

# The Clouds Microstructure and the Rain Stimulation by Acoustic Waves

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## Abstract

A simplified model of clouds' microstructure dynamics under turbulent conditions is presented. The nature of rain stimulation by acoustic waves, based on the drops injection in the region of turbulent coagulation, is described. The conditions for effective rain stimulating are estimated.

**Keywords:** Cloud, Drop, Turbulence, Condensation, Coagulation, Rain Stimulation

## 1. Introduction

The investigations of clouds' microstructure represent a significant step for studying the possibilities and prospects to stimulate rain and ultimately to control weather patterns. The most commonly used methods have been by "seeding" the cloud by various agents that causes a large scale fracture within its structure or by direct injecting of large drops of water into the cloud. However, these methods require the use of artillery and rocket systems or flying devices which can be pretty costly. Scientists have been searching for cost-effective alternative methods.

To bring the changes in the clouds' microstructure by using the impact of acoustic waves were an interesting idea from the start. Theoretical calculations have estimated that the impact of intense acoustic waves upon a cloud would initiate the coagulation of the drops which would create a shift of the maximum in the distribution of the drops towards larger sizes. The experiments, conducted with both, overland fog and artificial fog in a chamber have confirmed these estimations [1,2]. What became apparent, that the impact of acoustic waves has to be extremely intense (~140 dB) and has to last for a long time (minutes). However, it would not be realistic to obtain the required parameters of the acoustic waves in the clouds while the source of sound waves is on the ground. But the clouds in nature, comparative to an artificial fog or an overland fog may, intrinsically be "operating" under turbulent conditions. And the turbulent conditions combined with acoustic waves could sufficiently enhance the effectiveness of stimulating the rain.

There are many theoretical and empirical models describing the clouds' microstructure and its dynamics due to the drops' coagulation and condensational growth [1, 2]. These models represent various approaches to the movement of the drops and their interaction. In the present paper a simplified model of the drops' movement, coagulation and condensational growth under turbulent conditions is represented. On the basis of the represented model, the possibilities of rain stimulating by the impact of acoustic waves under turbulent conditions are examined.

## 2. The Motion of Drops under Turbulent Conditions in the Air

After the nucleation of drops in a supersaturated air the main processes that have been involved in the formation of the clouds microstructure are:

- Condensational growth and evaporation of drops;
- Coagulation of the drops by colliding;
- Pulverizing of the drops.

The speed of the drop relative to the air particles plays the main role in these dynamics. Due to the air viscosity the drop gets involved by the turbulent streams of the air. But the density of the drops is much greater than the air density and due to inertia of drops their trajectory deviate from the air particles trajectory. It means that drops move relative to the surrounding air. The relative velocity of a spherical drop in the viscous air is determined by the following equation:

$$\frac{\partial v_d}{\partial t} + \frac{v_d}{\tau_d} = -\frac{\partial v_a}{\partial t} \quad (1)$$

Here  $v_d$  is the drop velocity relative to the surrounding air particles,

$$\tau_d = \frac{2\rho_d}{9\mu\rho_a} r_d^2 \quad (2)$$

is the time of Stokes relaxation of the drop's velocity in the viscous air,  $v_a$  is the local velocity of the air particles at the position of the drop,  $\rho_d$  and  $\rho_a$  are the densities of the water and the air correspondingly,  $\mu$  is the kinematic viscosity of the air,  $r_d$  is the radius of the drop. The air resistance, proportional to the drop's acceleration is neglected. The mass of the air in a hydrodynamic boundary layer is neglected as well.

The pattern of the random turbulent flows is a superposition of numerous vortices of various spatial scales and the variance of the air particle velocity

$$\bar{v}_a^2 = \int_0^\infty E(k) dk$$

is determined by the spatial spectral density of the random field of rates  $E(k)$ , where  $k = 2\pi/l$  is the spatial wave number of the vortex,  $l$  is the vortex size. For atmospheric turbulence we have

$$k = 2\pi/lE(k) = 2\pi C_v k^{-5/3}, \quad k = [k_0, k_m],$$

where  $k_0$  and  $k_m$  are the spatial wave numbers which correspond to the outer  $L_0$  and the inner  $l_m$  scales of turbulence correspondingly.

