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A Five-Step P-Stable Method for the Numerical Integration of Third Order Ordinary Differential Equations

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Abstract

In this paper we derived a continuous linear multistep method (LMM) with step number k = 5 through collocation and interpolation techniques using power series as basis function for approximate solution. An order nine p-stable scheme is developed which was used to solve the third order initial value problems in ordinary differential equation without first reducing to a system of first order equations. Taylor's series algorithm of the same order was developed to implement our method. The result obtained compared favourably with existing methods.

Keywords

Continuous Collocation, Multistep Methods, Interpolation, Third Order, Power Series, Approximate Solution

1. Introduction

Linear multistep methods (LMM) for solving first order initial value problems (ivps) is of the form

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h \sum_{j=0}^{k} \beta_j f_{n+j} \tag{1}$$

where α_j and β_j are uniquely determined and $\alpha_0 + \beta_j \neq 0, \alpha_k = 1$

Conventionally, they are used to solve higher order ordinary differential equations by first reducing them to a system of first order. This approach has been extensively discussed in [1]-[5]. However the method of reducing to

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a system first order has some serious drawback which includes wastage of human effort and computer time [6].

The LMM in (1) generates discrete schemes which are used to solve first order odes. Various forms of this LMM have been developed [1]-[4]. Other researchers have introduced the continuous LMM using the continuous collocation and interpolation technique. This has led to the development of continuous LMM of form

$$Y(x)=\sum_{j=0}^{k}\alpha_j(t)y_{n+j}+h\sum_{j=0}^{k}\beta_j(t)f_{n+j} \tag{2}$$

α_j and β_j are expressed as continuous functions of t and are at least differentiable once.

The introduction of continuous collocation methods as against the discrete schemes enhances better global error estimation and ability to approximate solution at all interior points [6]-[10]. In this study, we shall develop continuous multistep collocation method for the solution of third order ordinary differential equations using power series as the basis function.

Power Series Collocation

In [6] [8] [9], some continuous LMM of Type (2) were developed using power series of form

$$Y(x)=\sum_{j=0}^{k}a_jx^j : \tag{3}$$

In [10] Chebyshev polynomial function of the form

$$Y(x)=\sum_{j=0}^{k}a_jT_j(x)\left(\frac{x-x_k}{h}\right) \tag{4}$$

where $T_j(x)$ are some Chebyshev function used to develop continuous LMM.

The use power series as basis function for derivation of continuous LMM are based on the property of analytic function that given the Taylor's polynomial of the form

$$P(x)=\sum_{r=0}^{n}\frac{y^r(x_0)(x-x_n)}{r!} \tag{5}$$

The approximate function $p(x)$ reduces to $Y(x)$ as $n\to\infty$

In this study we proposed the polynomial function of the form in [7]:

$$Y(x)=\sum_{j=0}^{k}a_j(x-x_n)^j \tag{6}$$

which is of Type (3) to develop a continuous LMM for the solution of initial value problem of the form:

$$y'''=f(x,y,y',y''),y(a)=y_0;\qquad y'(a)=\partial_0,y''(a)=\partial_N \tag{7}$$

This paper is organized as follows: Section 1 consists of introduction and background of study; Section 2, we derive a continuous approximation to $Y(x)$ for exact solution $y(x)$, and specific methods; Section 3 consists of the analysis and implementation followed by numerical examples.

2. Derivation of the Method

Consider the third order differential Equation (7), we proposed an approximate solution of the form:

$$y_k(x)=\sum_{j=0}^{M}a_j\psi_j(x) \tag{8}$$

where $\psi_j(x)=(x-x_n)^j, j\ge 0$.

The derivative of (8) up to the third order yield

$$y_k'''(x)=\sum_{j=0}^{k}j(j-1)(j-2)a_j\psi_{j-3}(x) \tag{9}$$

And $a \le x \le b$ where $a_j's$ are the parameters to be determined. By substituting (8) and (9) into (7) we have

$$\sum_{j=0}^{k} j(j-1)(j-2)a_j\psi_{j-3}(x) = f(x,y,y',y'') \tag{10}$$

Collocating (10) at $x = x_{n+i}, i = 0(1)m$ and interpolating (8) at $x = x_{n+i}, i = 0(1)m-1$ we obtained the system of equations given below

$$\sum_{j=0}^{M} a_j\psi_j(x_{n+i}) = y_{n+i} \tag{11}$$

$$\sum_{j=0}^{M} j(j-1)(j-2)a_j\psi_{j-3}(x) = f_{n+i} \tag{12}$$

The above equations are solved to obtain the values of $a_j's$ which when substituted into Equation (6) yield a method of the form in Equation (13)

$$Y(x) = \sum_{j=0}^{k-1}\alpha_j(t)y_{n+j} + h\sum_{j=0}^{k}\beta_j(t)f_{n+j} \tag{13}$$

