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A Finite Element Analysis on MHD Free Convection Flow in Open Square Cavity Containing Heated Circular Cylinder

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Abstract

The problem of Magnetohydrodynamic (MHD) free convection heat transfer in a square open cavity containing a heated circular cylinder at the centre has been investigated in this work. As boundary conditions of the cavity, the left vertical wall is kept at a constant heat flux, bottom and top walls are kept at different high and low temperature respectively. The remaining side wall is open. Finite element analysis based on Galerkin weighted Residual approach is used to visualize the temperature distribution and fluid flow solving two-dimensional governing mass, momentum and energy equations for steady state, natural convection flow in presence of magnetic field in side an open square cavity. A uniformly heated circular cylinder is located at the centre of the cavity. The object of this study is to describe the effects of MHD on the thermal fields and flow in presence of such heated circular cylinder by visualization of graph. The investigations are conducted for different values of Rayleigh number (*Ra*) and Hartmann number (*Ha*). The results show that the temperature field and flow pattern are significantly dependent on the above mentioned parameters.

Keywords

Free Convection, MHD, Heated Cylinder, Open Cavity, Finite Element Method

1. Introduction

The presence of magnetic field on the convective heat transfer and the natural convection flow of the fluid are of

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paramount importance in scientific and engineering research. Several numerical and experimental methods have been developed to investigate flow characteristics inside the cavities with and without obstacle. Because these types of geometries have practical engineering and industrial application, this type of problems of heat transfer attract significance attention of researchers since it's numerous application in the areas of energy conservations, cooling of electrical and electronic equipments, design of solar collectors, heat exchangers, etc. Many researchers have recently studied heat transfer in enclosures with partitions, fins and block which influence the convection flow nature. It is difficult to solve free convection problem in complicated bodies like it, which greatly influences the heat transfer process. A related application of MHD acceleration is to produce high energy wind tunnels for simulating hypersonic flight.

In the present work, we studied MHD free convection heat transfer and flow in a square open cavity containing a heated circular cylinder. The left vertical wall is kept at a constant heat flux. Bottom and top walls are kept at different high and low temperature respectively. The remaining side wall is open. Finite element analysis based on Galerkin weighted Residual method is used to solve the problem. Using the set of boundary conditions and values of parameters, it is observed that all isotherm lines are concentrated at right lower corner of the cavity and the magnetic field affects the heat flux inversely in the cavity.

Chan and Tien [1] investigated shallow open cavities and made a comparison study using a square cavity in an enlarged computational domain. In the result, they observed that for a square open cavity having an isothermal vertical side facing the opening and two adjoining adiabatic horizontal sides. Satisfactory heat transfer results could be obtained, especially at high Rayleigh numbers. Mohammad [2] investigated inclined open square cavities, by considering a restricted computational domain. The gradients of both velocity components were set to zero at the opening plane in that case which were different from that of Chan and Tien [1]. In the result, he found that heat transfer was not sensitive to inclination angle and the flow was unstable at high Rayleigh numbers and small inclination angles.

Ostrach [3], Davis [4], Hossain and Wilson [5], Hossain *et al*. [6] and Sarris *et al*. [7] studied MHD natural convection in a laterally and volumetrically heated square cavity. Their results show that the effect of increasing Hartmann number was not found to be straight forward connected with the resulting flow patterns. Roy and Basak [8] analyzed finite element method of natural convection flows in a square cavity with non-uniformly heated wall(s). S. Pervin and R. Nasrin [9], (2011) studied MHD free convection and heat transfer for different values of Rayleigh numbers *Ra* and Hartmann numbers *Ha* in a rectangular enclosure. Their results show that the flow pattern and temperature field are significantly dependent on the used parameters. Sheikh Anwar Hossain and Alim [10] studied Effects of Natural Convection from an open square cavity containing a heated circular cylinder.

S. Saha [11] studied thermo-magnetic convection and heat transfer of paramagnetic fluid in an open square cavity with different boundary conditions. His results show the Effects of Magnetic Rayleigh number, Prandtl number on the flow pattern and isotherm as well as on the heat absorption graphically. He found that the heat transfer rate is suppressed in decreased of the Magnetic Rayleigh number.

The study related to heat absorption or rejection in the confined rectangular enclosures has been well discussed in the literature C. Taylor and P. Hood [12], Chandrasekhar [13], Dechaumphai [14].

However, a comparatively little work has been done in the case of open square cavities. The reason might be the complexity on using the boundary conditions at the open side. The terms magneto hydrodynamic, hydrodynamics, magneto gas dynamics and magneto aerodynamics all are the branches of fluid dynamics that deals with the motion of electrically conducting fluids in presence of electric and magnetic fields. In a magnetic field the moving electric charge carried by a flowing fluid velocity and acting in the opposite direction, it is also very small. So the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from free stream pressure gradient.

2. Physical Model

A schematic diagram of the system considered in the present study is shown in **Figure 1**. The system consists of an open square cavity with sides of length L and heated circular cylinder of diameter D is located at the center of the cavity. A Cartesian co-ordinate system is used with origin at the lower left corner of the computational domain. A constant heat flux q is considered at the left wall of the cavity. The bottom wall is kept high temperature T_h and top wall is kept at low temperature T_c. The remaining right side wall is open. The temperature at the cylinder T_{h1} is less than that of the bottom wall. A magnetic field of strength B_0 is applied horizontally normal to the side walls.

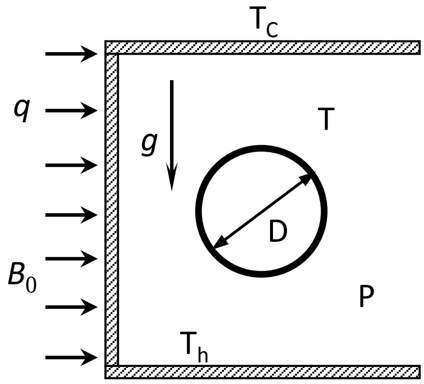


Figure 1. Schematic diagram of the problem.