The air particles being involved with vortices of various sizes have curvilinear trajectories of movement. So the acceleration of the air particles in some direction may be represented as a superposition of accelerations of various frequencies  $\omega_r(k)$  which depend on the vortex size

$$\frac{\partial v_a}{\partial t} = \int_0^\infty A(k) \sin[\omega_r(k)t] dk$$

Let's assume a model of dynamic turbulence. According to the model kinetic energy of the air particles is stationary and the particles have only tangential acceleration caused by the curvilinearity of their trajectories. In this case the magnitude of tangential acceleration equals to the air velocity in the vortex raised to the second power and divided by the size of the vortex, while the frequency equals to the air velocity in the vortex divided by the size of the vortex.

$$A(k) = C_v k^{-2/3}, \quad \omega_r(k) = \sqrt{6\pi C_v} k^{2/3}.$$

In this case the quasistationary solution of Equation (1) has the form

$$v_d = \int_{k_0}^{k_m} \frac{C_v k^{-2/3}}{\tau_d^{-2} + \omega_r^2(k)} \cdot \left[ \tau_d^{-1} \sin(\omega_r(k)t) - \omega(k) \cos(\omega_r(k)t) \right] dk \quad (3)$$

Let's analyze the obtained solution. For small drops the time of Stokes relaxation (2) is much less than the characteristic time of small-scale turbulent pulsations  $\tau_d \omega_r(k) \ll 1$  and the amplitude of relative velocity of the drop has the square-law dependence on the radius of the drop

$$v_d \approx 3C_v k_m^{1/3} \tau_d = \frac{2\rho_d C_v k_m^{1/3}}{3\mu\rho_a} r_d^2 \quad (4)$$

For larger drop if the time of Stokes relaxation is in the range of the turbulent pulsation's  $\omega_r(k_0) \ll \tau_d^{-1} \ll \omega_r(k_m)$  the dependence of the relative velocity on the radius of the drop became linear

$$v_d \approx (2\pi)^{1/4} (3C_v)^{3/4} \sqrt{\tau_d} = (2\pi)^{1/4} (3C_v)^{3/4} \sqrt{\frac{2\rho_d}{9\mu\rho_a}} r_d. \quad (5)$$

Very large drop with  $\tau_d \omega_r(k_0) \gg 1$  is too inertial to move with the air particles and its relative velocity corresponds to the velocity of the turbulent flows

$$v_d \approx \sqrt{6\pi C_v} k_0^{-1/3}.$$

### 3. Condensational Growth of the Drops

The clouds arise in a supersaturated air consequently to the heterogeneous nucleation of drops. The boundary between the water and the air is subjected to the molecular unevenness, so the kinetic coefficient of the condensation  $q$ , determining the dependency of the growth rate  $u$  on the supercooling would be quite high and we can disregard the kinetic supercooling and; accordingly a kinetic supersaturation. It means that the supersaturation at the growing surface corresponds to the equilibrium conditions.

The process of condensation gets always accompanied by the latent heat release, and the value of the drop's overheating, relatively to the air is at exactly the correct level, which is needed to transfer the latent heat of condensation from the growing surface of the drop to the cloud and can be obtained by the condition of the heat transfer

$$u(r_d) \rho_d H = c_a \rho_a K_a \frac{\Delta T_H}{\delta_T}. \quad (6)$$

Here  $u(r_d)$  is the growth rate of the drop,  $H$  is the specific heat of evaporation,  $c_a$  is the specific heat capacity of the air,  $K_a$  is the thermal diffusivity of the air,  $\delta_T$  is the thickness of the thermal boundary layer at the surface of the moving drop. So, the value of the steam concentration at the growing surface  $C_s$  differs from the concentration of saturation in the cloud  $C_0$  due to

the curvature of the drop's surface and the overheating of the drop

$$C_s = C_0 + C_0 \frac{2\sigma V_m}{r_d RT} + \Delta T_H \frac{dC_0}{dT}. \quad (7)$$

Here  $\sigma$  is the surface energy of the water-air boundary,  $V_m$  is the molar volume of the water,  $R$  is the gas constant,  $T$  is the absolute temperature.

The steam transfer from the cloud to the growing surface of the drop takes place due to the difference between the steam concentration in the cloud  $C_{eq}$  and at the drop's growing surface  $C_s$

$$\begin{aligned} u(r_d) \rho_d \\ = D_a \frac{C_{eq} - C_s}{\delta_D}. \end{aligned} \quad (8)$$

Here  $D_a$  is the steam molecular diffusion coefficient in the air,  $\delta_D$  is the thickness of the diffusive boundary layer at the surface of the moving drop.