The continuous polynomial obtained when the values of $a_j's$ are substituted into (6) and simplified is as follows

$$\begin{aligned}
Y(x) = {} & \frac{1}{59520h^{10}}\Big[59520h^{10} - 3305158h^9(x-x_n) + 5968895h^8(x-x_n)^2 - 8146800h^6(x-x_n)^4 \\
& +8869224h^5(x-x_n)^5 - 4685898h^4(x-x_n)^6 + 1424760h^3(x-x_n)h^7 - 253425h^2(x-x_n)^8 \\
& +24490h(x-x_n)^9 - 1024(x-x_n)^{10}\Big]y_n \\
& +\frac{1}{59520h^{10}}\Big[6211314h^9(x-x_n) - 11400365h^8(x-x_n)^2 + 156277600h^6(x-x_n)^4 \\
& -17084088h^5(x-x_n)^5 + 9069483h^4(x-x_n)^6 - 2770920h^3(x-x_n)^7 + 495075h^2(x-x_n)^8 \\
& -48030h(x-x_n)^9 + 1952(x-x_n)^{10}\Big]y_{n+1} \\
& +\frac{1}{59520h^{10}}\Big[898062h^9(x-x_n) - 158251h^8(x-x_n)^2 + 199800h^6(x-x_n)^4 - 1963080h^5(x-x_n)^5 \\
& +907074h^4(x-x_n)^6 - 235800h^3(x-x_n)^7 + 35325h^2(x-x_n)^8 - 2850h(x-x_n)^9 + 96(x-x_n)^{10}\Big]y_{n+2} \\
& +\frac{1}{59520h^{10}}\Big[-7311962h^9(x-x_n) + 13490545h^8(x-x_n)^2 - 18291600h^6(x-x_n)^4 \\
& +19701528h^5(x-x_n)^5 - 10278870h^4(x-x_n)^6 + 3085320h^3(x-x_n)^7 - 542175h^2(x-x_n)^8 \\
& +51830h(x-x_n)^9 - 2080(x-x_n)^{10}\Big]y_{n+3} \\
& +\frac{1}{399974400h^7}\Big[-160164924h^9(x-x_n) + 27846271h^8(x-x_n)^2 + 6666240h^7(x-x_n)^3 \\
& -50359720h^6(x-x_n)^4 + 518336784h^5(x-x_n)^5 - 269004260h^4(x-x_n)^6 + 81125040h^3(x-x_n)^7 \\
& -14366570h^2(x-x_n)^8 + 98910h(x-x_n)^9\Big] - 56000(x-x_n)^{10}\Big]f_n \\
& +\frac{1}{399974400h^7}\Big[-9863776238h^9(x-x_n) + 18418906560h^8(x-x_n)^2 - 25437064780h^6(x-x_n)^4 \\
& +27743628290h^5(x-x_n)^5 - 14672416610h^4(x-x_n)^6 - 4463635120h^3(x-x_n)^7 \\
& -794196650h^2(x-x_n)^8 + 5482910h(x-x_n)^9 - 3109568(x-x_n)^{10}\Big]f_{n+1}
\end{aligned}$$

$$+\frac{1}{399974400h^7}\Big[-2355609311h^9(x-x_n)+43809648630h^8(x-x_n)^2-59823556780h^6(x-x_n)^4$$
$$+64857656850h^5(x-x_n)^5-34104473030h^4(x-x_n)^6+10321466720h^3(x-x_n)^7$$
$$+1828259700h^2(x-x_n)^8+12572860h(x-x_n)^9-7107968(x-x_n)^{10}\Big]f_{n+2}$$
$$+\frac{1}{399974400h^7}\Big[-10969770700h^9(x-x_n)+20392494160h^8(x-x_n)^2-27749220780h^6(x-x_n)^4$$
$$+29987527570h^5(x-x_n)^5-157505754790h^4(x-x_n)^6+4732439520h^5(x-x_n)^5$$
$$+834535300h^2(x-x_n)^8+5715660h(x-x_n)^9-3219328(x-x_n)^{10}\Big]f_{n+3}$$
$$+\frac{1}{399974400h^7}\Big[-177230988h^9(x-x_n)+329512510h^8(x-x_n)^2$$
$$-449080800h^6(x-x_n)^4+486136784h^5(x-x_n)^5-255405332h^4(x-x_n)^6$$
$$+77358320h^3(x-x_n)^7+13746850h^2(x-x_n)^8+95110h(x-x_n)^9-54208(x-x_n)^{10}\Big]f_{n+4}$$
$$+\frac{1}{399974400h^7}\Big[-2399724h^9(x-x_n)+4458670h^8(x-x_n)^2-5990880h^6(x-x_n)^4+6383440h^5(x-x_n)^5$$
$$-3620852h^4(x-x_n)^6+941680h^3(x-x_n)^7-154930h^2(x-x_n)^8+950h(x-x_n)^9-448(x-x_n)^{10}\Big]f_{n+5} \quad (14)$$