3. Mathematical Formulation

The governing equation of MHD natural convection is given by the differential equation expressing conservation of mass or continuity equations, conservation of momentums and conservation of energy. In this case, flow is considered as steady, laminar, incompressible, two-dimensional and the buoyancy force. The Boussinesq approximation is used to relate density changes to temperature changes in the fluid properties and to couple in this way the temperature field to the flow field. The steady natural convection can be governed by the following differential equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\rho\beta\left(T_h-T_c\right)-\sigma B_0^2\upsilon \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Boundary conditions:

At Bottom wall: $u=v=0; T=T_h;\quad T_h\left(x,0\right)=374\text{ K}$ (say); $372\text{ K}\le T_h\le 375\text{ K}$.

At Top wall: $u=v=0; T=T_c;\quad T_c\left(x,t\right)=275\text{ K}$; $272\text{ K}\le T_c\le 278\text{ K}$.

At the Left wall: $u = v = 0$, and heat flux q =100 w/m^2, $p = 0$.

At the right side & open side: Convective Boundary Condition (CBC), $p = 0$, $u = v$.

At the circular cylinder $u\left(x,y\right)=v\left(x,y\right)=0$, $T\left(x,y\right)=T_{h_1}$.

$$\frac{\partial T\left(x,0\right)}{\partial y}=\frac{\partial T\left(x,L\right)}{\partial y}=\frac{\partial T\left(L,y\right)}{\partial x}=0.$$

At the circular cylinder $u\left(x,y\right)=v\left(x,y\right)=0$, $T\left(x,y\right)=T_{h_1}$.

At the right side & open side: Convective Boundary Condition (CBC), $p = 0$.

where x and y are the distances measured along the horizontal and vertical directions respectively; u and v are the velocity components in the x and y direction respectively; T denotes the temperature in Kelvin scale of measurement; γ and α are the kinematic viscosity and the thermal diffusivity respectively; p is the pressure and ρ is the density.

Governing Equations in Non-Dimensional Form

We can nondimensionalize the governing equations using the following scales.

Non-dimensional scales:

$$X=\frac{x}{L}, Y=\frac{y}{L}, U=\frac{u}{U_0}, V=\frac{v}{U_0},\quad P=\frac{pL^2}{\rho\alpha^2},\quad \theta=\frac{T-T_c}{T_h-T_c},\quad Pr=\frac{\nu}{\alpha},$$

$$Ra=\frac{gB\left(T_h-T_c\right)L^3}{\upsilon\alpha},\quad Ha=\sqrt{\frac{\sigma L^3B_0^2}{\mu}},\quad dr=\frac{D}{L},\quad \Delta T=\left(T_h-T_\infty\right),\quad \Delta T=\frac{qL}{K}.$$

Non-dimensional governing equations:

Continuity equation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{5}$$

Momentum equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+Pr\left(\frac{\partial^2U}{\partial X^2}+\frac{\partial^2U}{\partial Y^2}\right) \tag{6}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+Pr\left(\frac{\partial^2V}{\partial X^2}+\frac{\partial^2V}{\partial Y^2}\right)+RaPr\theta-Ha^2PrV \tag{7}$$

Energy equation:

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \tag{8}$$

where $Pr=\frac{\nu}{\alpha}$ is Prandtl number, $Ra=\frac{gB\left(T_h-T\right)L^3}{\upsilon\alpha}$ is Rayleigh number and $Ha=\sqrt{\frac{\sigma L^3B_0^2}{\mu}}$ is Hartmann number, $Gr=\frac{gB\left(T_h-T\right)L^3}{\upsilon^2}$.

Boundary conditions

At Bottom wall: $U=V=0;\theta=1$. At Top wall: $U=V=0;\theta=0$.

At the left wall: $U=V=0$; heat flux q = 100 w/m^2, p = 0.

At the right side & open side: Convective Boundary Condition (CBC), P = 0.

The Nusselt number for natural convection is a function of the Grash of number only. The local Nusselt number *Nu* can be obtained from the temperature field by applying the function

$$Nu=-\frac{1}{\theta\left(0,Y\right)}.$$

The overall or average Nusselt number was calculated by integrating the temperature gradient over the heated wall as follows:

$$Nu_{av}=-\int_0^1\frac{1}{\theta\left(0,Y\right)}\mathrm{d}y$$

Since the dimensionless Prandtl Number *Pr* is the ratio of kinematic viscosity to thermal diffusivity. So *Pr* is a heat transfer characteristics in the flow field of natural convection.

4. Numerical Technique

The numerical technique used in this study is based on the Galerkin weighted residual method of finite element formulation. The application of this technique is well described by Tailor and Hood [12] and Dechaumphai [14]. Here the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then, the nonlinear governing partial differential equations (*i.e.* mass, momentum and energy equations) are transferred into a system of integral equations by applying the Galerkin weighted residual method. In this case, the integration over each term of these equations is performed by using Gauss's quadrature method and nonlinear algebraic equations are obtained. These nonlinear algebraic equations are modified by imposing boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. At last, these linear equations are solved by using triangular factorization method.

5. Results and Discussion

Finite element simulation is applied to perform the analysis of laminar free convection heat transfer and fluid flow in an open square cavity containing a heated circular cylinder. Effects of the parameters Rayleigh number (*Ra*), Hartmann number (*Ha*) and heat flux *q* on heat transfer and fluid flow inside the cavity has been studied. The visualization focused on temperature and flow fields, which contains isotherms and streamlines for the cases. The range of *Ra* and *Ha* for this investigation vary from 10^3 to 10^4 and 0 to 400 respectively while $Pr = 0.72$ & heat flux $q = 100$.