The thickness of the boundary layers depends on the relative velocity of the drop. The estimations have shown that the airflow of the drops in the cloud has a laminar character and the following expression can be used

$$\delta_T = \sqrt{\frac{r_d K_a}{v_d}}, \quad \delta_D = \sqrt{\frac{r_d D_a}{v_d}}. \quad (9)$$

Substituting (7) and (8) in (6) and taking into account (9) the expression for the drop's growth rate can be obtained

$$\begin{aligned} u(r_d) = \frac{2C_0 \sigma V_m}{RT} \left( \frac{1}{r_{eq}} - \frac{1}{r_d} \right) \\ \left( \frac{\rho_d}{\sqrt{D_a}} + \frac{\rho_d H}{c_a \rho_a \sqrt{K_a}} \frac{dC_0}{dT} \right)^{-1} \frac{\sqrt{v_d}}{\sqrt{r_d}}. \end{aligned} \quad (10)$$

Here  $r_{eq}$  is the radius of the drop which is under the equilibrium conditions, *i.e.* do not grow and do not evaporate. It is obvious, that the condensational growth rate depends on the drop's velocity, relative to the air and the atmospheric turbulence considerably intensifies the condensational growth. However the defining factor remains in the difference of the values between the drop's radius and the critical radius. The value of the critical radius is determined by the value of the supersaturation

$$\Delta C_{eq} = C_{eq} - C_0 = C_0 \frac{2\sigma V_m}{r_{eq} RT}.$$

Although a size of an individual drop is minute, a combined surface adds up to a rather large amount. For example, in a cubic meter of the cloud with 2 g of water capacity and the average drop size about 20  $\mu\text{m}$ , the total

surface of the drops is about 0.3  $\text{m}^2$ . In the course of a few seconds, all the steam that makes the supersaturation condenses on the growing surface of the drops. The supersaturation in the cloud decreases to some quasi equilibrium level which is much less than the water volume content of the cloud  $M$ . It means that at the stationary conditions the growth of the larger drops occurs due to the evaporation of smaller drops and the water content of the cloud practically does not change.

$$\int_0^\infty 4\pi u(r_d) W(r_d) r_d^2 dr_d = 0 \quad (11)$$

and

$$\frac{\partial W(r_d)}{\partial t} = u(r_d) \frac{\partial W(r_d)}{\partial r_d}. \quad (12)$$

Here  $W(r_d)$  is the density of the size distribution of the drops,

$$\int_0^\infty W(r_d) dr_d = M.$$

The Equations (11) and (12) describe the dynamics of the size distribution of the drops at the stationary conditions. Estimations give that the condensational growth rate under the stationary conditions is very low and it is doubtful that condensational growth can provide a favorable conditions for the rain to occur. The rate of condensational growth under the stationary conditions does not rank over 1  $\mu\text{m}/\text{h}$  and it would take too much time for the rain drops to form. At the same time the condensational growth under the nonstationary conditions is very important and efficiently changes the drops' sizes.

#### 4. The Drops' Coalescence

The relative motion of the drops can lead to their collisions and coalescence. The disappearance of two colliding drops and the appearance of the drop with the common volume lead to the change in the size distribution of the drops. The Smoluchowski equation describing the dynamics of the size distribution due to drops collisions can be represented at the following form

$$\begin{aligned} \frac{\partial W(r_1)}{\partial t} = & -W(r_1) \int_0^\eta dr_2 P(r_1, r_2) W(r_2) \\ & -W(r_1) \int_\eta^\infty dr_2 P(r_2, r_1) W(r_2) \\ & + \int_0^\eta dr_2 P\left(r_2, \sqrt[3]{r_1^3 - r_2^3}\right) W(r_2) W\left(\sqrt[3]{r_1^3 - r_2^3}\right) \end{aligned} \quad (10)$$

Here the first term corresponds to the disappearance of the drops of radius  $r_1$  due to their collisions with the smaller drops; the second term corresponds to the disappearance of the drops of radius  $r_1$  due to their collisions with the larger drops, and the third term corresponds to

the appearance of the drops of radius  $r_1$  due to collisions of the smaller drops.