Evaluating (14) at $x = x_{n+5}$ the following discrete method is obtained

$$y_{n+5}-\frac{29}{31}y_{n+4}-\frac{68}{31}y_{n+3}+\frac{68}{31}y_{n+2}+\frac{29}{31}y_{n+1}-y_n$$
$$=h^3\left(\frac{21}{2480}f_{n+5}+\frac{1177}{2480}f_{n+4}+\frac{1921}{1240}f_{n+3}+\frac{1921}{1240}f_{n+2}+\frac{1177}{2480}f_{n+1}+\frac{21}{2480}f_n\right) \quad (15)$$

3. Analysis and Implementation of the Method

3.1. Basic Properties of the Method

The method (15) is a specific member of the conventional LMM which can expressed as

$$\sum_{j=0}^{k}\alpha_j y_{n+j}=h^3\sum_{j=0}^{k}\beta_j y'''_{n+j} \quad (16)$$

Following [1] [2], we define the local truncation error associated with (16) by the difference operator

$$L\big[y(x);h\big]=\sum_{j=0}^{k}\Big[\alpha_j y(x_n+jh)-h^3\beta_j y'''(x_n+jh)\Big] \quad (17)$$

where $y(x)$ is assumed to have continuous derivatives of sufficiently high order. Therefore expanding (23) in Taylor series about the point x to obtain the expression

$$L\big[y(x);h\big]=C_0y(x)+C_1hy'(x)+C_2hy''(x)+\cdots+C_ph^py^p(x)+C_{p+1}h^{p+1}y^{p+1}(x)+C_{p+2}h^{P+2}y^{p+2}(x) \quad (18)$$

where the $C_0, C_1, C_2, \cdots, C_{p+2}$ are defined as

$$C_0=\sum_{j=0}^{k}\alpha_j,\quad C_1=\sum_{j=0}^{k}j\alpha_j,\quad C_2=\frac{1}{2!}\sum_{j=1}^{k}j^2\alpha_j,\quad C_q=\frac{1}{q!}\left[\sum_{j=1}^{k}j^q\alpha_j-q(q-1)(q-2)\sum_{j=1}^{k}\beta_j j^{q-3}\right]$$

In the sense of [1], we say that the method (20) is of order p and error constant $C_{p+2}\neq 0$ if

$$C_0=C_1=C_2=\cdots=C_{p+1}=0, C_{p+2}\neq 0$$

Using the concept above, the method (19) has order $p=9$ and error constant given by

$$C_{p+2} = -0.005403C_{p+2}$$

3.2. Zero-Stability of the 5-Step Method

Considering the first characteristics polynomial of the method of Equation (15) given as

$$\rho(r) = r^5 - \frac{29}{31}r^4 - \frac{68}{31}r^3 + \frac{68}{31}r^2 + \frac{29}{31}r - 1 = 0$$

Putting $r = 1, \rho(r) = 0$ implying that $r-1$ is a factor. Therefore solving the polynomial it is found that $(r-1)^3$ is also a factor of the polynomial and $r = 1,1,1$. The other roots which are called spurious roots are $r = -0.776216472$ and -1.28829956.

3.3. Region of Absolute Stability of the 5-Step Scheme

Applying the boundary locus method, we have that

$$\rho(r) = r^5 - \frac{29}{31}r^4 - \frac{68}{31}r^3 + \frac{68}{31}r^2 + \frac{29}{31}r - 1 = 0$$

$$\sigma(r) = \frac{h^3}{2480}\left(21r^5 + 1177r^4 + 3842r^3 + 3842r^2\sigma(r) + 1177r + 21\right)$$

In the spirit of Lambert (1973),

$$h(r) = \frac{\rho(r)}{\sigma(r)} = \frac{\rho(\mathrm{e}^{\mathrm{i}\theta})}{\sigma(\mathrm{e}^{\mathrm{i}\theta})}$$