The flow with all *Ra* in this work has been affected by the buoyancy force. **Figures 2(a)-8(a)** illustrate temperature field in the flow region and in **Figures 2(b)-8(b)** illustrate streamlines in the flow field. In **Figures 2(a)-8(a)**, the high temperature region remains below the circular cylinder and the isothermal lines are nonlinear for all *Ra* used in this work and they occupied almost right half of the region in the cavity. The influence of $Ha = 0$, 75, 150, 225 on isotherms as well as on streamlines for the present configuration at $Ra = 1000$, $q = 100$ has been demonstrated in **Figure 2**. In **Figure 2(b)**, a recirculation is formed around the cylinder and one small vortex is formed below the cylinder in the cavity for $Ha = 225$. The recirculation region is decreased for $Ha = 150$. For this case, isothermal lines are concentrated at the right corner of bottom side. **Figure 3** shows the effects for $Ha = 100$, 200, 300, 400 on isotherms as well as on streamlines for the present configuration at $Ra = 10^3$, $q = 100$. In this case, the isothermal lines are concentrated at the right corner of the bottom side. In **Figure 3(b)**, a recirculation is formed around the cylinder and one small vortex is formed below the cylinder in the cavity for $Ha = 200$, 400. The recirculation region is decreased for $Ha = 400$. **Figure 4** shows the effects of $Ha = 75$, 150, 225, 300 on isotherms as well as on streamlines for the present configuration at $Ra = 10^3$, $q = 100$. In **Figure 4(b)**, a recirculation is formed around the cylinder at every *Ha* and one small vortex is formed in the cavity for $Ha = 225$. In **Figure 4(a)**, the isothermal lines are concentrated at the right lower corner of the cavity for all *Ha*. **Figure 5** shows the effects for $Ha = 0$, 100, 200 & 300 on isotherms as well as on streamlines for the present configuration at $Ra = 10^4$, $q = 100$. In **Figure 5(b)**, a recirculation is formed around the cylinder for every *Ha*. The recirculation region is increased for $Ha = 100$. The isothermal lines are concentrated at the right lower corner of the cavity for every *Ha*. **Figure 6** shows the effects for $Ha = 100$, 200, 300, 400 on isotherms as well as on streamlines for the present configuration at $Ra = 10^4$, $q = 100$. In **Figure 6(b)**, a recirculation is formed around the cylinder at every *Ha* and one small vortex is formed below the cylinder for $Ha = 200$. The recirculation region is increased for $Ha = 200$, 400. The isothermal lines are concentrated at the right corner of bottom side for every *Ha*. **Figure 7** shows the effects of $Ha = 75$, 150, 225 & 300 at $Ra= 10^4$ and heat flux $q = 100$. In this case, the isotherm lines are concentrated at right lower corner of the cavity for every *Ha* and the isotherm lines are located in the right half of the cavity. A recirculation is formed around the cylinder for every *Ha*. One small vortex is formed below the cylinder for $Ha = 225$. The recirculation region is increased for $Ha = 75$. The isothermal lines are concentrated at the right corner of bottom side for every *Ha*. **Figure 8** shows the effects for $Ha = 80$, 160, 240 & 320 on isotherms as well as on streamlines for the present configuration at $Ra = 10^4$, $q = 100$. Here, isotherm lines concentrate at lower right corner of the cavity for every *Ha*. In **Figure 8**, on small vortex is formed for $Ha= 240$, 320. One recirculation is formed around the cylinder for every *Ha*.

To evaluate how the presence of magnetic field affects the heat flux along the heated surface it is observed in the figures from **Figures 9-11** that Hartmann number inversely affects on heat flux. That is heat flux is maximum when *Ha* is minimum. If *Ha* rises then heat flux decreases. **Figure 11** shows that heat flux is highest when *Ha* is lowest, because the magnetic field tends to retard the fluid motion.

6. Conclusion

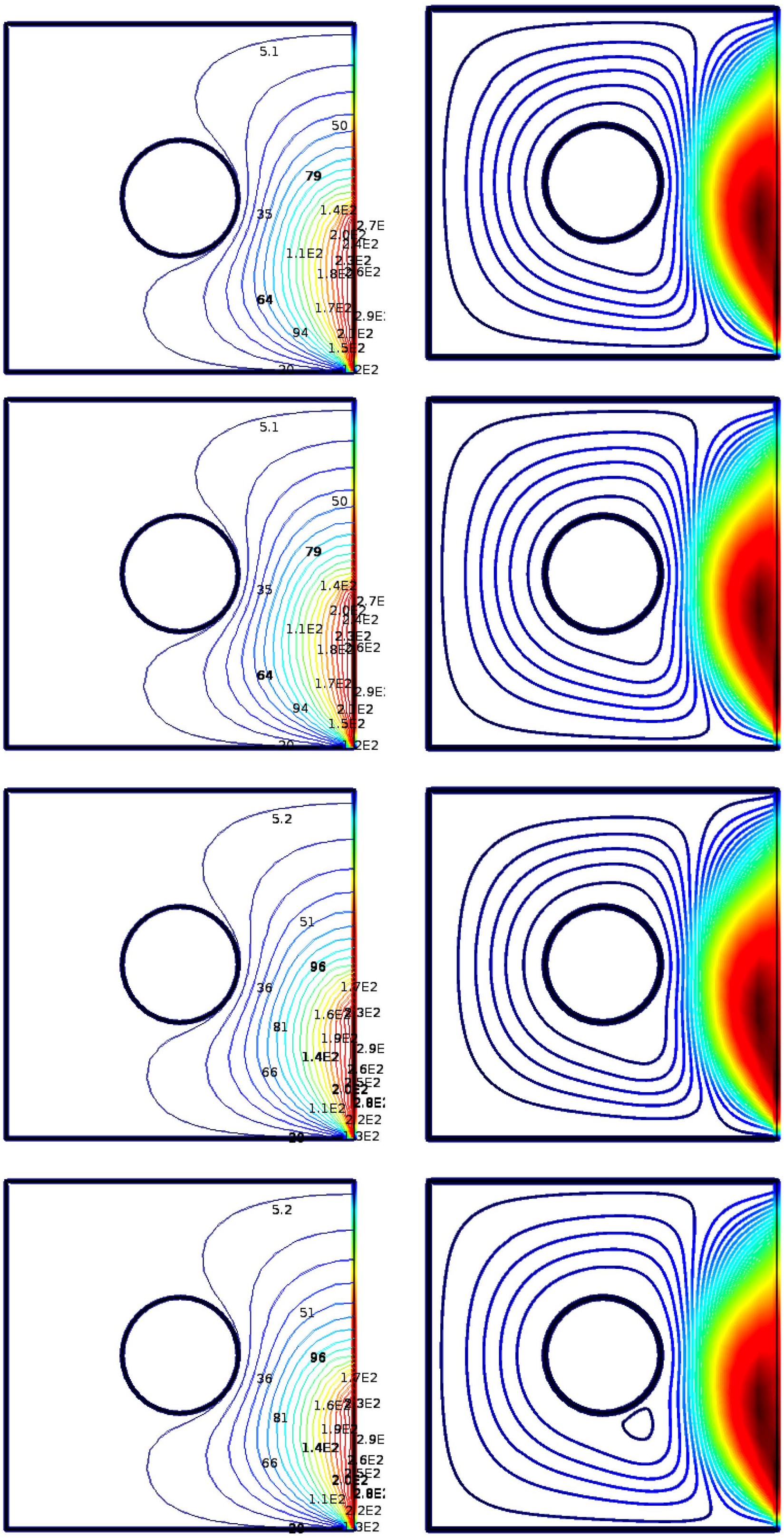
Finite element method is used to solve the present physical problem and analyze the effects of Hartmann number *Ha*, Rayleigh number *Ra*, heat flux *q* for steady-state, incompressible, laminar and MHD free convection flow in a square open cavity containing a heated circular cylinder. The flow with all *Ra* in this work has been affected by the buoyancy force. Temperature fields are illustrated in the flow region. The high temperature region remains