The function  $P(r_1, r_2)$  corresponds to the space volume that in time unit becomes free of the drops of radius  $r_2$  due to their capture by a larger drop of radius  $r_1$ . In general the determination of  $P(r_1, r_2)$  is a very knotty problem because it is necessary to take into accounting the interaction between colliding drops. The case of the drops movement caused by the atmospheric turbulence is simpler because the radius of spatial correlation of turbulent random field of velocities is considerably greater than a distance between drops, participated in coalescence. It means that the drops that have a possibility to collide move at the same direction and the larger drop having greater velocity runs down the smaller one. In that simplified model a function  $P(r_1, r_2)$  can be represented as

$$P(r_1, r_2) = (v_1 - v_2) \pi R_{\max}^2, \quad (14)$$

Here  $v_1$  and  $v_2$  are the velocities of the larger and the smaller drops correspondingly,  $R_{\max}$  is the maximal transversal distance between trajectories of drops which could have a collision. If a hydrodynamic interaction between the drops is neglected then  $R_{\max} = r_1 + r_2$ . However the moving drop in a real viscous air is surrounded by a laminar hydrodynamic boundary layer with thickness

$$\delta_1 = \sqrt{\frac{\mu r_1}{v_1}}.$$

The opposite air particles and drops have to flow round the moving drop. The tangential shift of the inertialess drop equals

$$S_a = r_1 + r_2 + \delta_1 + \delta_2,$$

where  $\delta_2$  is the hydrodynamic boundary layer thickness of the opposite drop. Taking into account inertia of the opposite drop and assuming for the simplicity the circular motion of air round the larger drop we have the less value of its tangential shift

$$S_d = \frac{(r_1 + r_2 + \delta_1 + \delta_2) v_1}{(v_1 - v_2)(1 + r^2)} \left[ 1 + \gamma \exp\left(-\frac{\pi}{2r}\right) \right],$$

where

$$\gamma = \frac{\pi \tau_2 (v_1 - v_2)}{2(r_1 + r_2 + \delta_1 + \delta_2)},$$

$\tau_2$  is the time of Stokes relaxation of the opposite drop. Then the maximal transverse distance between trajectories of drops which could have a collision equals:

$$\begin{aligned} R_{\max} &= r_1 + r_2 - S_d \\ &= r_1 + r_2 - \frac{(r_1 + r_2 + \delta_1 + \delta_2) v_1}{(v_1 - v_2)(1 + r^2)} \left[ 1 + \gamma \exp\left(-\frac{\pi}{2r}\right) \right] \end{aligned} \quad (15)$$

Thus the possibility of collision depends on the drops' sizes and velocities. The velocities of the drops depend on the intensity of turbulent mixing and for any intensity of turbulence; there exists a minimal radius for the drop that allows it to absorb smaller drops. For example, for the following parameters of turbulence  $L_0 = 200$  m,  $L_m = 10$  mm and the root mean square velocity of the turbulent flows  $\bar{v}_a = 3$  m/s, the low borderline of the diapason for the turbulent coalescence  $r_c$  equals  $43 \mu\text{m}$ , while at  $\bar{v}_a = 5$  m/s we have  $r_c = 32 \mu\text{m}$ . and at  $\bar{v}_a = 10$  m/s -  $r_c = 20 \mu\text{m}$ . If the drops in the cloud are distributed in the range of diapason of the radiuses, smaller than  $r_c$ , the coalescence does not take place, the cloud remain stable and the drops have continue to grow slowly through the condensational mechanism. The rate of growth in such a cloud could have pick up for example, with the changed temperature or increased intensity of turbulence.

The rain condition can be caused also by injecting the cloud with drops of the radiuses substantially larger than  $r_c$ . The required number of injected drops that can intake all the water content of the cloud is about 200 in a cubic meter. If the radius of the injected drop is significantly greater than the average radius of water drops in the cloud its exponential growth would have occurred.

## 5. The Rain Stimulation by Acoustic Waves

A propagating sound wave evokes a wavy motion of air particles.

$$v_a = \frac{P_s}{\rho_a c_s} \sin\left(\frac{2\pi t}{\tau_s}\right). \quad (16)$$

Here  $P_s$  is the sound wave's pressure,  $\tau_s$  is the wave period of sound,  $c_s$  is the sound speed. Due to air viscosity the drops have a wavy motion as well. The amplitude of relative velocity of the drop is

$$v_d = \frac{P_s}{\rho_a c_s} \frac{\tau_d}{\sqrt{\tau_d^2 + \tau_s^2}} \quad (17)$$