By letting $r = \mathrm{e}^{\mathrm{i}\theta} = \cos\theta + \mathrm{i}\sin\theta$ and substituting this into the express above to yield

$$x(\theta) = \frac{2480\{\cos 5\theta - 29\cos 4\theta - 68\cos 3\theta + 68\cos 2\theta + 29\cos\theta + 1\}}{21\cos 5\theta + 1177\cos 4\theta + 3842\cos 3\theta + 3842\cos 2\theta + 1177\cos\theta}$$

At $\theta = 0^\circ$ and $\theta = 180^\circ$ for $0^\circ \le \theta \le 180^\circ$ at an interval of 30° we have that $X(\theta) = (0,\infty)$. The method is therefore said to be p-stable.

3.4. Implementattion

Single step method can be used to solve higher order ordinary differential equations directly without the need to first reducing it to an equivalent system of first order.

Consider the initial value problem in (7). For our method of order $p = 9$, Taylor series expansion is used to calculate.

$y_{n+1}, y_{n+2}, y_{n+3}$ and their first, second, third derivatives up to order $p = 9$.

$$\begin{aligned} y_{n+j} &\equiv y(x_n + jh) \\ &\cong y(x_n) + jhy'(x_n) + \frac{(jh)^2 y''(x_n)}{2!} + \frac{(jh)^3 f_n}{3!} + \frac{(jh)^4 f_n'}{4!} \\ &\quad + \frac{(jh)^5 f_n''}{5!} + \frac{(jh)^6 f_n'''}{6!} + \frac{(jh)^7 f_n^{iv}}{7!} + \frac{(jh)^8 f_n^{v}}{8!} + \frac{(jh)^9 f_n^{vii}}{9!} \\ j &= 1(1)4 \end{aligned}$$

$$\begin{aligned} y'_{n+j} \equiv y'(x_n + jh) \cong y'(x_n) + jhy''(x_n) + \frac{(jh)^2 f_n}{2!} + \frac{(jh)^3 f_n'}{3!} + \frac{(jh)^4 f_n''}{4!} + \frac{(jh)^5 f_n'''}{5!} + \frac{(jh)^6 f_n^{iv}}{6!} \\ + \frac{(jh)^7 f_n^{v}}{7!} + \frac{(jh)^8 f_n^{vi}}{8!} + \frac{(jh)^9 f_n^{vii}}{9!} + \cdots \end{aligned}$$

$$y''_{n+j} \equiv y''(x_n + jh)$$
$$\cong y''(x_n) + jhf_n + \frac{(jh)^2 f'_n}{2!} + \frac{(jh)^3 f''_n}{3!}$$
$$+ \frac{(jh)^4 f'''_n}{4!} + \frac{(jh)^5 f^{iv}_n}{5!} + \frac{(jh)^6 f^{v}_n}{6!} + \frac{(jh)^7 f^{vi}_n}{7!} + \frac{(jh)^8 f^{vii}_n}{8!} + \frac{(jh)^9 f^{viii}_n}{9!}$$
$$y'''_{n+j} \equiv y'''(x_n + jh)$$
$$\cong f_n + jhf'_n + \frac{(jh)^2 f''_n}{2!} + \frac{(jh)^3 f'''_n}{3!} + \frac{(jh)^4 f^{iv}_n}{4!}$$
$$+ \frac{(jh)^5 f^{v}_n}{5!} + \frac{(jh)^6 f^{vi}_n}{6!} + \frac{(jh)^7 f^{vii}_n}{7!} + \frac{(jh)^8 f^{viii}_n}{8!} + \frac{(jh)^9 f^{ix}_n}{9!} + \cdots$$

Then the known values of x_n and y_n are substituted into the differential equations. Next the differential equation is differentiated to obtain the expression for higher derivatives using partial differentiation as follows

$$y''' = f(x, y, y', y'') = f_j$$
$$y^{iv} = f_x + y'f_y + y''f_{y'} + ff_{y''} = \left(\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} + y''\frac{\partial}{\partial y'} + f\frac{\partial}{\partial y''}\right) = Df_j$$
$$y^{v} = f_{xx} + (y')^2 f_{yy} + (y'')^2 f_{y'y'} + f^2 f_{y''y''} + 2y'ff_{xy} + 2y''f_{xy'} + 2ff_{xy''}$$
$$+ 2y'y''f_{yy'} + 2y'ff_{yy''} + 2y''ff_{y'y''} + Df_j(f_{y'}) + f_j(y'' + f_{y'})$$
$$= D^2 f_j + (f_{y''})Df_j + f_j(y'' + f_{y'})_j = D^2 f_j + (f_{y''})Df_j + f_j(y'' + f_{y'})_j$$

where

$$D = \left(\frac{\partial}{\partial x} + y'\frac{\partial}{\partial y} + y''\frac{\partial}{\partial y'} + f\frac{\partial}{\partial y''}\right) \text{ and } D^2 = D(D)$$

⋮

$D^P f_j$ where p is the order of the method.