Ha = 0

Ha = 75

Ha = 150

Ha = 225



(a) (b)

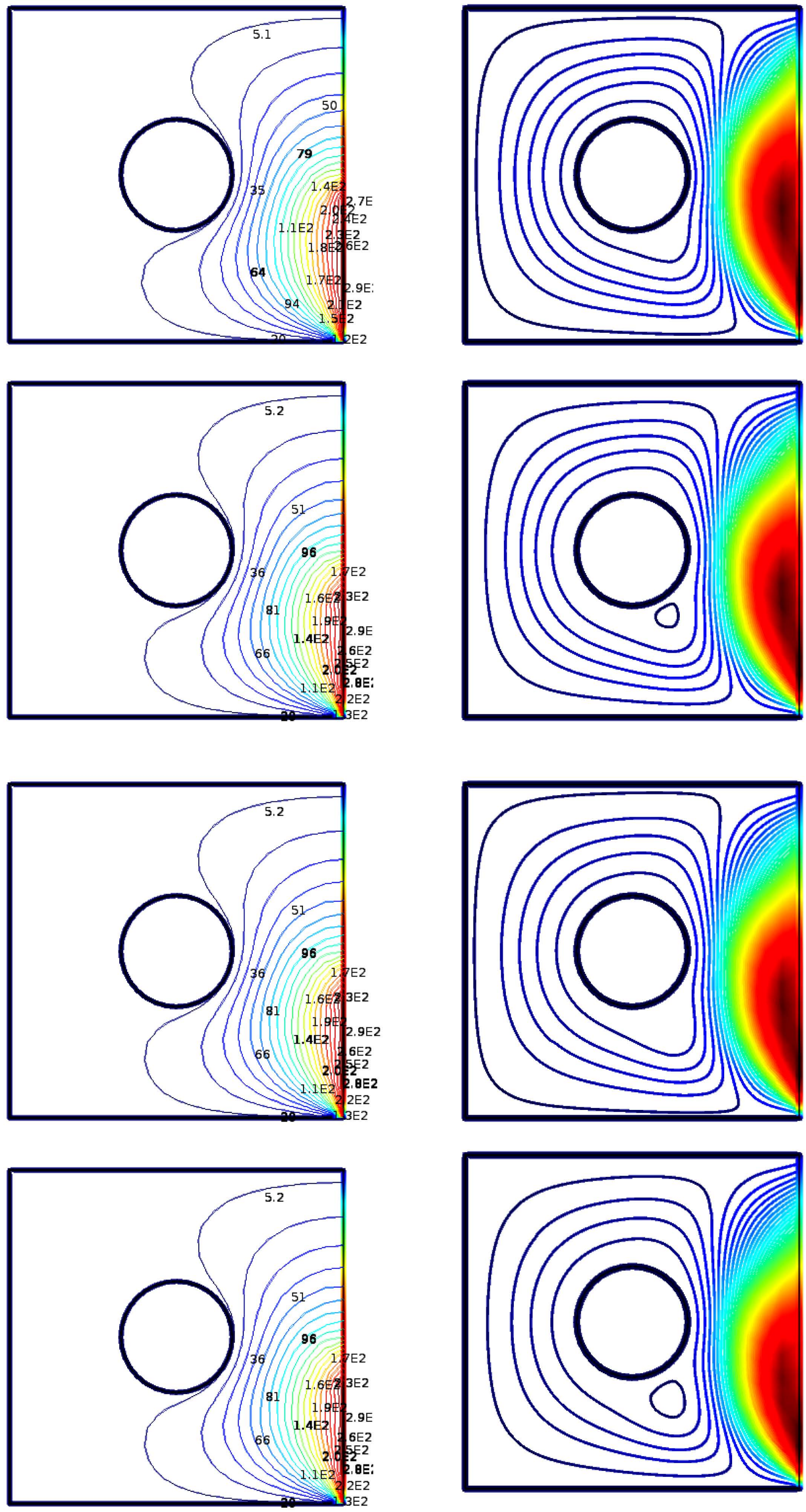
Figure 2. Isotherms (a) & streamlines (b) for various *Ha* and *Ra* = 1000, *q* = 100 in the cavity.

Ha = 100

Ha = 200

Ha = 300

Ha = 400



(a) (b)

Figure 3. (a) Isotherms & (b) streamlines in the cavity for various Ha and $Ra = 1000$, $q = 100$.