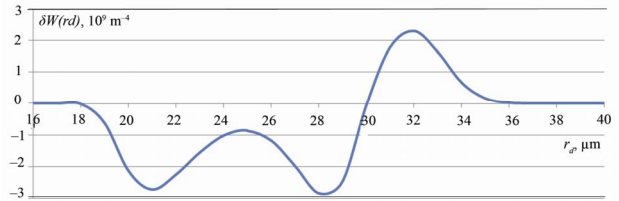
Awakened by sound wave, the movement of the drops intensifies the heat and the mass transfer and according to (11) increases the condensational growth rate. However, the effect of sound wave could be even more indispensable for the coagulation of the drops. The mechanism that is involved in the coalescence under the impact of acoustic wave is actually similar to the mechanism that is involved in the coalescence under the turbulent conditions. For example, the effectiveness of the coales-

cence under turbulence with parameters  $C_v = 0.265 \text{ m}^{4/3} \cdot \text{s}^{-2}$  and  $L_m = 10 \text{ mm}$  can be matched up by the sound wave with  $P_s = 180 \text{ Pa}$  and frequency  $17 \text{ Hz}$ . The coalescence of the drops under the effect of acoustic wave can be described by the same Equations (13-15), as it were under the turbulent conditions, however, the value of the drops' velocities have to correspond to (17).

It is evident, the best result in the coalescence can be achieved if the period of the sound equals to the time of Stokes velocity relaxation of the larger of the two coalescing drops. That would secure the maximum relative velocity for the larger drop, provide the minimum thickness for its boundary hydrodynamic layer and ensure the maximum contrast of velocities between the larger and the smaller drops. Note, that the turbulence frequency of  $17 \text{ Hz}$  is the optimum for the drops with the radius  $67 \mu\text{m}$ . For the smaller drops the effectiveness of turbulent coalescence decreases (proportional to  $r^2$ ) and acoustic waves with the correspondent frequency would be more effective.

As it were in the case of the coalescence under the effect of turbulence, for any intensity of the sound wave, there exists a minimum radius for the drop that is able to consume a smaller drop. Or, in other words, for a drop of any size, there exists a threshold point of intensity for the sound wave which would have just ensured a consumption of smaller drops. For example, for the drops with the radius  $30$ , the threshold point of intensity for the sound wave has to be  $50 \text{ Pa}$  with the frequency  $120 \text{ Hz}$ , for the drops with the radius  $25 \mu\text{m}$  we have  $60 \text{ Pa}$  with the frequency  $175 \text{ Hz}$  and for the drops with the radius  $20 \mu\text{m}$  we have  $80 \text{ Pa}$  with the frequency  $265 \text{ Hz}$ .

The impact of sound wave is especially effective for the sizes of the drops, which are unable to coalesce under turbulent conditions, in other words for the drops, which are in the range for the stable cloud. In **Figure 1** the change of the density of the size distribution of the drops  $\delta W(r_d)$  ( $10^9 \text{ m}^{-4}$ ) under the impact of acoustic wave is represented. In the represented case the water capacity is  $2 \text{ g/m}^3$ , the radiuses of the drops were initially distributed in the range from  $20 \mu\text{m}$  to  $30 \mu\text{m}$ . Acoustic wave has the intensity  $P_s = 100 \text{ Pa}$ , frequency is  $130 \text{ Hz}$ , the impact duration is  $1 \text{ s}$ . It can be seen the disappearance of the drops in the ranges from  $27$  to  $29 \mu\text{m}$  and from  $24$  to  $25 \mu\text{m}$ . Their coalescence leads to formation of the drops with the radius around  $32 \mu\text{m}$ . A total number of appeared single coalesced drops is about  $6800$  in a cubic meter. If the size of appeared drops is in the range of an effective coalescence under turbulence they could represent the base for the follow up rain. Note that at  $P_s =$



**Figure 1.** The change of the density of the size distribution of the drops  $\delta W(r_d)$  under the impact of acoustic wave.

$120 \text{ Pa}$  the number of the newly formed in a cubic meter drops came to  $10000$  and at  $P_s = 140 \text{ Pa}$  came to  $13500$ .

Thus the stimulating effect of acoustic wave results in producing not actually the rain drops, but the drops which are good and ready to coalesce under the turbulent conditions. The impact of acoustic waves becomes especially effective with a near-rain cloud. It means that the drops are distributed in a close proximity to the borderline of the diapason for a successful turbulent coalescence and a single coalescence would be sufficient.

The main concern in the realization of the acoustic wave rain stimulation is the actual delivery of the right intensity sound wave to the cloud. For example, the intensity of  $100 \text{ Pa}$  with the radius of the acoustic spot of  $200 \text{ m}$  corresponds to the acoustic power of  $3 \text{ MW}$ . That power could not very likely be achieved by the means of electro-mechanical translators. However, a certain perspectives have become visible with a connection to the sound wave's generation by means of the fuel gas explosion. For example, the energy of a single impulse of indicated intensity and the duration correspondent to necessary frequency is about  $30 \text{ kJ}$ , which corresponds to mechanical energy, released from the explosion of  $2 \text{ g}$  of propane.

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