfrac{\tilde{z}}{\tilde{v}^z}$, $\tilde{v}^z$ is the simulation velocity, $a_{12} = \dfrac{1}{2}\kappa_1\tilde{\alpha}_{12}s\left(\cos\Delta_2\right)$,
$a_{13} = -\dfrac{1}{2}\kappa_1\tilde{\alpha}_{13}s\left(\cos\Delta_1\right)$, $a_{21} = -\dfrac{1}{2}\kappa_2\tilde{\alpha}_{21}s\left(\cos\Delta_2\right)$, $a_{31} = -\dfrac{1}{2}\kappa_3\tilde{\alpha}_{31}s\left(\cos\Delta_1\right)$, $p = \left(a_{12}a_{21} + a_{13}a_{31}\right)/a_{31}$,
$\dfrac{1}{\kappa_{0,1,2,3}} = \dfrac{1}{\tilde{v}_{0,1,2,3}^z} - \dfrac{1}{\tilde{v}^z}$, $\sin\Phi = \left(-2C_0\tilde{\xi}\right)\Big/\left(1 + \left(C_0\tilde{\xi}\right)^2\right), C_0 = 2\sqrt{a_{31}p}.$

The behaviour of the latter system is analyzed in terms of phase planes. As an example, **Figure 5** shows the phase plane of the following system

$$\frac{d\tilde{\Delta}_1}{d\tilde{\xi}} = \left(\sin\tilde{\Delta}_1 + \sin\tilde{\Delta}_2\right)\sin\Phi, \tag{21}$$

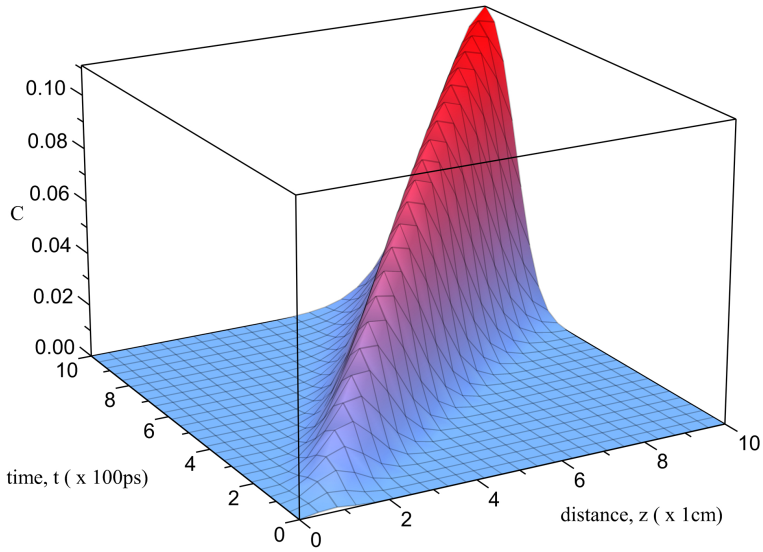

Figure 3. The generation of the soliton at frequency ω_2.