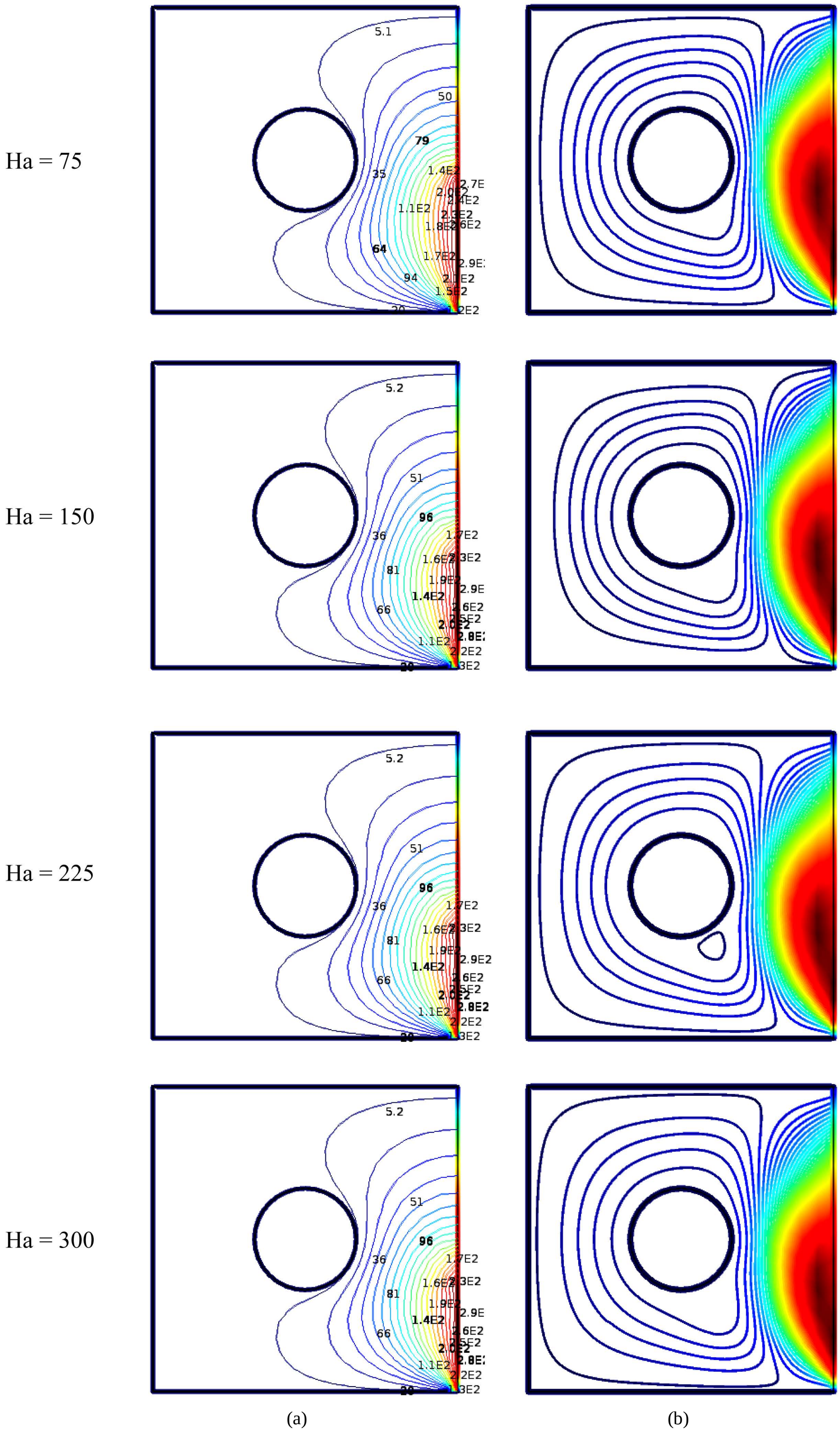


Figure 4. (a) Isotherms and (b) streamlines for various *Ha* while *Ra* = 1000 & heat flux = 100.