3.5. Numerical Experiments

Our methods of order $p = 9$ were used to solve some initial value problems of both general and special nature using Taylor's series. Our results were compared with the results of other researchers in this area as seen in **Table 1**. In **Table 2** and **Table 3**, the accuracy of our method is seen in the small error values.

The following initial value problems were used as our test problems:

3.6. Problem 1

$$y''' = -y, y(0) = 1, y'(0) = -1, y''(0) = 1.$$
$$o \le x \le 1, h = 0.1$$

Exact solution: $y(x) = \mathrm{e}^{-x}$.

3.7. Problem 2

$$.y''' - 2y'' - 3y' + 10y = 34x\mathrm{e}^{-2x} - 16\mathrm{e}^{-2x} - 10x^2 + 6x + 34.$$
$$y(0) = 3, y'(0) = 0, y''(0) = 0, 0 \le x \le 1$$

Exact solution: $y(x) = x^2 e^{-2x} - x^2 + 3$.

3.8. Problem 3

$$y''' = e^x, \quad y(0) = 3, y'(0) = 1, y''(0) = 5$$

Exact solution: $2 + 2x^2 + e^x$.

Table 1. Showing the result of test problem 1.

X	Exact solution	New result method (P = 9)	Error in our for (P = 9)	Error in [11] (P = 9)
0.1	0.904837	0.940837	0.0000+00	2.1760E−12
0.2	0.818731	0.818731	2.7756E−14	1.3935E−11
0.3	0.740818	0.740818	1.5838E−12	3.4443E−11
0.4	0.670320	0.670320	2.7879E−11	6.4477E−11
0.5	0.606531	0.606531	2.9477E−11	1.0316E−10
0.6	0.548812	0.548812	8.5048E−11	1.4979E−10
0.7	0.496585	0.496585	8.0357E−11	2.0486E−10
0.8	0.449329	0.449329	1.6601E−10	2.6756E−10
0.9	0.406570	0.406570	1.1176E−10	6.9382E−10
1.0	0.367879	0.367879	1.4871E−10	1.4224E−10

Table 2. Showing the result of test problem 2.

X	Exact solution	New result method (P = 9)	Error in our for (P = 9)
0.1	0.299819E+01	0.299819E+01	6.6218E−13
0.2	0.298681E+1	0.298681E+01	6.2238E−11
0.3	0.295939E+01	0.295939E+01	3.5134E−09
0.4	0.291189E+01	0.291189E+01	6.1100E−07
0.5	0.284197E+01	0.284197E+01	6.4183E−07
0.6	0.274843E+01	0.274843E+01	1.8082E−06
0.7	0.263083E+01	0.263083E+01	1.3511E−06
0.8	0.248921E+01	0.248921E+01	1.3367E−06
0.9	0.232389E+01	0.232390E+01	7.9041E−06
1.0	0.213534E+01	0.213537E+01	3.7360E−05

Table 3. Showing the result of test problem 3.

X	Exact solution	New result method (P = 9)	Error in our for (P = 9)
0.1	0.321517E+01	0.321517E+01	0.0000E+00
0.2	0.330140E+01	0.330140E+01	2.8422E−13
0.3	0.352980E+01	0.352980E+01	1.6729E−12
0.4	0.381182E+01	0.381182E+01	2.9983E−11
0.5	0.414872E+01	0.414872E+01	3.1673E−11
0.6	0.454212E+01	0.454212E+01	9.1899E−11
0.7	0.499375E+01	0.499375E+01	8.9531E−11
0.8	0.550554E+01	0.550554E+01	1.9168E−10
0.9	0.607960E+01	0.607960E+01	2.1110E−10
1.0	0.671828E+01	0.671828E+01	4.9398E−10
1.1	0.742417E+01	0.742417E+01	8.6728E−10
1.2	0.820012E+01	0.820012E+01	2.3764E−09

4. Discussion of Result

We have developed and implemented our methods using Taylor series of the same order as the schemes that we developed. Some special and general third order initial value problems (ivps) were used to test the efficiency of our methods. Our method was found to be zero stable, consistent and convergent. The better accuracy of our method can be shown from the numerical examples.

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