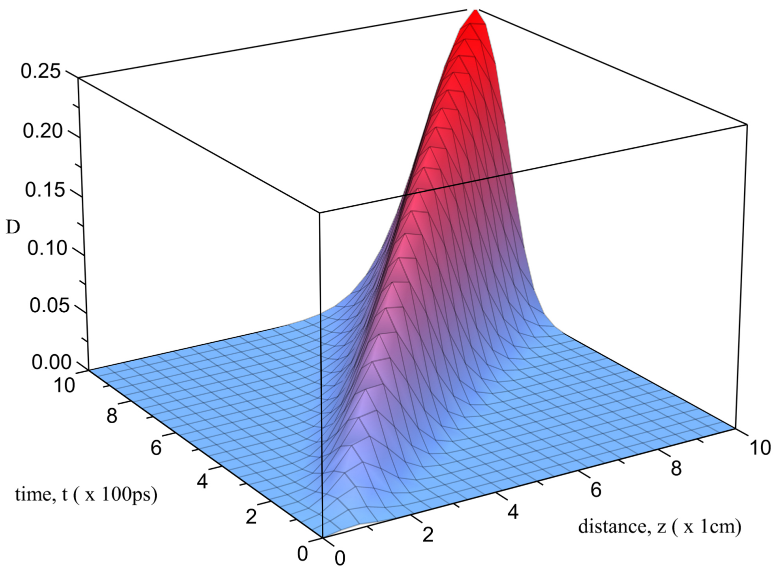

Figure 4. The generation of the soliton at frequency ω_3.

$$\frac{d\tilde{\Delta}_2}{d\tilde{\xi}} = \left(-\sin\tilde{\Delta}_1 + \sin\tilde{\Delta}_2\right)\sin\Phi, \tag{22}$$

at 11 different initial conditions for $\tilde{\Delta}_{1,2}$.

5. Conclusion

The space-time evolution of the optical pulses by using the computer simulation of transition regimes of four-photon resonant parametric processes in case of two-photon resonance is investigated. The computer simulation was based on application of the finite difference methods to the system of nonlinear equations modeling the foregoing interactions. It is shown that at certain boundary conditions (those result from the "area theorem" (see, e.g. [30])) the incoming laser pulses at frequencies $\omega_{0,1}$ first generate new waves at $\omega_{2,3}$, and then all become simultons of Lorentzian shape. It has also been shown that upon the conditions of phase locking ($\Delta_{1,2} = 2\pi n + \tilde{\Delta}_{1,2}$

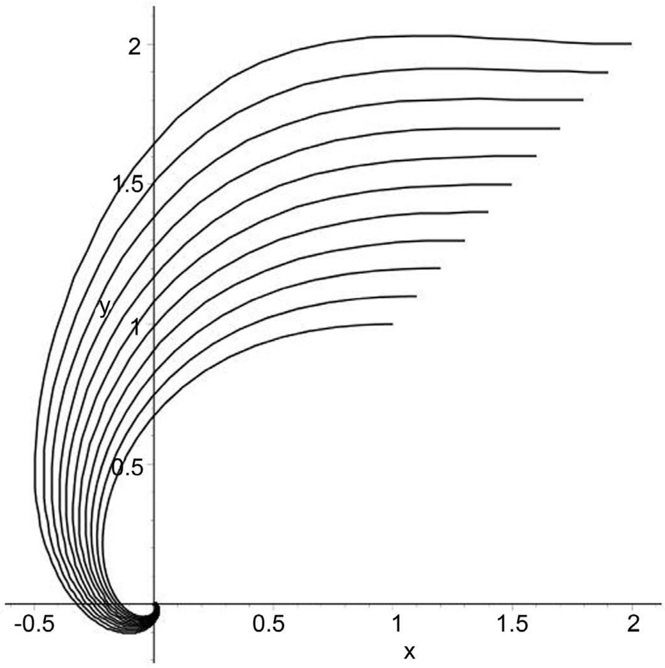

Figure 5. The phase plane of the system (21) and (22) ($x = \tilde{\Delta}_1$ and $y = \tilde{\Delta}_2$) for 11 consecutive initial conditions for $\tilde{\Delta}_{1,2}$: $\tilde{\Delta}_{1,2}$ = 2.0, 2.0; 1.9, 1.9; 1.8, 1.8; 1.0, 1.0.

or $\Delta_{1,2} = (2n+1)\pi + \tilde{\Delta}_{1,2}$) and synchronism ($k_3^7 \approx 2k_0^7 - k_1^7$ and $k_2^7 \approx 2k_0^7 + k_1^7$) in wide range of typical values of polarizabilities, the simultons are stable. These results could be useful for the applications related with designing the lossless communication systems using the tunable frequencies ranging from IR (ω_0) to UV ($2\omega_0 \div \omega_1$).

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