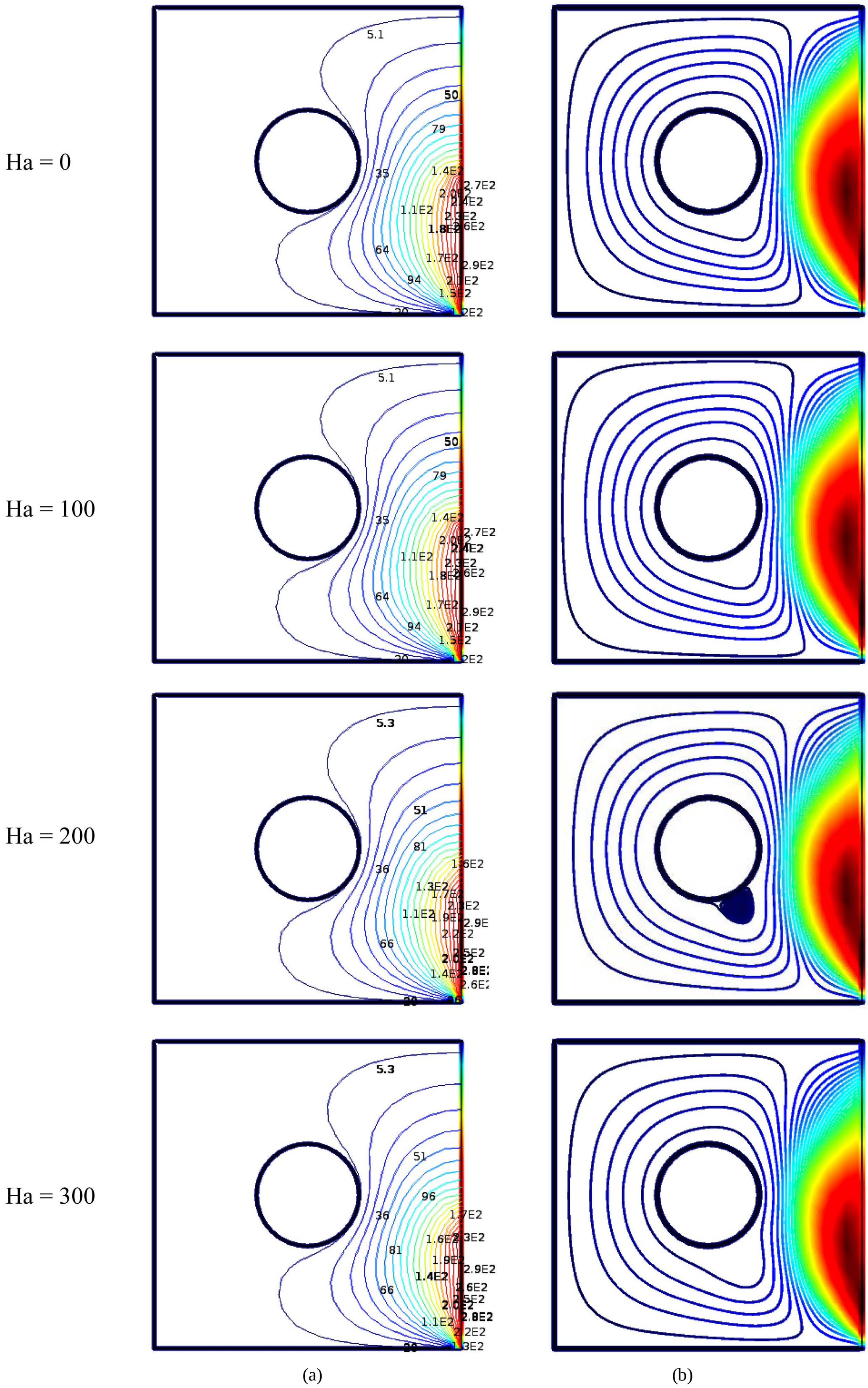


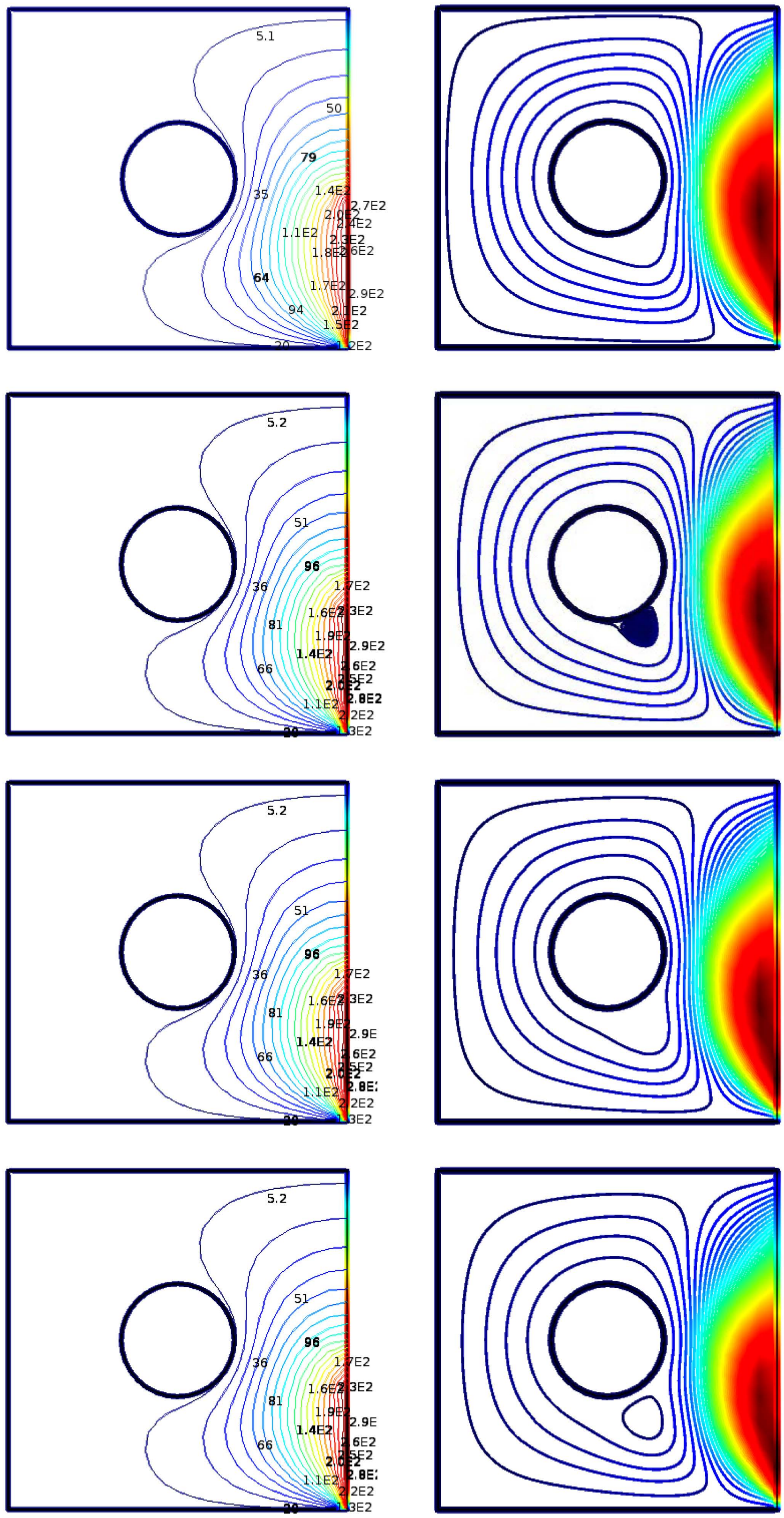
Figure 5. (a) Isotherms and (b) streamlines for various *Ha* while *Ra* = 10,000 & heat flux = 100.

Ha = 100

Ha = 200

Ha = 300

Ha = 400



(a) (b)

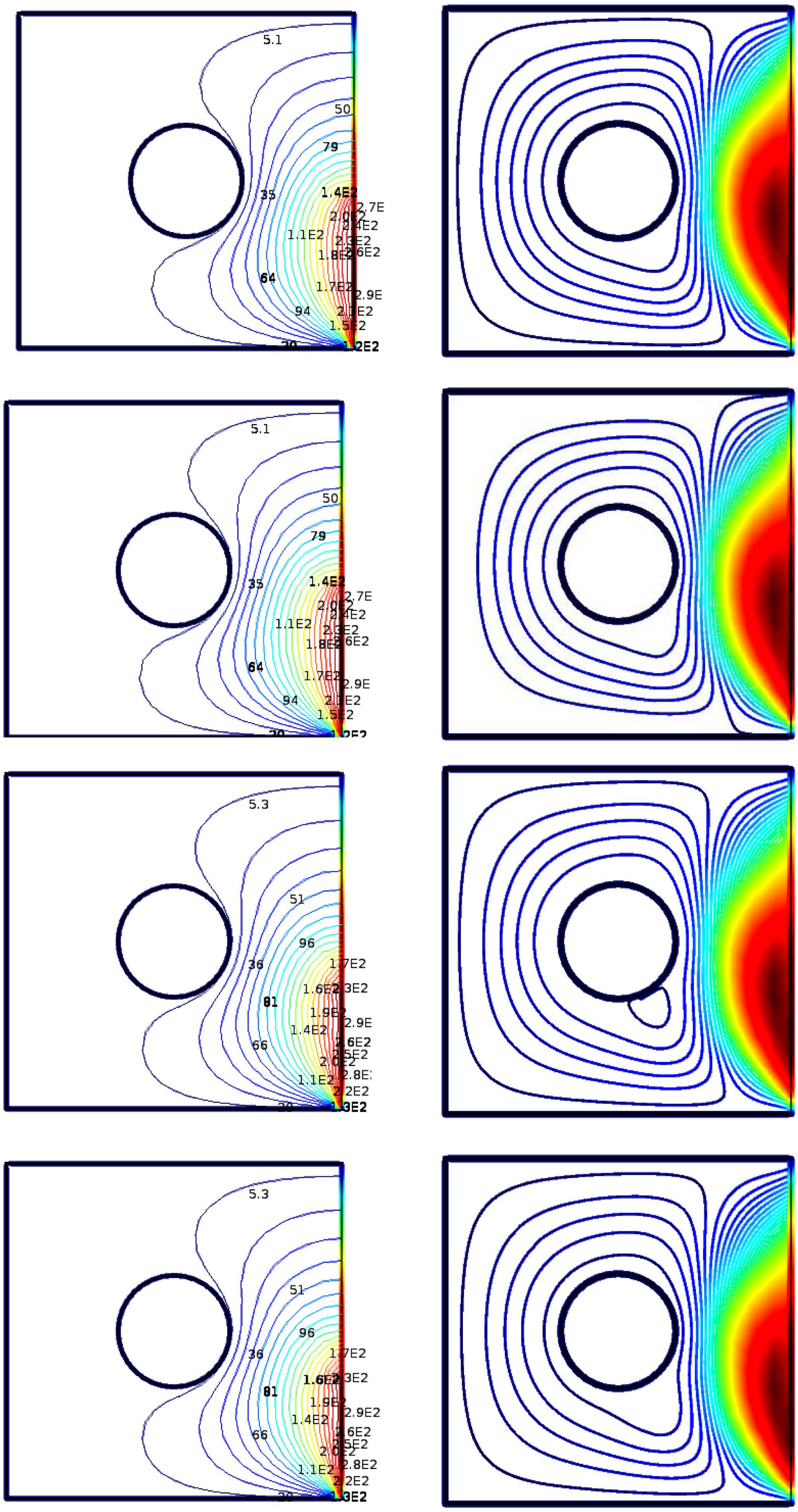
Figure 6. (a) Isotherms and (b) streamlines for various *Ha* while *Ra* = 10,000 & heat flux = 100.

Ha = 75

Ha = 150

Ha = 225

Ha = 300



(a) (b)

Figure 7. (a) Isotherms and (b) streamlines for various *Ha* while *Ra* = 10,000 & heat flux = 100.

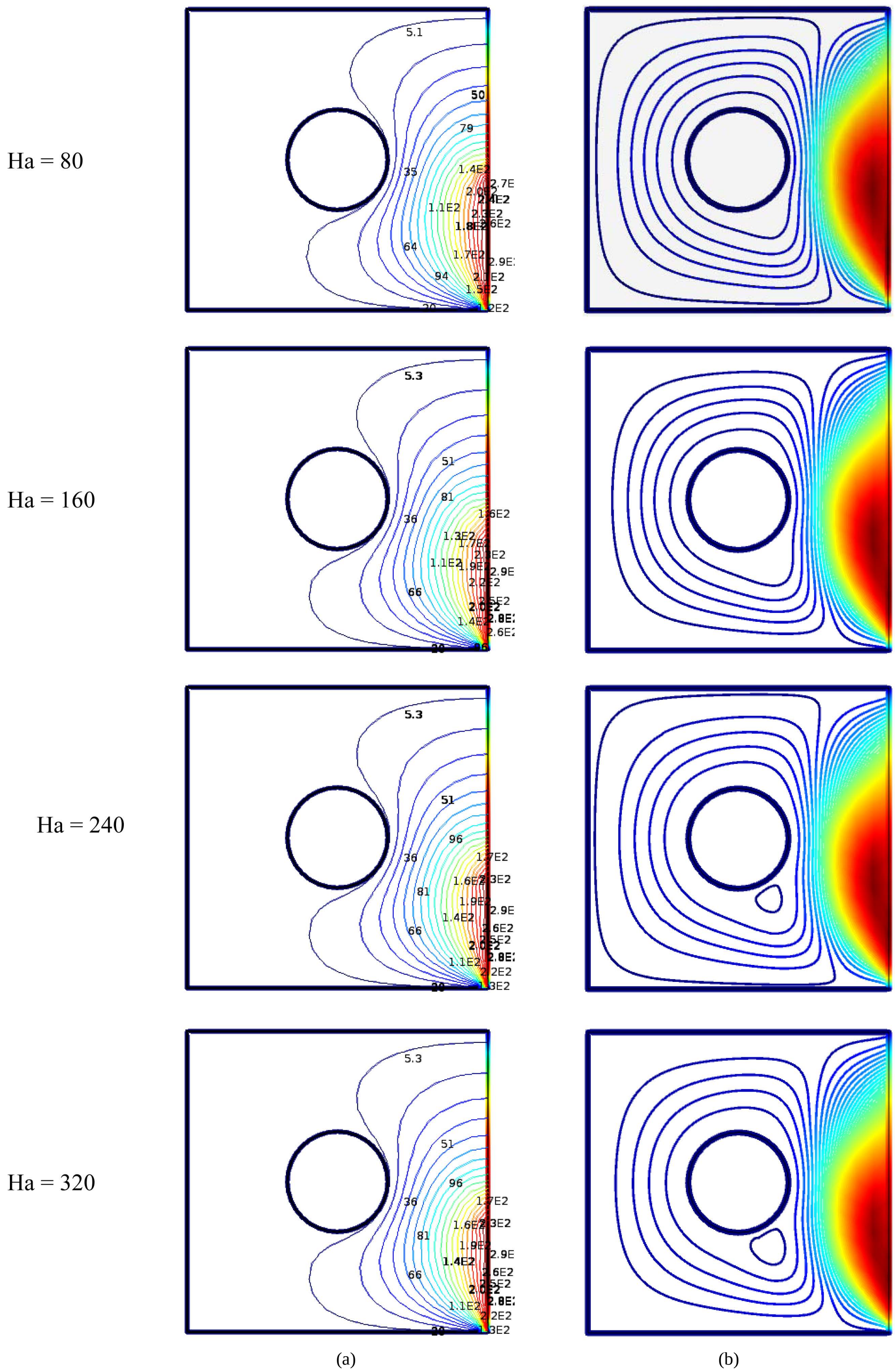


Figure 8. (a) Isotherms and (b) streamlines for various *Ha* & *Ra* = 1000, q = 100.

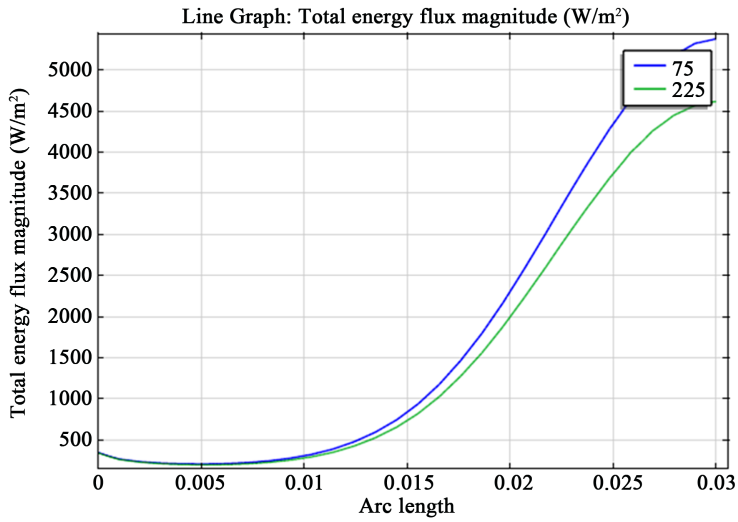


Figure 9. Line graph of heat flux at upper wall for *Ha* = 75,225 & *Ra* = 1000.

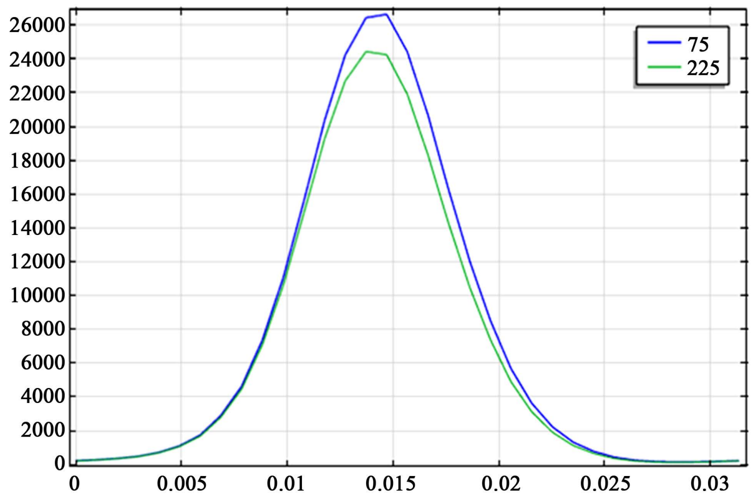


Figure 10. Line graph of heat flux at the cylinder for *Ha* = 75,225 & *Ra* = 1000.

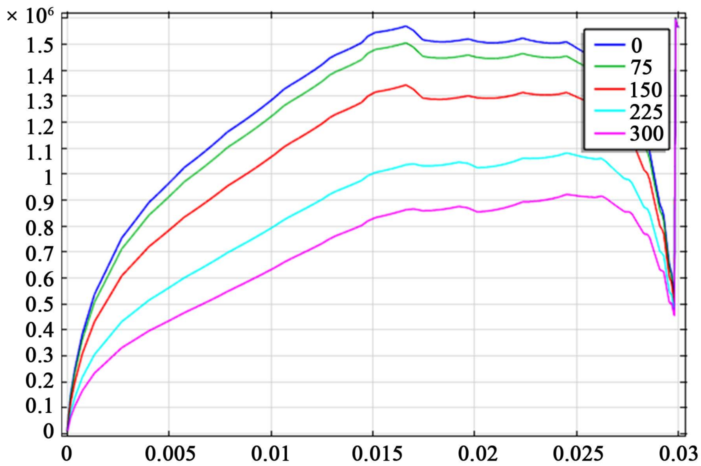


Figure 11. Line graph of heat flux at the open side for *Ha* = 0, 75, 150, 225, 300 & *Ra* = 10,000.

at the lower portion near the open side of the cavity and the isothermal lines are nonlinear for all *Ra* used in this works and they occupy almost half of the region of the cavity near the open side. The significant findings of this work are that for all cases of *Ha* and *Ra* the isothermal lines concentrate to the right lower corner of the cavity and there is a recirculation around the cylinder and one vortex has been created in the flow field. Magnetic field (*Ha*) inversely affects on heat flux. That is heat flux is maximum when *Ha* is lowest. If *Ha* rises, then heat flux decreases. That is the magnetic field tends to retard the fluid flow and the rate of heat transfer.

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